

MA377 EXAMPLE SHEET I

Q. 1. Find all rings in which $0 = 1$.

Q. 2. What is your favourite ring?

Q. 3. Let S be a set and R the set of all subsets of S . Define addition to be the symmetric difference of sets and define multiplication to be intersection. Check the ring conditions.

Q. 4. Interval arithmetic is used in numerical analysis. Let R be the set of intervals in \mathbb{R} , so R is defined by

$$R = \{[x, y] \subset \mathbb{R} \mid x \leq y\}$$

Define the ring operations setwise so that

$$A + B = \{x + y \mid x \in A, y \in B\}$$

$$AB = \{xy \mid x \in A, y \in B\}$$

Check the ring conditions.

Q. 5. Fix a ring R and $n > 0$. Construct inverse isomorphisms between $M_n(R[x])$ and $M_n(R)[x]$.

Q. 6. Fix a ring R and $m, n > 0$. Construct inverse isomorphisms between $M_m(M_n(R))$ and $M_{mn}(R)$.