Name: .... Supervisor: ....

Class Teacher: ....

MA131 - Analysis 1
Workbook 5 Assignments

Due in 3rd Nov

Assignment 1 Prove that there is a rational number between any two real numbers.

Assignment 2 Let $a < b$. Prove that there is an infinite number of rational numbers in the open interval (a, b).

Assignment 3 Prove that between any two distinct *rational* numbers there is an irrational number.

Assignment 4 Prove that between any two distinct real numbers there is an irrational number.

Assignment 5 Prove that a set A can have at most *one* least upper bound.

Assignment 6 Suppose a set A is non-empty and bounded above. Given $\epsilon > 0$, prove that there is an $a \in A$ such that $\sup A - \epsilon < a \leq \sup A$.

Assignment 7

Suppose A is a non-empty set of real numbers which is bounded below. Define the set $-A = \{-a : a \in A\}$. Sketch two such sets A and $-A$ on the real line. Mark in the position of $\inf A$. Prove that $-A$ is a non-empty set of real numbers which is bounded below, and that $\sup(-A) = -\inf A$. Mark $\sup(-A)$ on the diagram.

Assignment 8 Prove that every bounded increasing sequence is convergent.

Assignment 9 Prove that every bounded decreasing sequence is convergent.

Assignment 10 Consider the sequence (a_n) defined by $a_1 = \frac{5}{2}$ and $a_{n+1} = \frac{1}{5}\left(a_n^2 + 6\right)$. Show by induction that $2 < a_k < 3$. Show that (a_n) is decreasing. Finally, show that (a_n) is convergent and find its limit.

Assignment 11 If (a_n) is an increasing sequence that is *not* bounded above, show that $(a_n) \to \infty$. Make a rough sketch of the situation.

Assignment 12 Calculate the distance $\mathrm{d}(A, B)$ between the following pairs of subsets A and B. You may benefit by sketching A and B quickly first.

1. $A = \{x \in \mathbb{R}^2 : \|x\| < 1\}$, $B = \{(1, 1)\}$ (B contains just one point).
2. $A = \{x \in \mathbb{R}^2 : \|x\| \leq 1\}$, $B = \{(1, 1)\}$.
3. $A = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \geq 1 + x_1^2\}$, $B = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \leq 0\}$.
4. $A = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \geq e^{x_1}\}$, $B = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \leq 0\}$.

In each case state whether there exists $a \in A$ and $b \in B$ such that $\mathrm{d}(A, B) = |a - b|$.

Assignment 13 Show that the set A is non-empty.

Assignment 14 Show that $a_n^k \to b^k$ and conclude that $b^k \geq a$.

Assignment 15 Now achieve a contradiction by showing that $b - \delta \in A$.