

# SNAPPY TUTORIAL

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**Problem 0.1.** Install SnapPy onto your laptop. There are instructions for how to download and install SnapPy for various systems here:

<https://www.math.uic.edu/t3m/SnapPy/installing.html>

SnapPy is also included as a part of Sage. Use `import snappy` to import snappy. If you run SnapPy from a Sage notebook, then the graphical tools will also work. You can also run SnapPy from inside of the Sage notebook available at CoCalc. Here, however, the graphical tools are not available.

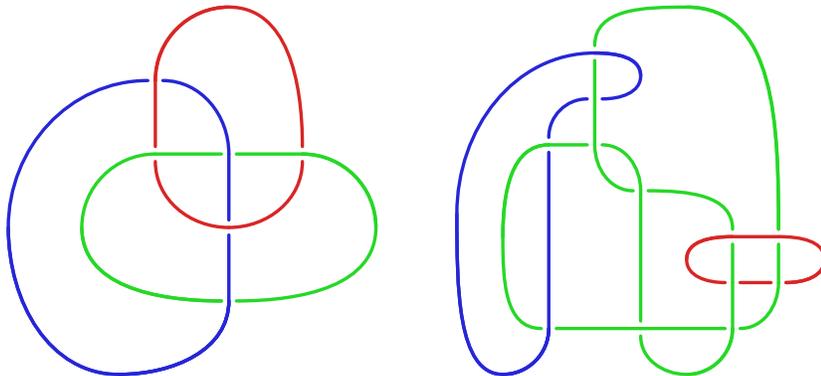


FIGURE 0.2. On the left we have the Borromean rings,  $B$ , a three-component link. On the right we have  $AF$ , an *augmentation* of the figure-eight knot.

**Problem 0.3.** Give better drawings of the links  $B$  and  $AF$  shown in Figure 0.2. Prove that the link complements  $S^3 - B$  and  $S^3 - AF$  are homeomorphic. You can ask SnapPy for the drawing window by typing `B = Manifold()` (or `B = snappy.Manifold()` in Sage). Here are some methods of the SnapPy Manifold class which may be useful: `volume`, `identify`, `is_isometric_to`. Recall that you can ask for documentation by typing `?`  after a method.

Challenge: Give a rigorous, computer-aided proof. The command `isometry_signature`, run inside of Sage, will be useful.

Challenge: Give a computer-free proof.

**Problem 0.4.** Prove that there is no isotopy of  $S^3$  taking  $AF$  to  $B$ . That is, there is no continuous motion of  $S^3$  that takes  $AF$  onto  $B$ .

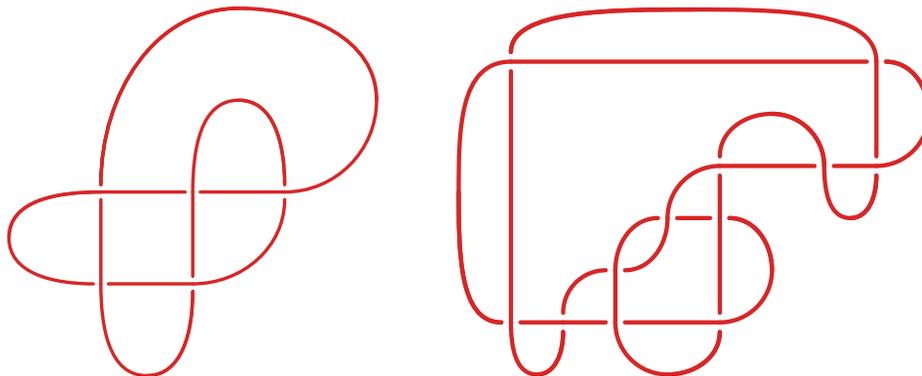


FIGURE 0.5. On the left we have  $T_3$ , the third *twist knot*. The trefoil and the figure-eight are the first and second twist knots, respectively. On the right we have a somewhat strange drawing of  $P = P(-2, 3, 7)$ , a *pretzel knot*.

**Problem 0.6.** In the diagrams for  $T_3$  and  $P$  in Figure 0.5, shade all *bigons*: components of the the complement of the diagram in the plane which are bounded by exactly two arcs. Use this to justify the naming schemes in the figure and also to draw better figures.

**Problem 0.7.** Give a rigorous, computer-aided proof that that  $S^3 - T_3$  is not homeomorphic to  $S^3 - P$ .

**Problem 0.8.** [Hard.] Give a rigorous, computer-aided proof that  $S^3 - T_3$  and  $S^3 - P$  have identical volumes. The command `tetrahedron_shapes` will be useful as will `gluing_equations`.

**Problem 0.9.** Find all manifolds in the `OrientableCuspedCensus` with volume in the range  $[2.0, 2.2]$ . The method `name` may be useful.

**Problem 0.10.** Let  $T_k$  be the  $k^{\text{th}}$  twist knot. Compute the volume of  $X_k = S^3 - T_k$  for values of  $k$  as large as your computer can handle. The command `RationalTangle` may be useful. Graph the volume of  $X_k$  as a function of the number of twists and guess the asymptotic behaviour (and the rate of convergence).

**Problem 0.12.** Compute the volume of the Whitehead link,  $W$ , shown in Figure 0.11. Give a plausability argument that the volumes of the  $X_k = S^3 - T_k$  converge to the volume of  $W$  from below. Understand the tetrahedra shapes may be useful; do this via the `browse` method, and look at the “cusp neighborhood”.

**Problem 0.13.** Find as many non-hyperbolic Dehn fillings of the figure-eight knot complement as you can. Identify these non-hyperbolic manifolds. The method `dehn_fill` will be useful.

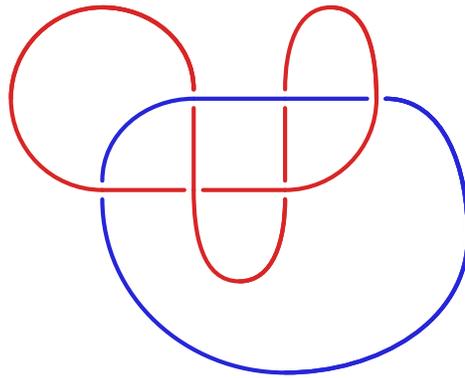


FIGURE 0.11. The Whitehead link  $W$