Please let me know if any of the problems are unclear or have typos.

Exercise 1.1. Prove $\mathbb{S}^n$, $\mathbb{P}^n$ and $\mathbb{T}^n$ are manifolds.

Exercise 1.2. Prove no pair of $\mathbb{S}^n$, $\mathbb{P}^n$, $\mathbb{T}^n$ are homeomorphic, when $n \geq 2$. What happens in dimensions zero and one?

Exercise 1.3. Recall S_g is the closed orientable surface with g handles and N_k is the closed non-orientable surface with k cross caps. Prove S_g double covers N_{g+1}.

Exercise 1.4. Suppose $\pi\colon M^n \to N^n$ is a covering map of compact manifolds, of degree d. Prove $\chi(M) = d \cdot \chi(N)$. (You may restrict to the case $n = 2$.)

Exercise 1.5. Show $\chi(S_g) = 2 - 2g$ and $\chi(N_k) = 2 - k$.

Exercise 1.6. [Reading] Look up the Gauss-Bonnet Theorem, understand it, and reproduce the statement. Suppose $F = X/\Gamma$ is a surface modelled on the geometry X. Prove $\chi(F)$ determines X. (Typically $\chi(F)$ will not determine Γ.)

Exercise 1.7. [Medium] Suppose M^3 is a closed three-manifold. Prove $\chi(M) = 0$. Deduce generally, when M is compact, $\chi(M) = \frac{1}{2}\chi(\partial M)$.

Exercise 1.8. [Hard] Prove the circle $\mathbb{S}^1$ and the interval I are the only compact connected 1–manifolds, up to homeomorphism. For a detailed outline of the argument, see David Gale's article "The classification of 1-manifolds: a take-home exam", in the American Mathematical Monthly.