Generalized Tits Conjecture for Artin groups

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Goal

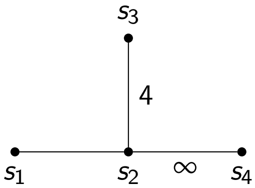
Artin groups: in general not well-understood

Certain naturally defined subgroups: well-behaved, as free as possible

Artin groups

$$A = \langle s_1, \dots, s_k \mid \underbrace{s_i s_j s_i \dots}_{m_{ij}} = \underbrace{s_j s_i s_j \dots}_{m_{ij}} \rangle$$

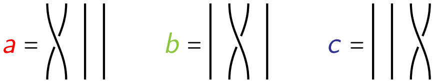
where $m_{ij} = m_{ji} \in \{2, 3, \dots, \infty\}$

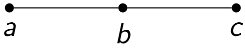


- RAAGs: all $m_{ij} = 2$ or ∞
- Spherical Artin groups: the Coxeter group $W = A/\langle\langle s_i^2 \rangle\rangle$ is finite

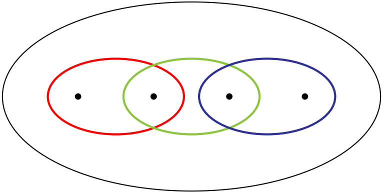
Example: Braid groups

$Br_4 = \langle a, b, c \mid aba = bab, bcb = cbc, ac = ca \rangle$





$Br_4 = Mod(D^2, \{4 \text{ points}\})$



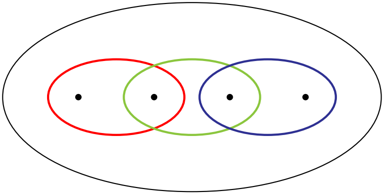
Tits conjecture

Theorem (Crisp-Paris, 2001)

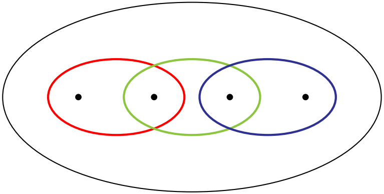
For every $n \geq 2$, the subgroup $\langle a_1^n, \ldots, a_k^n \rangle$ of A is a RAAG where $[a_i^n, a_j^n] = 1 \iff [a_i, a_j] = 1$.

Example

$\langle a^n, b^n, c^n \rangle = \mathbb{Z} * \mathbb{Z}^2 \subseteq Br_4$



What if we consider Dehn twists around more curves?



$\langle a^n, b^n, c^n, {\Delta_{ab}}^n, {\Delta_{bc}}^n, {\Delta_{abc}}^n \rangle = ???$

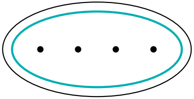
Garside element of a spherical Artin group

Fact

Every spherical Artin group A has the center $Z(A) \simeq \mathbb{Z}$.

Denote a generator of $Z(A)$ by Δ, and call it the *Garside element* of A.

Example (Δ in a braid group)



More generally

$A = \langle S \rangle$ be any Artin group.

For each irreducible spherical subset $U \subseteq S$, $A_U = \langle U \rangle$ is a spherical Artin group with the Garside element Δ_U.

$R_n = \langle \Delta_U^n : U \text{ a spherical subset} \rangle \subseteq A$

Question

Is R_n the obvious RAAG?

$$[\Delta_U^n, \Delta_V^n] = 1 \iff \begin{array}{l} U \subset V, \text{ or} \\ V \subset U, \text{ or} \\ [v, u] = 1 \text{ for all } v \in V, u \in U \end{array}$$

If YES, we say A satisfies the *generalized Tits conjecture*.

Generalized Tits conjecture

Theorem (J.-Schreve)

Spherical Artin groups of all types except for (possibly) E_7, E_8, H_4 satisfy the generalized Tits conjecture for sufficiently large n.

Many examples of 2-dimensional Artin groups satisfy the generalized Tits conjecture for sufficiently large n.

Applications: surface subgroups

We answer a question of Gordon-Long-Reid about hyperbolic surface subgroups:

Theorem (J.-Schreve)

Artin group of type H_3 (A_{235}) contains a hyperbolic surface subgroup.

Corollary (Gordon-Long-Reid 2003, J.-Schreve)

The only spherical Artin groups that do not contain hyperbolic surface subgroups are of type A_1 and $I_2(m)$.

Theorem (J.-Schreve)

Artin groups of type $\widetilde{C}_2$ (A_{244}) and $\widetilde{G}_2$ (A_{236}) contain a hyperbolic surface subgroup.

Applications: coherence

A group is *coherent* if every finitely generated subgroup is finitely presentable.

Gordon classified coherent Artin groups, with a single exception. The remaining case was completed by Wise. We give a new proof.

Theorem (Wise 2013, J.-Schreve)

Artin group of type H_3 is incoherent.

Corollary (Gordon 2004, Wise 2013)

There is a complete classification of coherent Artin groups.

Thank you!