Reversible Biholomorphic Germs

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Abstract

A reversible map is a special kind of invertible map: it is one that is conjugate to its own inverse, so that the associated dynamical system behaves in the same way when run forwards or backwards in time. We describe this situation by saying that "the past is the future of an alternative present".

Apart from the obvious examples in classical mechanics, one finds that reversible maps arise in diverse areas such as polynomial convexity, real and complex polynomial approximation, and other areas of functional analysis. These examples motivated our interest in reversibility.

The natural abstract context is the theory of groups. Let G be a group. We say that an element $f \in G$ is *reversible in* G if it is conjugate to its inverse, i.e. there exists $g \in G$ such that $g^{-1}fg = f^{-1}$. We denote the set of reversible elements by $R(G)$. For $f \in G$, we denote by $R_f(G)$ the set (possibly empty) of *reversers* of f, i.e. the set of $g \in G$ such that $g^{-1}fg = f^{-1}$.

For some time, we have studied reversibility in specific groups of mappings connected to real and complex analysis and geometry, including Möbius groups, groups of formal power series, groups of biholomorphic maps, homeomorphism groups and diffeomorphism groups. There are many open questions about reversibility, and about the related subjects of involutions and products of involutions.

In this talk, we characterise the elements of $R(G)$ and describe each $R_f(G)$, where G is the the group of biholomorphic germs in one complex variable. That is, we determine all solutions to the equation $f \circ g \circ f = g$, in which f and g are holomorphic functions on some neighbourhood of the origin, with $f(0) = g(0) = 0$ and $f'(0) \neq 0 \neq g'(0)$. This is joint work with Patrick Ahern, of U. Wisc.