Please work in groups of two or three. Please explain all answers carefully.

Be sure to acknowledge any and all help you receive with your write-up. As usual, you can *think* with other people but you should not *write* with anybody. Please give your answer in complete English sentences. Helpful figures are always welcome.

Problem 6.1 (Warm-up). The constraint $x^2 + 2y^2 \leq 1$ defines a closed and bounded set in $\mathbb{R}^2$. Thus, the function $f(x, y) = xy$ attains a maximum value on that set. What is this maximum value? Be sure to analyze carefully and completely any system of equations you solve.

Problem 6.2 (Warm-up). Suppose that a and b are fixed positive real numbers. The constraint $ax^2 + by^2 \leq 1$ defines a closed and bounded set in $\mathbb{R}^2$. Thus, the function $f(x, y) = xy$ attains a maximum value on that set. What is this maximum value? Explain.

Problem 6.3. Let G be the graph of the function $z = xy$, in $\mathbb{R}^3$. Sketch G. For every real number a, let $P_a = (0, 0, a)$. Now find the minimal distance between P_a and G. (Hint: it is a bit easier to minimize the square of the distance.) Discuss the geometric meaning of all of the non-minimal solutions found by the Lagrange multiplier method.

Problem 6.4. Let G be the graph of the function $z = x^2 + 2y^2$. Sketch G in $\mathbb{R}^3$. Let $P_a = (0, 0, a)$ and find the minimal distance between P_a and G. Explain geometrically the various other solutions given by the Lagrange method. In particular, why do different values of a lead to different numbers of critical points?

Problem 6.5. Suppose that, in the problem above, G is the graph of $z = x^2 + y^2$. Draw a picture. What do you expect to happen in this case? How many critical points will there be when a is large and positive? When a is small and positive?

Now solve the problem using Lagrange. At what value of a does the behavior change? Why? Explain geometrically.