Please work in groups of two or three. These exercises are not to be turned in.

Problem 1.1. Sketch the following sets. Compute their dimensions.

1. $\{x \in \mathbb{R} \mid x = 1\}$.
2. $\{x \in \mathbb{R} \mid x^2 = 1\}$.
3. $\{x \in \mathbb{R} \mid x^2 \leq 4\}$.
4. $\{x \in \mathbb{R} \mid x^2 \geq 1 \text{ and } x^2 \leq 4\}$.
5. $\{x \in \mathbb{R} \mid x^2 \leq 1 \text{ and } (x-1)^2 \leq 1\}$.
6. $\{x \in \mathbb{R} \mid (x - 1/2)^2 \leq 1/4\}$.

Problem 1.2. Sketch the following sets. Compute their dimensions.

1. $\{(x, y) \in \mathbb{R}^2 \mid x + y = 1\}$.
2. $\{(x, y) \in \mathbb{R}^2 \mid x + y = 1 \text{ and } x - y = 1\}$.
3. $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$.
4. $\{(x, y) \in \mathbb{R}^2 \mid x = y^2\}$.
5. $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$.
6. $\{(x, y) \in \mathbb{R}^2 \mid y^2 > x\}$.
7. $\{(x, y) \in \mathbb{R}^2 \mid y^2 > x \text{ and } x^2 > y\}$.
8. $\{(x, y) \in \mathbb{R}^2 \mid x + y \leq 1\}$.
9. $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \text{ and } x + y = 1\}$.
10. $\{(x, y) \in \mathbb{R}^2 \mid y = x^2 \text{ and } x = -1\}$.
11. $\{(x, y) \in \mathbb{R}^2 \mid y = x^2 \text{ and } y = a\}$, where a is a fixed, unknown constant. Describe the various possibilities qualitatively.
12. $\{(x, y) \in \mathbb{R}^2 \mid y = 6x^3 - 6x \text{ and } x = 2y^2 - 2\}$. (A calculator may be useful.)

Problem 1.3. Sketch the following sets. Compute their dimensions.

1. $\{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 1\}$.
2. $\{(x, y, z) \in \mathbb{R}^3 \mid x + y = 1\}$.

3. $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$.
4. $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$.
5. $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1\}$.
6. $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1 \text{ and } z \geq 0\}$.
7. $\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1 \text{ and } x + y + z = 1\}$.
8. $\{(x, y, z) \in \mathbb{R}^3 \mid x + y - z = 1 \text{ and } x - y + z = 1 \text{ and } -x + y + z = 1\}$.

Problem 1.4. Is there a relationship between the dimension of a set and the number of equalities required to specify the set? (Ignore all of the examples involving inequalities.)