Math 311: Section 3.

Workshop 11: Random integrals. (Or: "Why the Mean Value Theorem remains, to this day, your friend.")

The following are workshop-level problems on integrals. You will not be turning any of them in; however, you may want to solve one or two of them to familiarize yourself with the material.

Problem 11.1 (Natural Logarithm - Problem 7.5.4 (mostly) from Abbott)**.** Let

$$L(x) = \int_1^x \frac{dt}{t},$$

where we consider only $x > 0$.

- What is $L(1)$? Find $L'(x)$.
- Show that $L(x)$ is strictly increasing; that is, show that if $0 < x < y$ then $L(x) < L(y)$.
- Show that $L(1/x) = -L(x)$. (What are the derivatives of the left and right hand sides?)
- Show that $L(cx) = L(c) + L(x)$. (Think of c as a constant and differentiate $g(x) = L(cx)$.)
- Can you find a power series expansion for $L(1+x)$? (Note that I am *not* asking for a power series expansion of $L(x)$.) What is the radius of convergence?
- (Harder) Does the power series you obtain converge to $L(x)$?

Problem 11.2. Consider the integral $\int_0^1 x^2 dx$.

- What is the lower sum for the partition $P_{1/2} = \{0, 1/2, 1\}$?
- Suppose $P_t = \{0, t, 1\}$. Compute the lower sum $L(x^2, P_t)$. What value of t maximizes $L(x^2, P_t)$?
- (Harder.) Suppose $P = P_{s,t} = \{0, s, t, 1\}$. Compute $L(x^2, P)$. What values of s and t gives the maximum?

Problem 11.3. Compute $\int_0^{1/10} \frac{dx}{1-x^3}$ to nine decimal places. By hand. (Hint: the geometric series $\sum t^n$ converges uniformly to $\frac{1}{1-t}$ in $[0, 1/10]$. Also, $1/7 = 0.142857...$)