Please work in groups of two or three.

Problem 3.1 (Warm-up). Let H be the set of points (x, y, z) in $\mathbb{R}^3$ satisfying the equation $x^2 + y^2 = 1 + z^2$. Sketch at least five representative z–*traces* (*ie* cross sections of H by planes of the form $z = c$). Now give a sketch of H in $\mathbb{R}^3$. What is this surface called?

Problem 3.2 (Warm-up). Let P be the set of points (x, y, z) in $\mathbb{R}^3$ satisfying the equation $xy = z$. Sketch at least five representative z–traces. Now give a sketch of P in $\mathbb{R}^3$. What is this surface called?

Problem 3.3. Let $L(t) = (t, 0, -1)$ and $M(s) = (0, s, 1)$ be the parametrizations of a pair of lines in $\mathbb{R}^3$. Let S be the set of points *equidistant* from L and from M: that is, every point $p \in S$ has the *same* distance to both L and to M. Give a sketch of S. Describe S implicitly, as the set of solutions to some equation. What are the z–traces of S? What is this surface called?

Problem 3.4 (Repeat). Suppose we are given vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $\mathbb{R}^3$. Define

$$P(\mathbf{u}, \mathbf{v}, \mathbf{w}) := \{r\mathbf{u} + s\mathbf{v} + t\mathbf{w} \mid 0 \leq r \leq 1,\ 0 \leq s \leq 1,\ 0 \leq t \leq 1\}.$$

This is a *parallelepiped*. Draw a picture of $P(\mathbf{i}, \mathbf{j}, \mathbf{k})$. Draw a picture of $P(\mathbf{i}, \mathbf{i}+\mathbf{j}, \mathbf{i}+\mathbf{k})$. Compute the volumes.

Now, what does a "general" parallelepiped look like? How would you find its volume?

Problem 3.5. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be vectors in $\mathbb{R}^3$, based at the origin. The *tetrahedron* spanned by these vectors is the set

$$T(\mathbf{u}, \mathbf{v}, \mathbf{w}) = \{a\mathbf{u} + b\mathbf{v} + c\mathbf{w} \mid a, b, c \geq 0,\ a + b + c \leq 1\}.$$

Give a careful, labelled sketch of T. Compute the volume of T. Compute the surface area of T: the sum of the areas of the four faces. (It may help to warm up with an easy case, like $T(\mathbf{i}, \mathbf{j}, \mathbf{k})$.)

Problem 3.6 (Challenging). Show that for any ellipsoid $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ there is a plane P so that the cross section of E by P is a round circle. (Hint: it is enough to consider planes going through the origin.)

More generally, which quadric surfaces have a round cross section? Which do not? (Eg, does an elliptic paraboloid have a round cross section? Play around with a graphing program like Maple!)