

Name:

Supervisor:

Class Teacher:

MA131 - Analysis 1
Workbook 5 Assignments

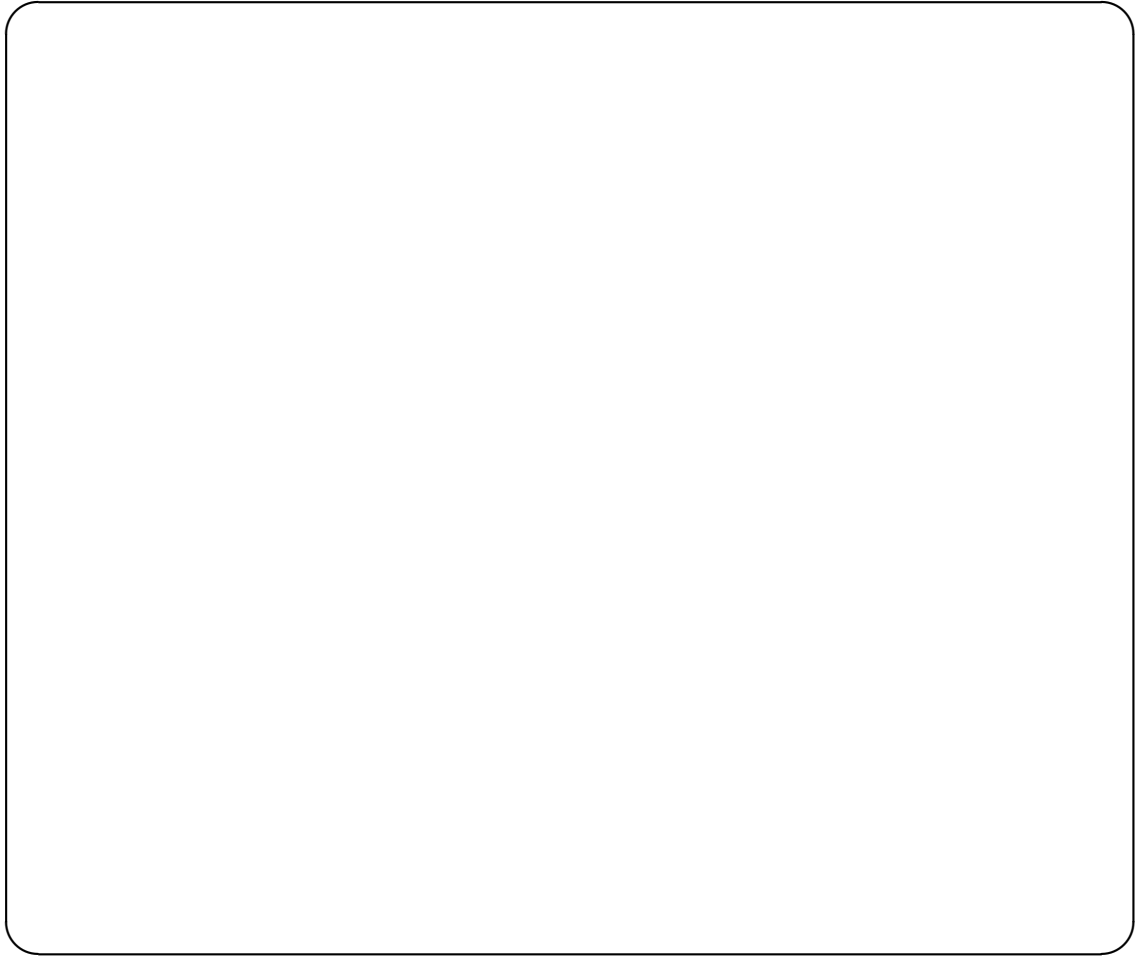
Due in 3rd Nov

Assignment 1 Prove that there is a rational number between any two real numbers.



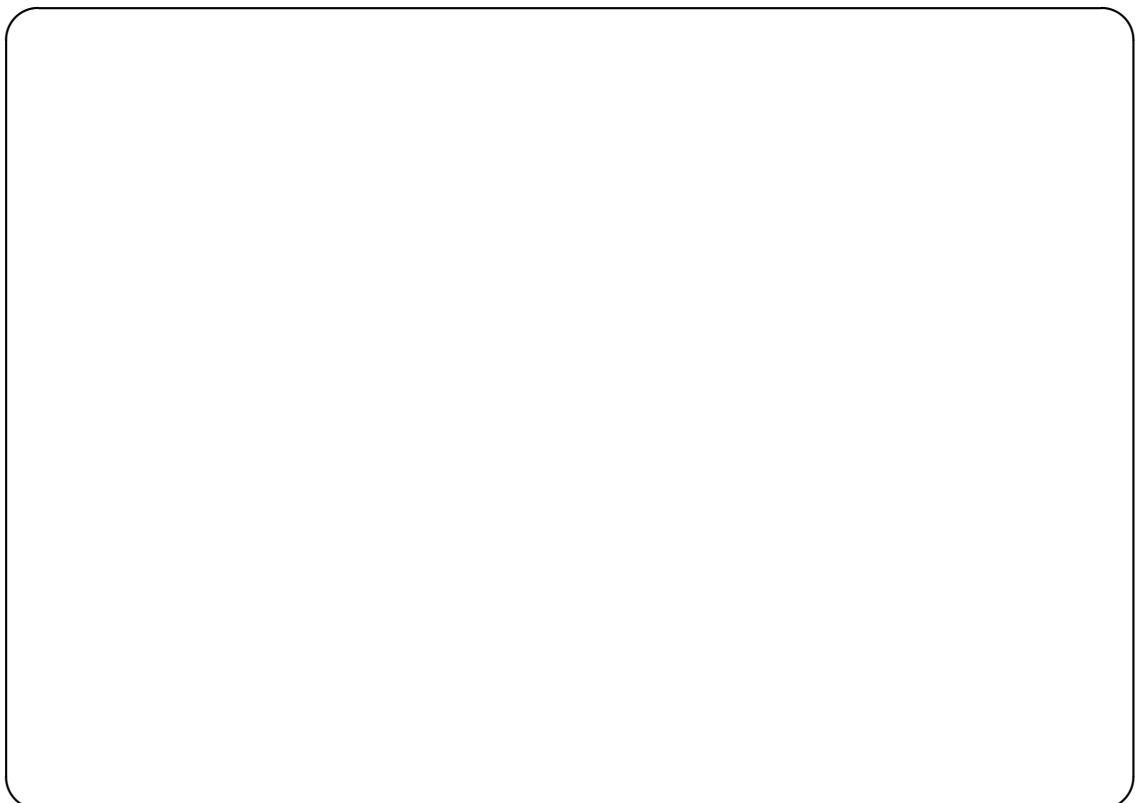
Assignment 2

Let $a < b$. Prove that there is an infinite number of rational numbers in the open interval (a, b) .

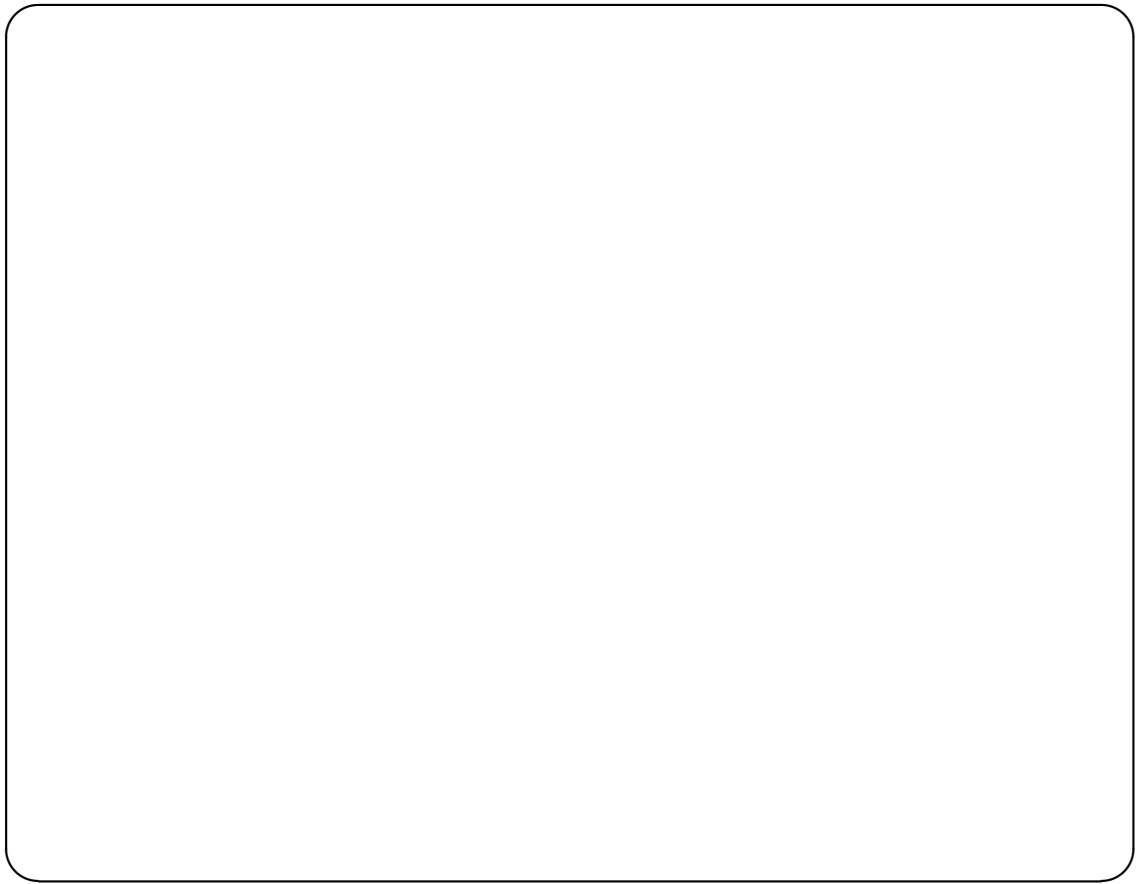


Assignment 3

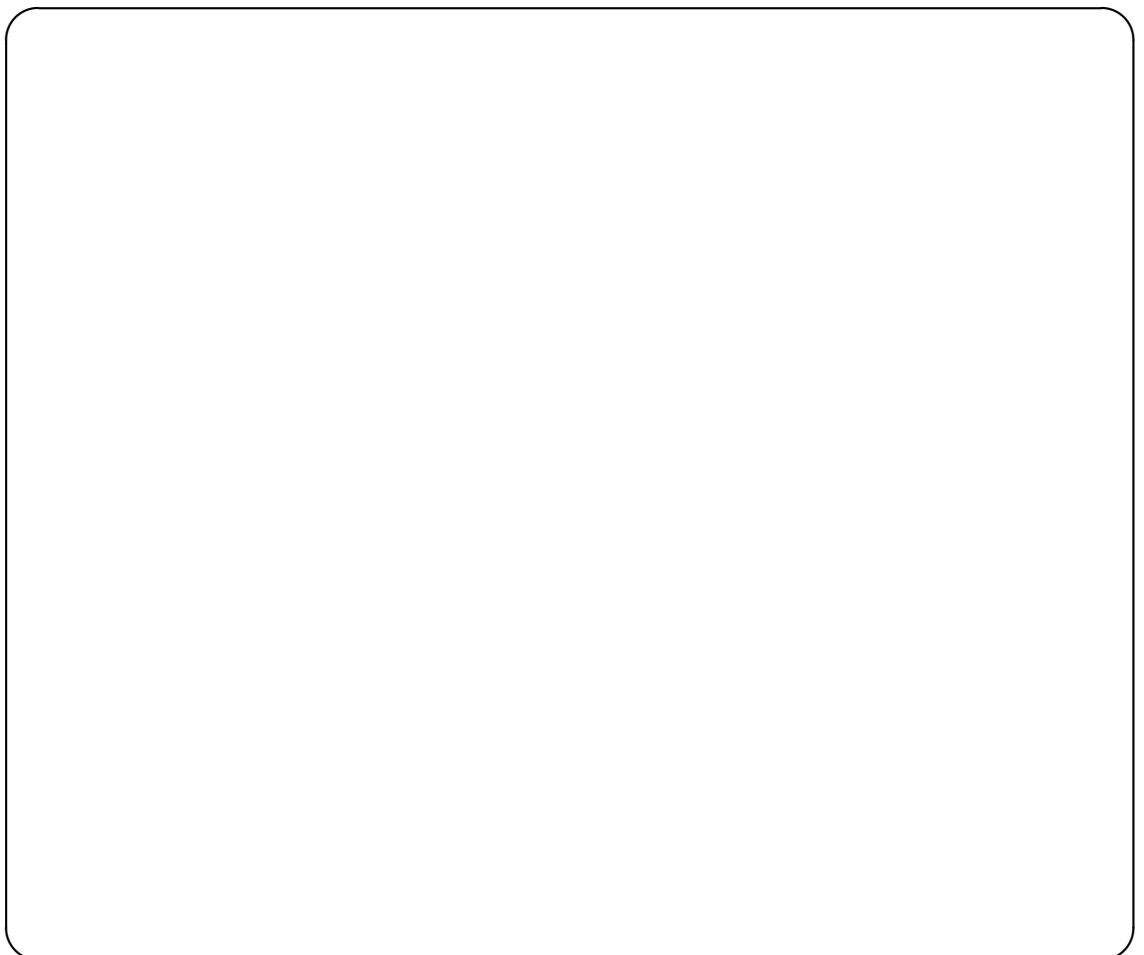
Prove that between any two distinct *rational* numbers there is an irrational number.



Assignment 4 Prove that between any two distinct real numbers there is an irrational number.

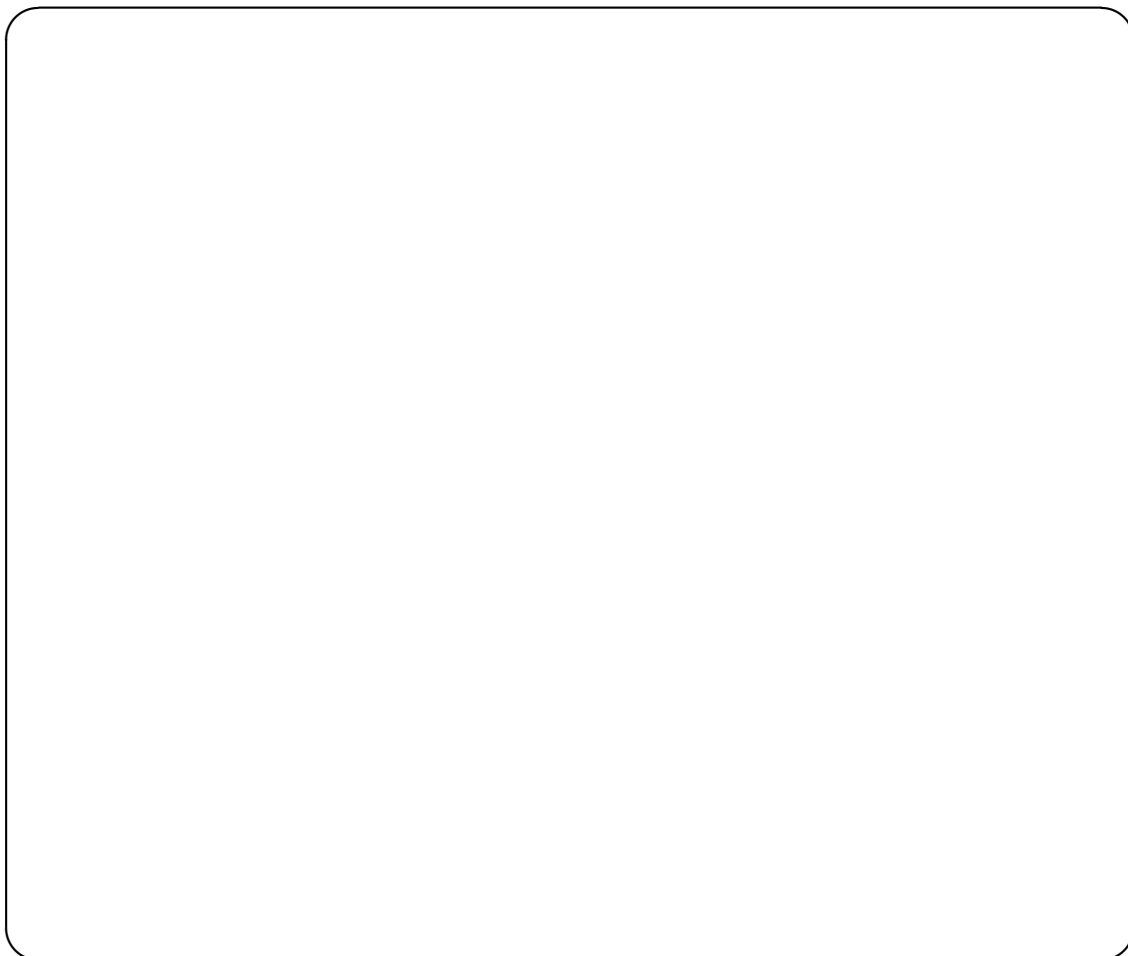


Assignment 5 Prove that a set A can have at most *one* least upper bound.



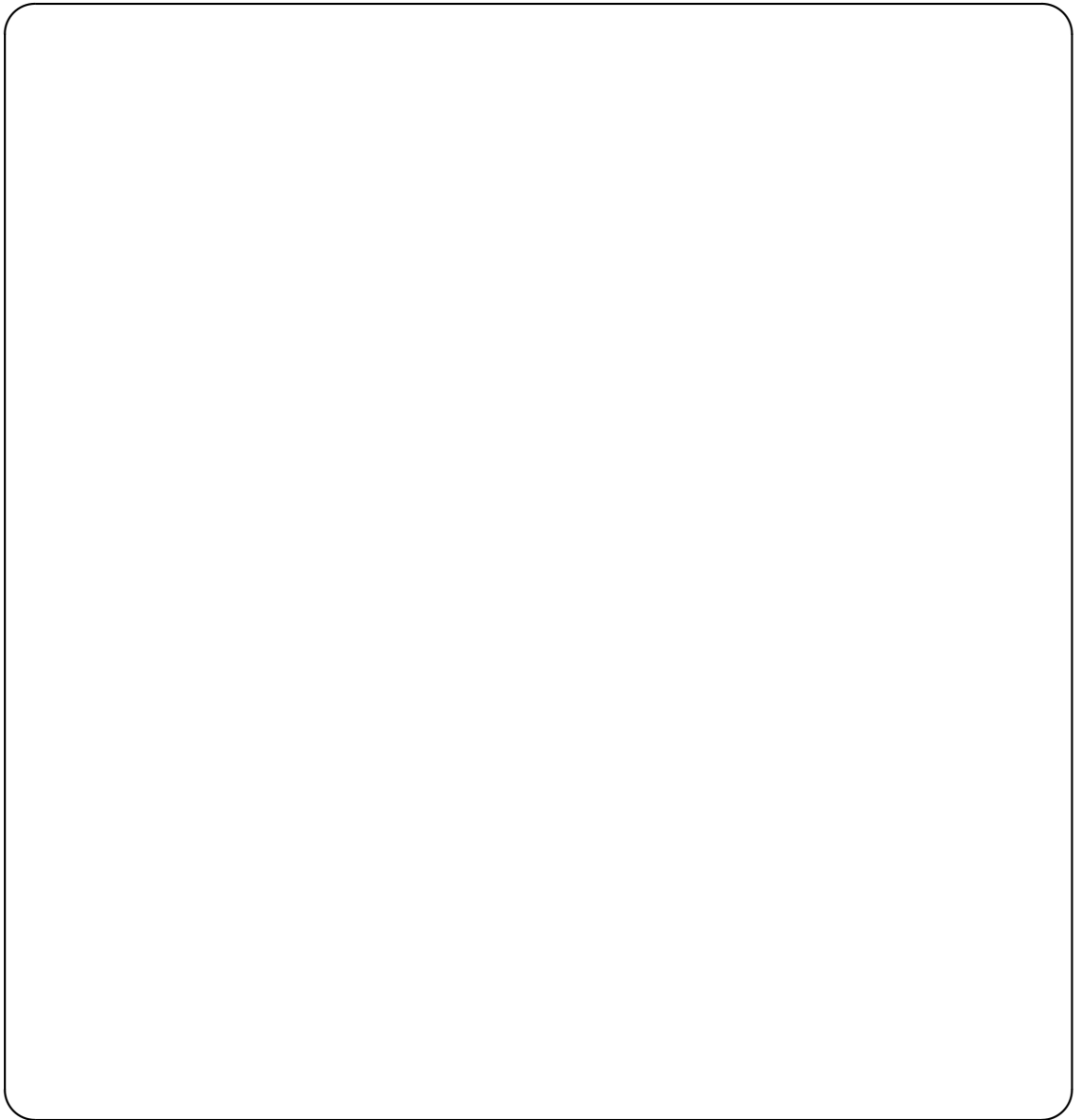
Assignment 6

Suppose a set A is non-empty and bounded above. Given $\epsilon > 0$, prove that there is an $a \in A$ such that $\sup A - \epsilon < a \leq \sup A$.



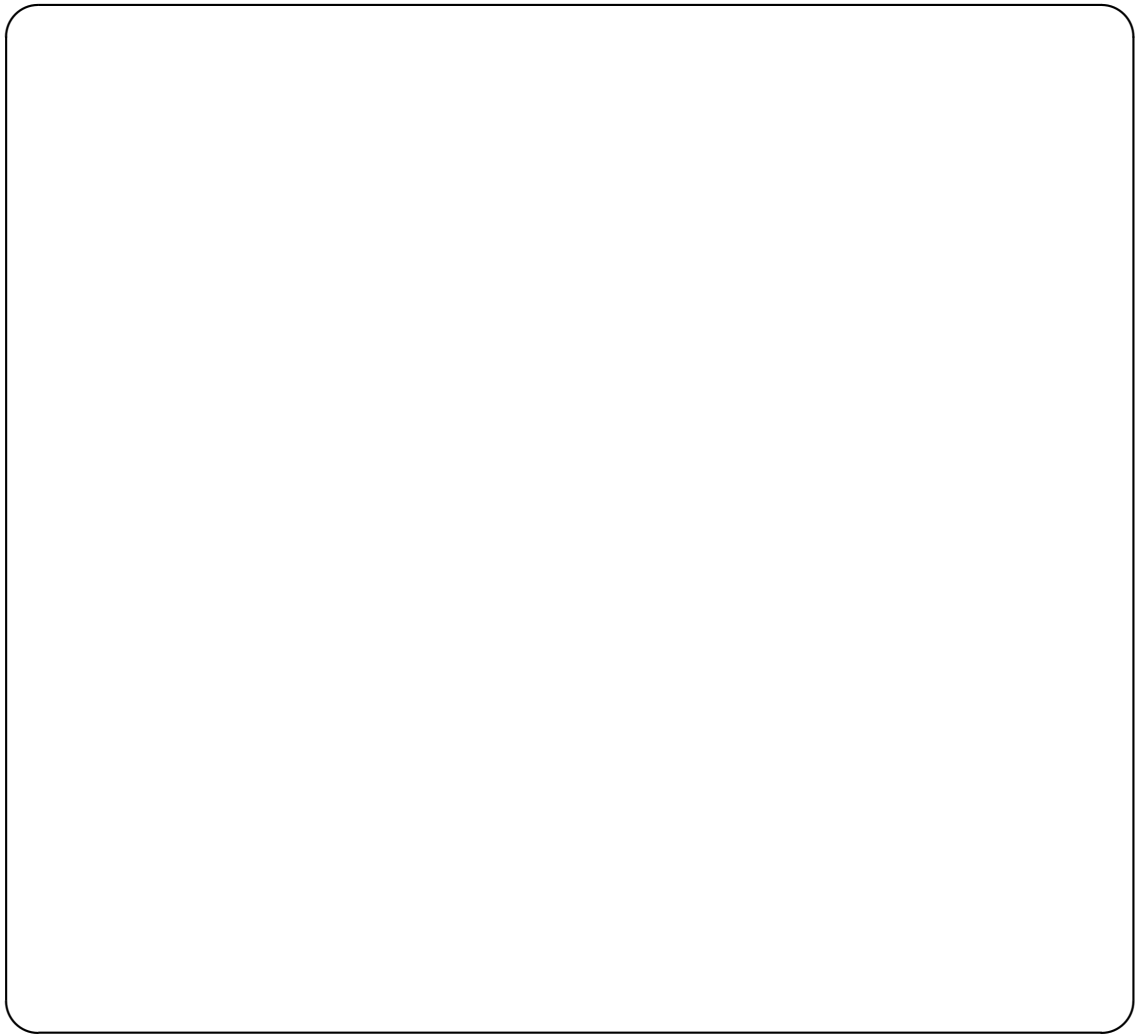
Assignment 7

Suppose A is a non-empty set of real numbers which is bounded below. Define the set $-A = \{-a : a \in A\}$. Sketch two such sets A and $-A$ on the real line. Mark in the position of $\inf A$. Prove that $-A$ is a non-empty set of real numbers which is bounded below, and that $\sup(-A) = -\inf A$. Mark $\sup(-A)$ on the diagram.



Assignment 8

Prove that every bounded increasing sequence is convergent.



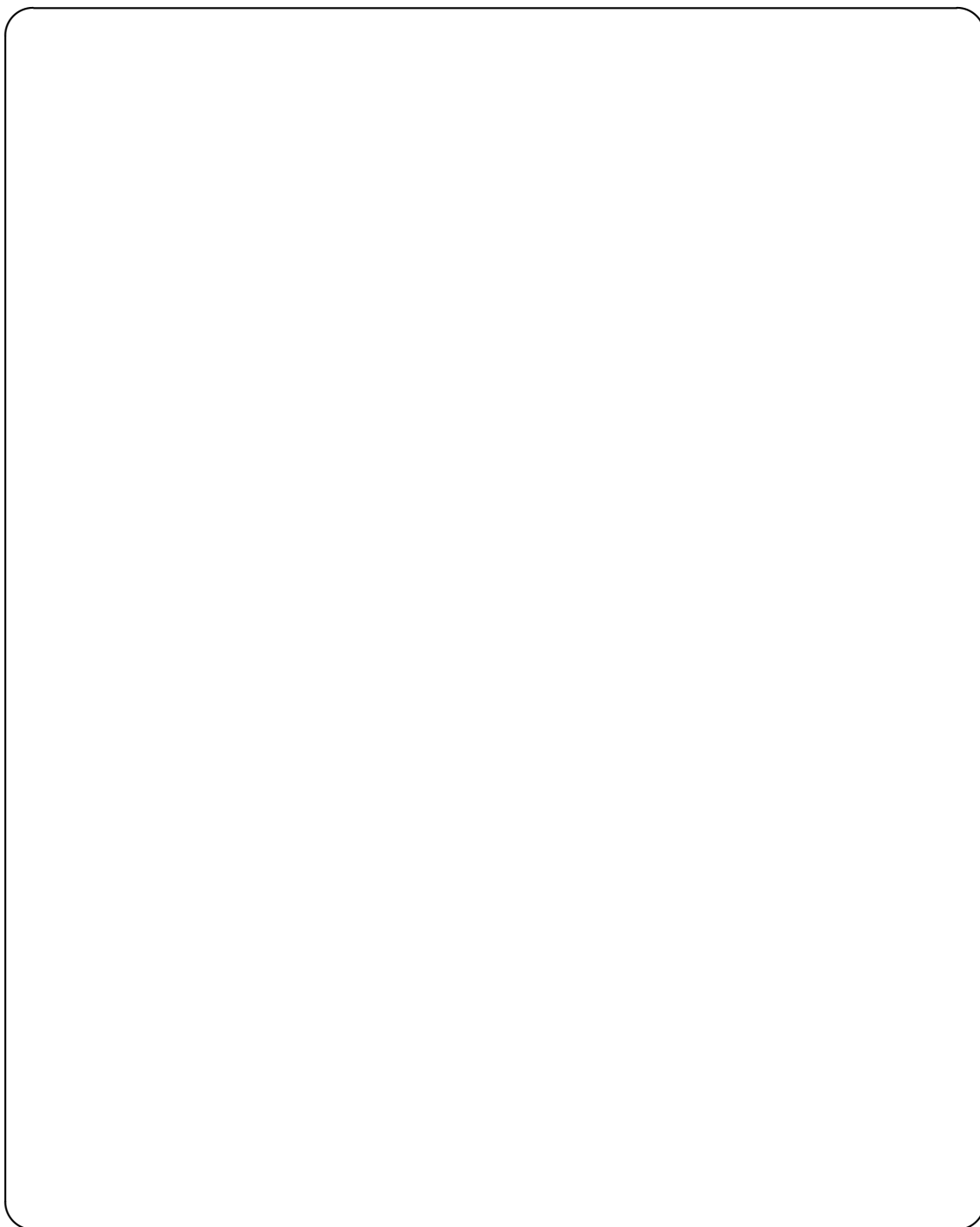
Assignment 9

Prove that every bounded decreasing sequence is convergent.



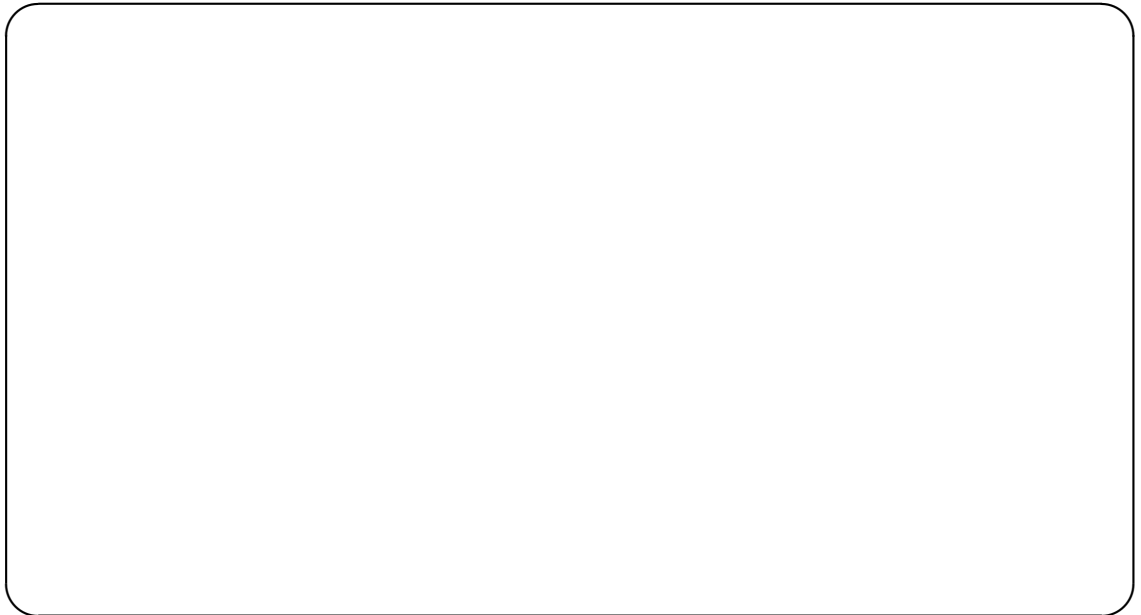
Assignment 10

Consider the sequence (a_n) defined by $a_1 = \frac{5}{2}$ and $a_{n+1} = \frac{1}{5}(a_n^2 + 6)$. Show by induction that $2 < a_k < 3$. Show that (a_n) is decreasing. Finally, show that (a_n) is convergent and find its limit.



Assignment 11

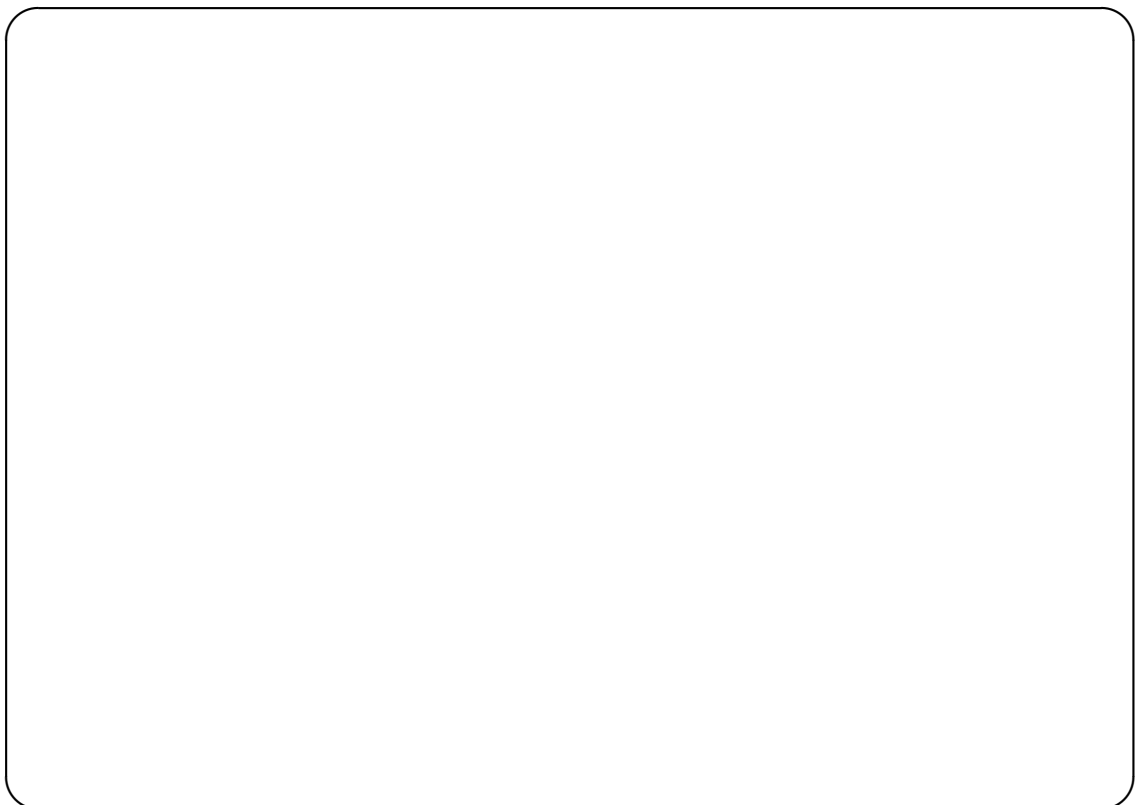
If (a_n) is an increasing sequence that is *not* bounded above, show that $(a_n) \rightarrow \infty$. Make a rough sketch of the situation.

**Assignment 12**

Calculate the distance $d(A, B)$ between the following pairs of subsets A and B . You may benefit by sketching A and B quickly first.

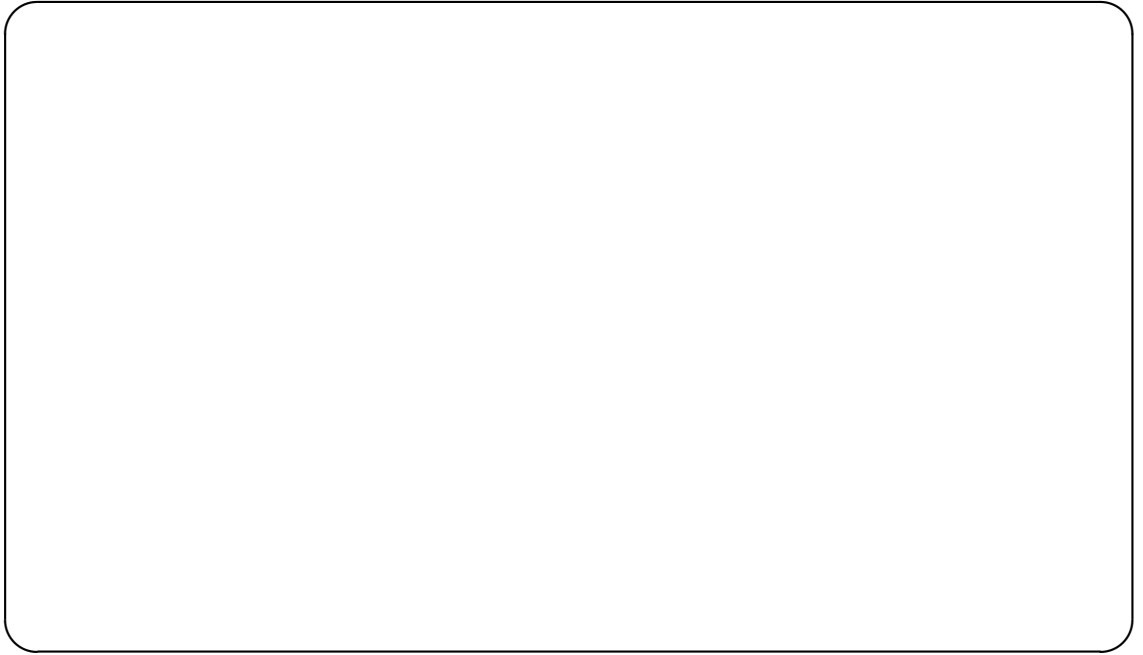
1. $A = \{x \in \mathbb{R}^2 : \|x\| < 1\}$, $B = \{(1, 1)\}$ (B contains just one point).
2. $A = \{x \in \mathbb{R}^2 : \|x\| \leq 1\}$, $B = \{(1, 1)\}$.
3. $A = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \geq 1 + x_1^2\}$, $B = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \leq 0\}$.
4. $A = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \geq e^{x_1}\}$, $B = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \leq 0\}$.

In each case state whether there exists $a \in A$ and $b \in B$ such that $d(A, B) = |a - b|$.



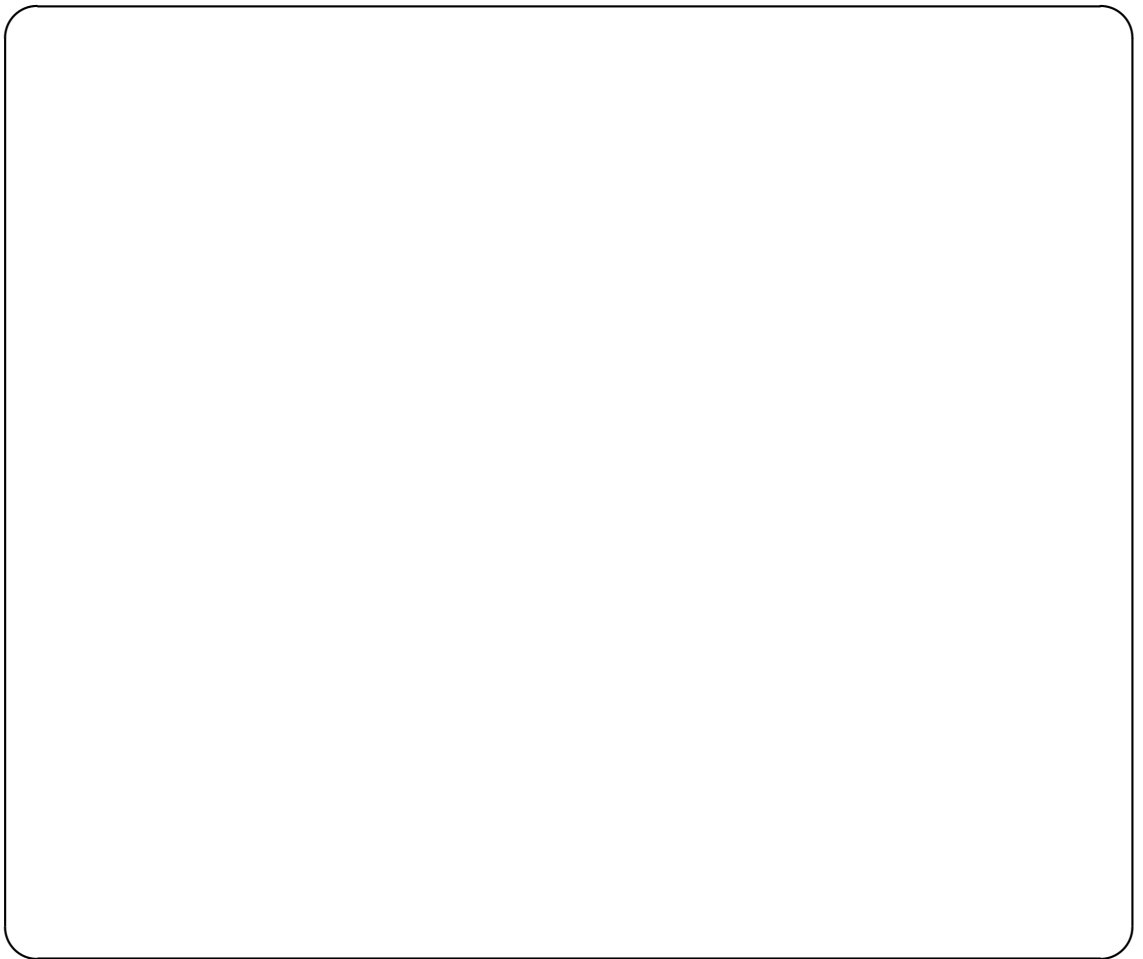
Assignment 13

Show that the set A is non-empty.



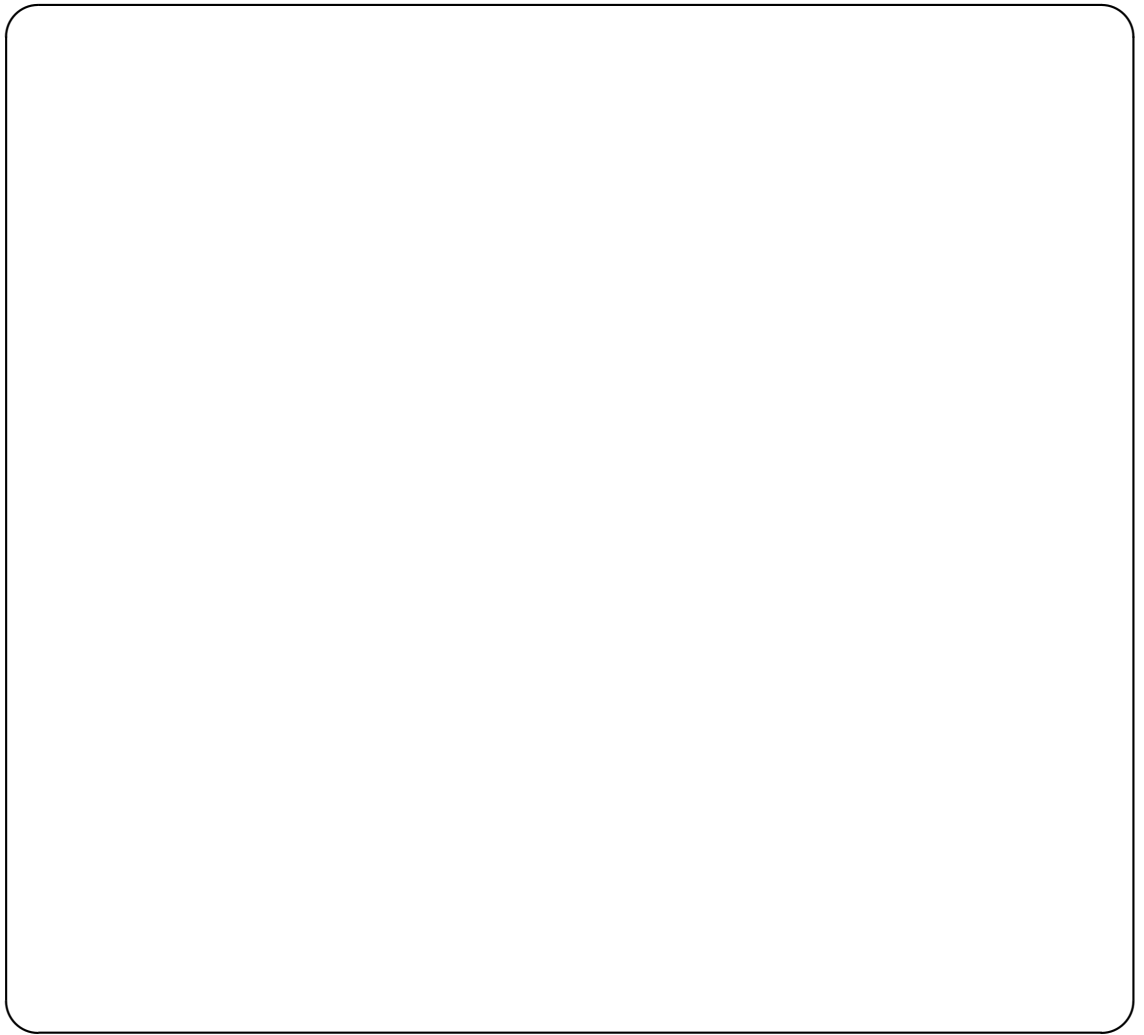
Assignment 14

Show that $a_n^k \rightarrow b^k$ and conclude that $b^k \geq a$.



Assignment 15

Now achieve a contradiction by showing that $b - \delta \in A$.

A large, empty rounded rectangular box with a thin black border, intended for the student to write their proof. The box is centered on the page and occupies most of the vertical space below the text.