

THE UNIVERSITY OF WARWICK

FOURTH YEAR EXAMINATION: JUNE 2004

REPRESENTATION THEORY

Time Allowed: 3 hours

Read carefully the instructions on the answer book and make sure that the particulars required are entered.

ANSWER 4 QUESTIONS.

If you have answered more than the required 4 questions in this examination, you will only be given credit for your 4 best answers.

1. Let A be the subspace of $M_3(\mathbb{C})$ of matrices (a_{ij}) which satisfy $a_{ij} = 0$ for $i < j$ and $a_{11} = a_{33}$.
- a) Show that A is an algebra. [5]
 - b) Give the definition of a composition series for a representation. [5]
 - c) Consider the inclusion of A in $M_3(\mathbb{C})$ as a representation and find a composition series. [5]
 - d) Hence find the dimension vector of this representation. [5]
 - e) Find a nilpotent endomorphism of this representation. [5]
-
2. a) Define the nilpotent radical of an algebra. [5]
- Let A be the algebra $\mathbb{R}[x]/\langle p(x) \rangle$ where $p(x)$ is a polynomial.
- b) Find the radical of A for:
 - (i) $p(x) = (x - 2)^2$.
 - (ii) $p(x) = x^2 + 9$.[6]
 - c) Show, that in each case, the radical satisfies the definition you have given in a). [6]
 - d) Give the definition of a local algebra. [3]
 - e) Show that each algebra in b) is a local algebra. [5]
-
3. a) State the classification of semisimple algebras over \mathbb{C} . [5]
- b) Give the classification of semisimple algebras over \mathbb{C} of dimension at most three. [6]

- c) Show that there are just five three dimensional algebras over \mathbb{C} (up to isomorphism). You may use any results from the course without proof but these should be clearly stated. [14]

4. a) Give the definition of a projective module. [6]
 b) Give the definition of a projective cover of a module M . [7]
 c) Prove that any two projective covers of M are isomorphic. [12]

5. Let A be the quiver algebra of the quiver

$$e \bullet \begin{array}{c} \xrightarrow{\alpha} \\ \xleftarrow{\beta} \end{array} \bullet f$$

with the relation $\alpha\beta\alpha = 0$.

- a) Find a basis of A where each element is a path. [3]
 b) Find the simple modules. [7]
 c) Find the indecomposable projective modules. [7]
 d) Give the definition of the Cartan matrix of an algebra. [4]
 e) Find the Cartan matrix of A . [4]