MA472	Term 2
Local Fields (Reading Module)	18 CATS

Status: List C .

Commitment: Read the set chapters of the textbook and work hard on the exercises.

Prerequisites: MA222 Metric Spaces and either MA3D5 Galois Theory or MA3A6 Algebraic Number Theory.

Orientation:

One knows, for example, that the equation $x^2 + y^2 = -1$ does not have rational solutions for the obvious reason that $x^2 + y^2$ is non-negative. Here we have used the fact that rationals are real numbers and we have also used the properties of the real numbers (e.g. squares are non-negative). The equation $x^2 + y^2 = 3$ also does not have rational solutions. The reason is less obvious, but you can prove this by looking at the equation modulo 8. Thus before studying rational solutions to equations, it is sensible to look at the real solutions and at solutions modulo integers. The integers modulo a prime power p^m are awkward to work with since they do not form a field for $m > 1$. This problem is solved by completing the rationals with respect to the p-adic topology and arriving at the p-adic numbers $\mathbb{Q}_p$. An equation has solutions in $\mathbb{Q}_p$ if and only if it has solutions modulo p^m for arbitrarly large values of m.

The book covers p-adic numbers and their extensions (local fields), and gives substantial Diophantine applications.

As the preface to the book points out, "p-adic methods have become a natural and indespensible tool in many areas not merely of number theory, but also, for example, of representation theory and algebraic topology".

Content: The set textbook is *Local Fields*, by Cassels. The students are expected to study Chapters 1–2, 4, 6–7 and work hard on the exercises for these chapters. In particular I recommend the following homework problems:

- Pages 74–77: Q5,6,9,10,14
- Page 112: Q3, 4, 5
- Page 139: Q10, 11

Leads to: Ph.D. studies in number theory. Useful, but by no means essential for MA426 Elliptic Curves module.

Books:

J. W. S. Cassels, *Local Fields*, London Mathematical Society Student Texts **3**, Cambridge University Press, 1986.

Assessment: 100% by 3-hour examination.

Moderator: Samir Siksek