

## Appendix

### Exceptional Dehn Surgery Spaces

This appendix shows the Dehn surgery spaces for knots of 9 or fewer crossings which have nonhyperbolic integer points off the horizontal axis. At each hyperbolic integer point the volume is given. If the volume is not in brackets, then the solution was obtained with positively oriented simplices and is completely reliable. A volume in square brackets represents a solution involving some negatively oriented simplices; in these cases the arguments of the cross-ratios are all between  $-\pi/2$  and  $3\pi/2$ , and should be fairly reliable. Users of SNAP-PEA will occasionally encounter volumes in curly brackets. Curly brackets mean that the analytic continuation feature was activated, which allows the arguments of the cross-ratios to extend beyond the range  $-\pi/2$  to  $3\pi/2$ . These are the most suspicious solutions of all. None appear in this appendix.

Solutions of zero volume are generally Seifert fibered spaces. The notation DEC labels points where an incompressible surface decomposes the space.

knot41  
number of simplices = 2      complete volume = 2.029883

1.79	1.77	1.76	1.75	1.74	1.73	1.73	1.74	1.75	1.76	1.77	1.79
1.59	1.53	1.48	1.44	1.41	1.40	1.39	1.41	1.44	1.48	1.53	1.59
1.28	0.98	DEC	0.00	0.00	0.00	0.00	0.00	0.00	DEC	0.98	1.28
--- 1.22---[ 0.94]---[ 0.51]---[ 0.51]---[ 0.94]--- 1.22---											

knot52  
number of simplices = 3      complete volume = 2.828122

2.47	2.48	2.50	2.53	2.55	2.58	2.61	2.63	2.65	2.67	2.69	2.70	2.72
2.06	2.03	2.03	2.07	2.13	2.23	2.32	2.41	2.48	2.53	2.58	2.61	2.64
[ 1.41]	[ 0.98]	DEC	-0.00	0.00	0.00	DEC	1.40	1.84	2.10	2.27	2.39	2.47
--- 2.04--- 1.72---[ 1.19]---[ 0.31]---[ 0.31]---[ 1.19]--- 1.72--- 2.04---												

knot61  
number of simplices = 4      complete volume = 3.163963

3.03	3.01	2.99	2.97	2.94	2.91	2.88	2.84	2.81	2.78	2.76	2.75	2.74
2.95	2.91	2.86	2.80	2.72	2.63	2.52	2.41	2.31	2.25	2.24	2.27	2.35
2.79	2.68	2.53	2.29	1.94	1.40	DEC	0.00	0.00	0.00	DEC	[ 1.61]	2.13
--- 2.47---[ 2.18]---[ 1.65]---[ 0.65]---[ 0.65]---[ 1.65]---[ 2.18]--- 2.47---												

knot62  
number of simplices = 5    complete volume = 4.400833

4.21	4.23	4.24	4.25	4.27	4.28	4.29	4.30	4.31	4.32	4.32	4.33	4.33
3.75	3.81	3.86	3.92	3.98	4.03	4.07	4.11	4.15	4.18	4.21	4.23	4.25
3.59	3.34	[ 2.95]	[ 2.27]	DEC	[ 1.65]	[ 2.13]	[ 2.50]	[ 2.83]	[ 3.12]	3.38	3.60	3.77
4.16	4.11	4.05	3.97	3.86	3.69	3.44	[ 3.03]	[ 2.32]	[ 1.03]			

knot72  
number of simplices = 4    complete volume = 3.331744

2.88	2.89	2.91	2.94	2.99	3.03	3.07	3.10	3.13	3.16	3.18	3.20	3.22
2.43	2.36	2.34	2.37	2.45	2.58	2.72	2.84	2.94	3.01	3.07	3.11	3.15
[ 2.19]	[ 1.61]	DEC	0.00	-0.00	-0.00	DEC	[ 1.73]	2.29	2.60	2.79	2.92	3.01
2.70	2.43	[ 1.94]	[ 0.93]						[ 0.93]	[ 1.94]	2.43	2.70

knot73  
number of simplices = 5    complete volume = 4.592126

4.55	4.55	4.54	4.54	4.54	4.53	4.53	4.52	4.51	4.51	4.50	4.49	4.48
4.50	4.49	4.48	4.46	4.45	4.43	4.40	4.37	4.34	4.30	4.25	4.20	4.13
4.22	4.13	4.01	3.84	3.61	3.28	[ 2.81]	[ 2.11]	DEC	[ 1.75]	[ 2.31]	2.76	3.16
			[ 1.29]	[ 2.62]	3.32	3.71	3.94	4.09	4.20	4.27	4.33	4.37

knot74  
number of simplices = 6      complete volume = 5.137941

4.94	4.92	4.89	4.87	4.84	4.82	4.80	4.79	4.79	4.80	4.81	4.83	4.85	
4.81	4.76	4.69	4.61	4.52	4.42	4.35	4.33	4.37	4.44	4.51	4.58	4.65	
4.59	4.44	4.23	3.92	[ 3.45]	[ 2.63]	DEC	[ 2.23]	[ 2.92]	[ 3.40]	3.77	4.07	4.31	
—	4.22	—	[ 3.04]	—	[ 1.57]	—	—	—	[ 1.57]	—	[ 3.04]	—	4.22

knot81  
number of simplices = 5      complete volume = 3.427205

3.31	3.29	3.27	3.25	3.22	3.18	3.15	3.10	3.06	3.02	2.98	2.96	2.95	
3.24	3.20	3.16	3.10	3.02	2.91	2.78	2.63	2.50	2.42	2.40	2.43	2.52	
[ 3.10]	[ 3.01]	2.88	2.67	2.33	[ 1.73]	DEC	0.00	-0.00	0.00	DEC	[ 1.81]	[ 2.39]	
—	[ 2.83]	—	[ 2.11]	—	[ 1.13]	—	—	—	[ 1.13]	—	[ 2.11]	—	[ 2.83]

knot82  
number of simplices = 6      complete volume = 4.935243

4.38	4.46	4.52	4.58	4.64	4.68	4.71	4.74	4.77	4.79	4.81	4.82	4.83							
4.44	4.31	4.12	3.83	[ 3.37]	[ 2.56]	DEC	[ 1.88]	[ 2.45]	2.91	3.33	3.71	4.02							
—	4.84	—	4.82	—	4.80	—	4.78	—	4.76	—	4.73	—	4.73	—	4.47	—	4.33	—	4.11
-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6							

[all but two near origin]

knot83  
number of simplices = 6      complete volume = 5.238684

4.84	4.77	4.68	4.59	4.50	4.42	4.39	4.59	4.68	4.77	4.84
4.58	4.38	4.10	3.72	[ 3.19]	[ 2.41]	DEC	[ 3.19]	[ 2.41]	4.38	4.58
— 4.37— [ 3.98]— [ 3.24]— [ 1.76]—   — [ 1.76]— [ 3.24]— [ 3.98]— 4.37—										

knot84  
number of simplices = 8      complete volume = 5.500486

4.87	4.95	5.03	5.10	5.16	5.21	5.25	5.34	5.36	5.37	5.39
[ 4.57]	[ 4.23]	[ 3.72]	[ 2.81]	DEC	[ 2.46]	[ 3.25]	[ 3.78]	[ 4.19]	4.91	5.04
— 5.29— [ 5.25]— [ 5.20]— [ 5.13]— [ 5.03]— [ 4.88]— [ 4.65]— [ 4.25]— [ 3.51]— [ 1.97]—   —										

knot85  
number of simplices = 9      complete volume = 6.997189

6.96	6.95	6.95	6.95	6.94	6.94	6.94	6.93	6.92	6.91	6.90
6.89	6.87	6.86	6.84	6.82	6.80	6.77	6.74	6.66	6.62	6.50
[ 6.10] [ 5.86] [ 5.56] [ 5.20] [ 4.75] [ 4.17] DEC 4.73 5.39 5.81 6.09 6.29 6.43										
— 5.99— [ 6.26]— [ 6.44]— [ 6.56]— [ 6.64]— [ 6.70]— [ 6.75]— [ 6.79]— [ 6.82]— [ 6.84]— [ 6.86]— [ 6.88]—										
6 7 8 9 10 11 12 13 14 15 16 17 18										

[ ALL BUT 2 NEAR ORIGIN ]

knot820  
number of simplices = 5      complete volume = 4.124903

3.90	3.87	3.82	3.78	3.73	3.68	3.64	3.62	3.62	3.62
3.75	3.66	3.54	3.40	3.24	3.08	2.99	3.00	3.12	3.12
3.42	[ 3.12 ]	[ 2.65 ]	[ 1.91 ]	DEC	0.00	0.00	[ 1.44 ]	2.57	2.57
— 2.77— 1.71—                     1.71— 2.77—									

knot92  
number of simplices = 5      complete volume = 3.486660

3.00	3.01	3.04	3.07	3.12	3.17	3.21	3.25	3.29	3.31	3.34	3.36	3.37
2.55	2.46	2.43	2.46	2.55	2.69	2.85	2.99	3.09	3.17	3.23	3.27	3.31
[ 2.42 ]	[ 1.81 ]	DEC	0.00	0.00	0.00	DEC	[ 1.86 ]	2.45	2.77	[ 2.97 ]	3.09	3.18
— 2.91— 2.66— [ 2.22 ]— [ 1.29 ]—                     [ 1.29 ]— [ 2.22 ]— 2.66— 2.91—												

knot93  
number of simplices = 6      complete volume = 4.994856

4.97	4.96	4.96	4.96	4.96	4.96	4.95	4.95	4.95	4.95	4.95	4.95	4.95
4.91	4.90	4.89	4.88	4.87	4.85	4.83	4.80	4.77	4.74	4.74	4.77	4.81
4.14	3.83	[ 3.34 ]	[ 2.52 ]	DEC	[ 1.90 ]	[ 2.50 ]	2.98	3.43	3.43	3.43	3.43	3.43
— 4.55— 4.64— 4.71— 4.76— 4.80— 4.82— 4.85— 4.87— 4.88—												
8	9	10	11	12	13	14	15	16	16	16	16	16

[all but 2 near origin]



knot942  
number of simplices = 5      complete volume = 4.056860

4.00	3.99	3.98	3.97	3.96	3.95	3.93	3.91	3.88	3.85	3.82	3.77	3.72
3.95	3.93	3.90	3.88	3.84	3.79	3.72	3.63	3.51	3.35	3.16	2.98	2.87
3.79	3.72	3.61	3.45	3.18	2.71	1.84	0.00	0.00	0.00	DEC	[ 2.32]	2.98
— 2.88 — [ 1.97] —   —   — [ 1.97] —   —   — 2.88 —   —   — 3.30 —   —   — 3.53 —   —   — 3.67 —   —   — 3.76 —												

knot946  
number of simplices = 6      complete volume = 4.751702

4.57	4.53	4.50	4.45	4.40	4.34	4.28	4.23	4.20	4.20	4.23	4.29	4.35
4.48	4.42	4.33	4.22	4.08	3.91	3.71	3.54	3.50	3.62	3.84	4.05	4.20
4.35	4.21	4.00	[ 3.67]	[ 3.13]	[ 2.26]	DEC	0.00	DEC	[ 2.67]	3.45	3.87	4.13
— 4.19 —   —   — 3.94 —   —   — [ 2.39] —   —   — [ 2.39] —   —   — 3.46 —   —   — 3.94 —   —   — 4.19 —												



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