

Name:

Supervisor:

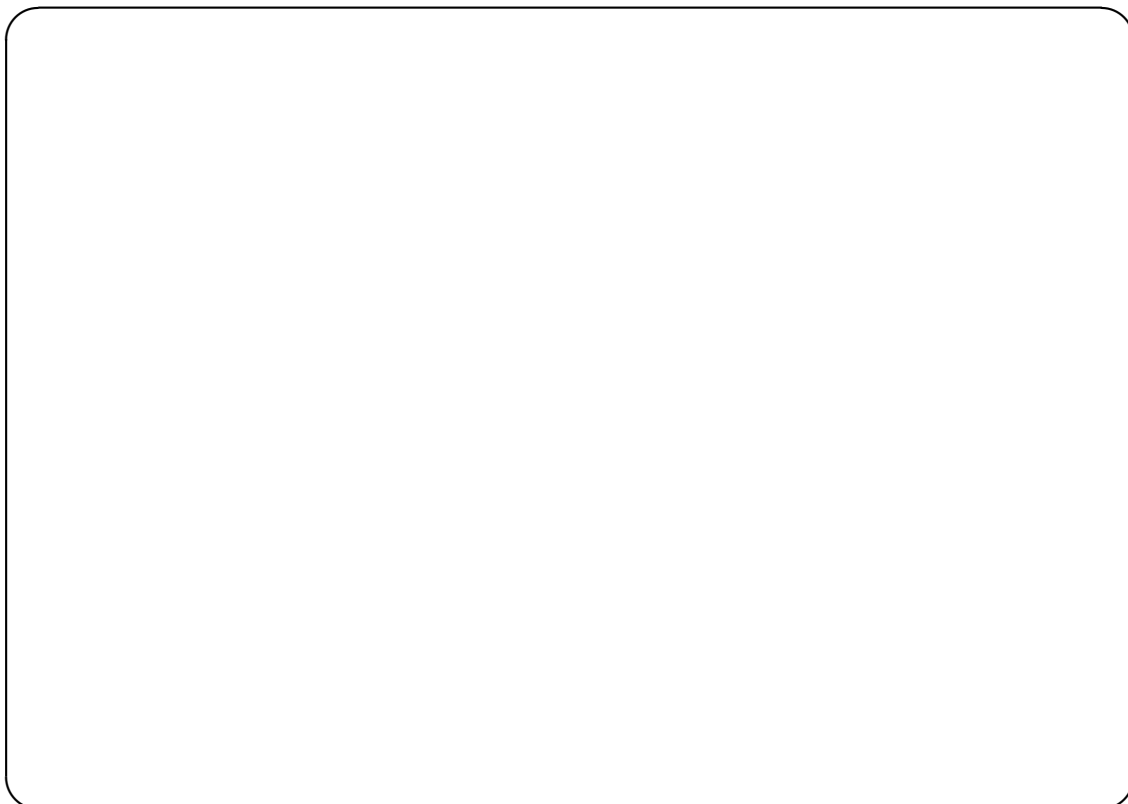
Class Teacher:

MA131 - Analysis 1
Workbook 4 Assignments

Due in 27th Oct

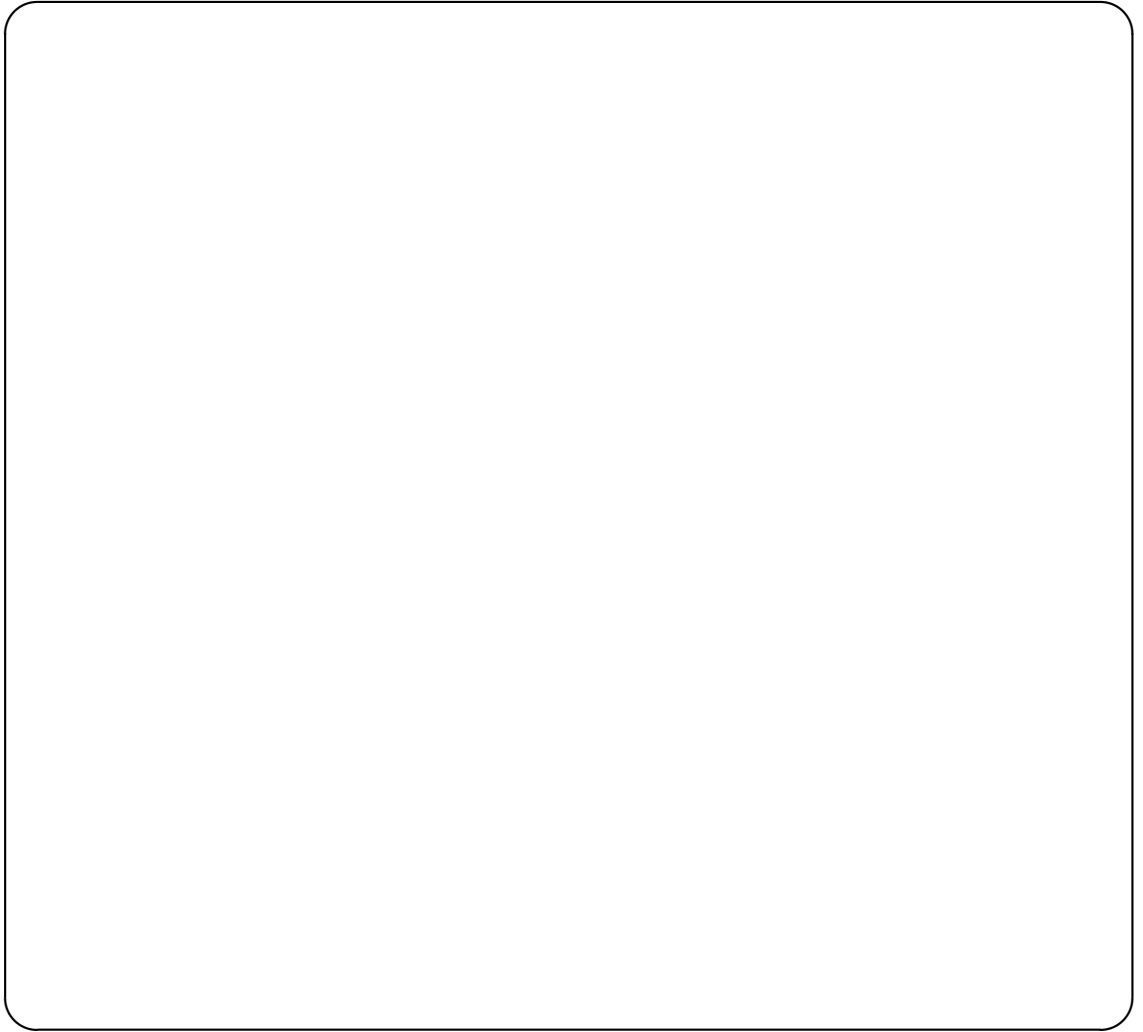
Assignment 1

Finish off the proof of Bernoulli's Inequality for $x > -1$ using *mathematical induction*. Note down where you use the fact that $x > -1$.

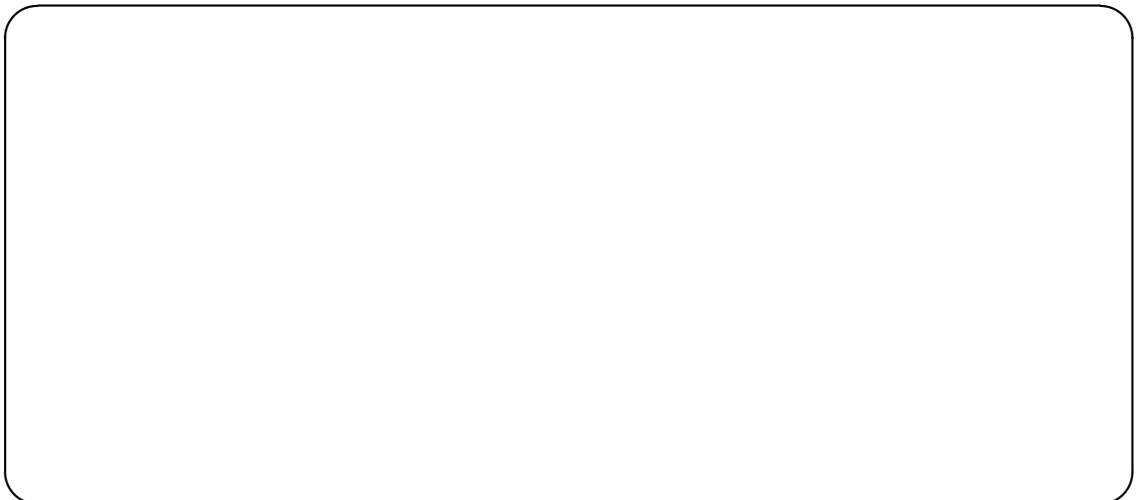


Assignment 2

Prove that if $x > 0$ then $(x^{1/n}) \rightarrow 1$.

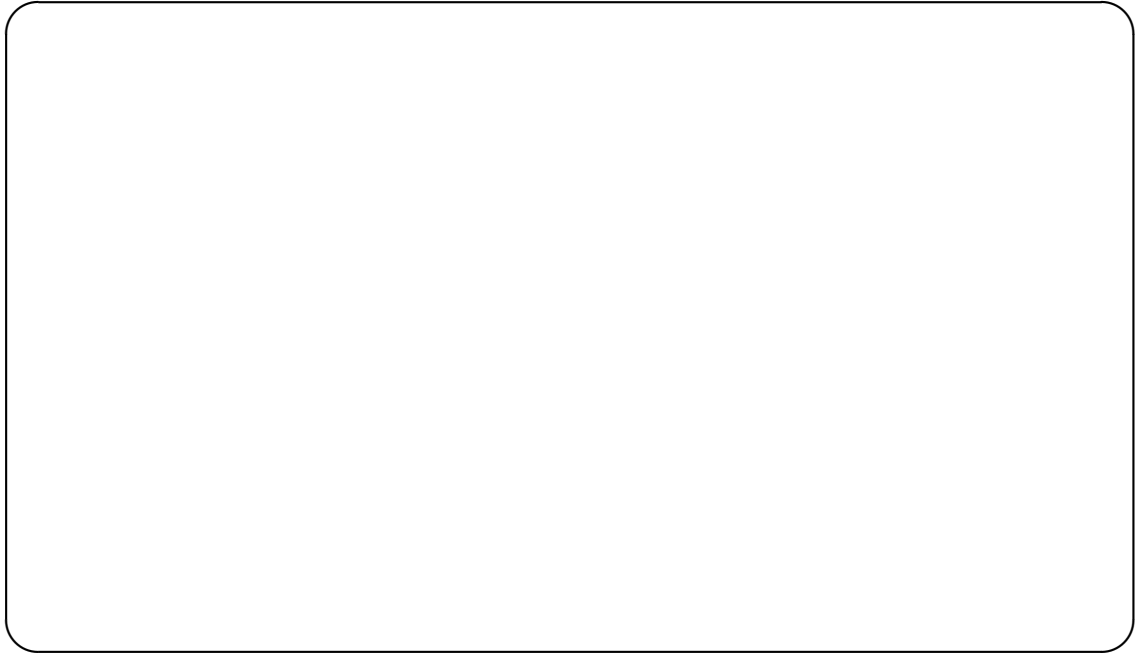
**Assignment 3**

Use a calculator to explore the limit of $(2^n + 3^n)^{1/n}$. Now find the limit of the sequence $(x^n + y^n)^{1/n}$ when $0 \leq x \leq y$.



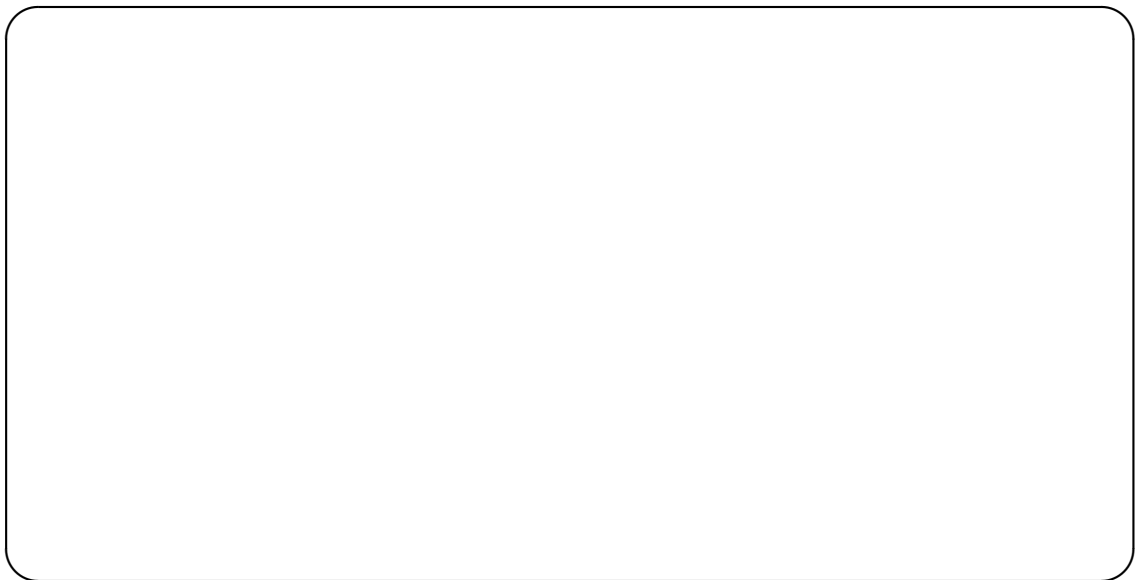
Assignment 4

Explore, with a calculator if necessary, and then write down a conjectured limit for the power sequence (x^n) .



Assignment 5

Prove the first version of the Ratio Lemma by first using induction to show $a_n \leq l^n a_0$ and using the fact that (l^n) is a null sequence.



Assignment 6

Prove the second version of the Ratio Lemma by using its first version and then using the Shift Rule.



Assignment 7

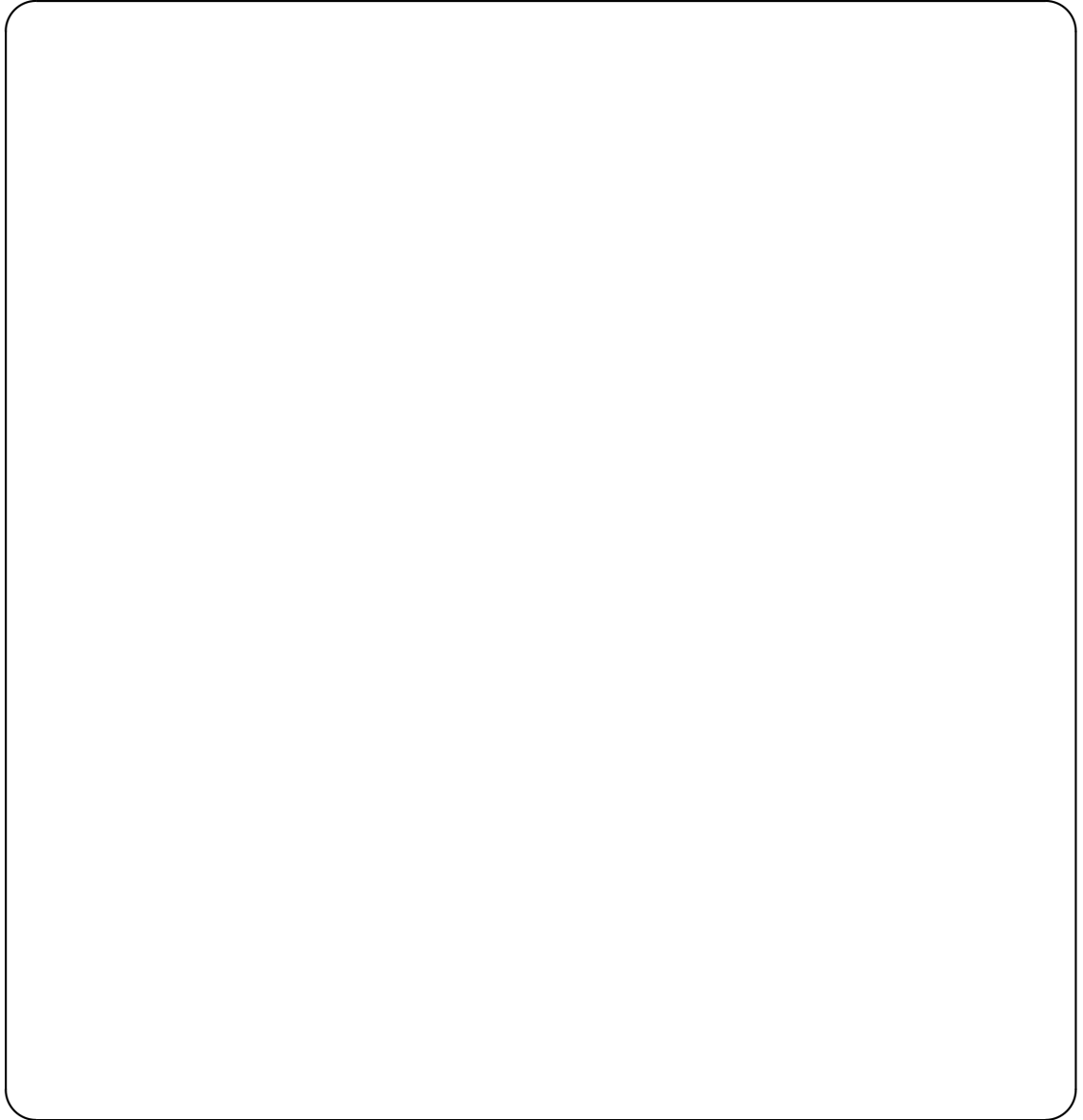
State whether the following sequences tend to zero or infinity. Prove your answers:

1. $\frac{n^{1000}}{2^n}$

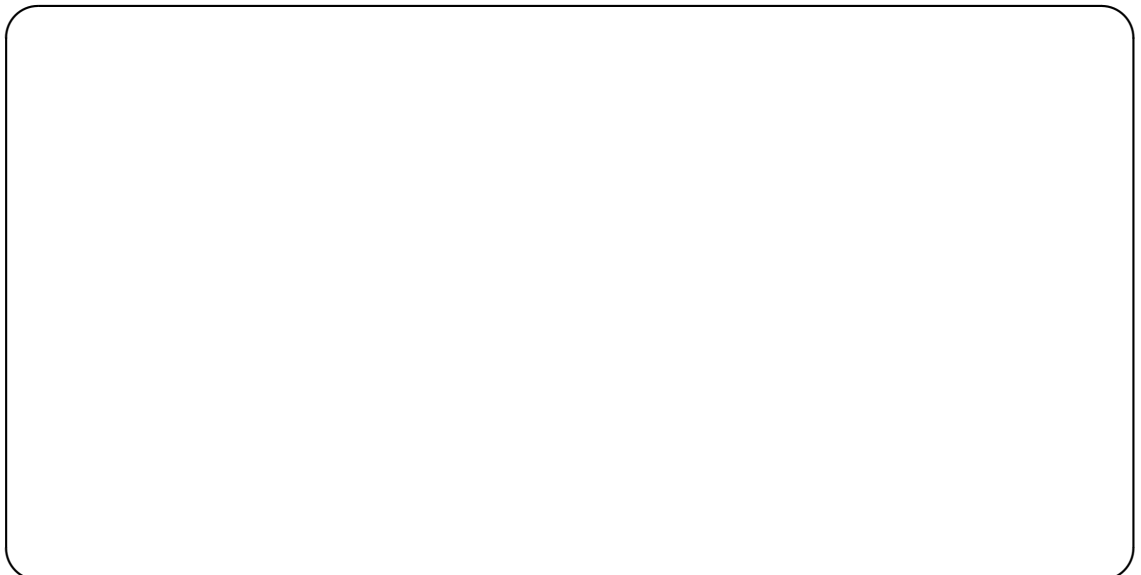
2. $\frac{1.0001^n}{n}$

3. $\frac{n!}{n^{1000}}$

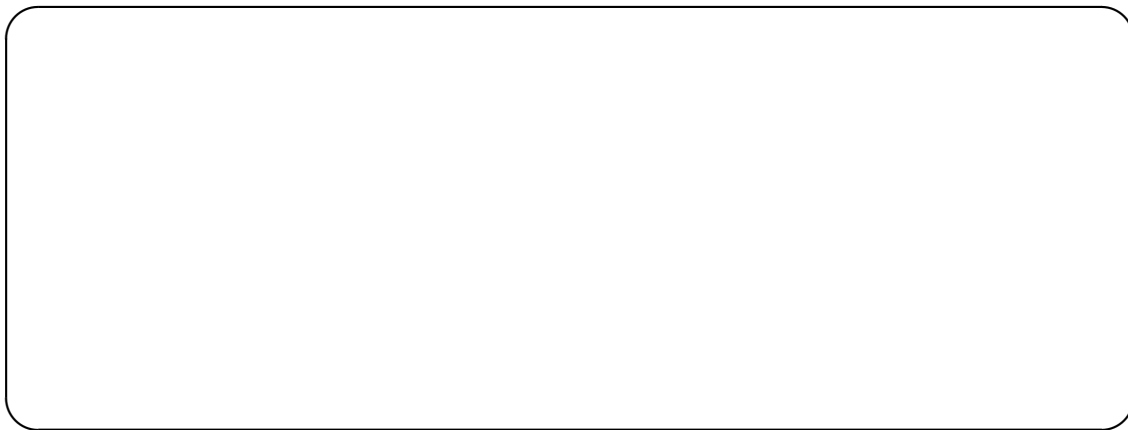
4. $\frac{(n!)^2}{(2n)!}$

**Assignment 8**

Prove that $\left(\frac{x^n}{n!}\right) \rightarrow 0$ for all values of x .



Assignment 9 Prove that $\left(\frac{n!}{n^n}\right) \rightarrow 0$ as $n \rightarrow \infty$.



Assignment 10 Find the limit of the sequence $\left(\frac{x^n}{n^k}\right)$ as $n \rightarrow \infty$ for all values of $x > 0$ and $k = 1, 2, \dots$



Assignment 11

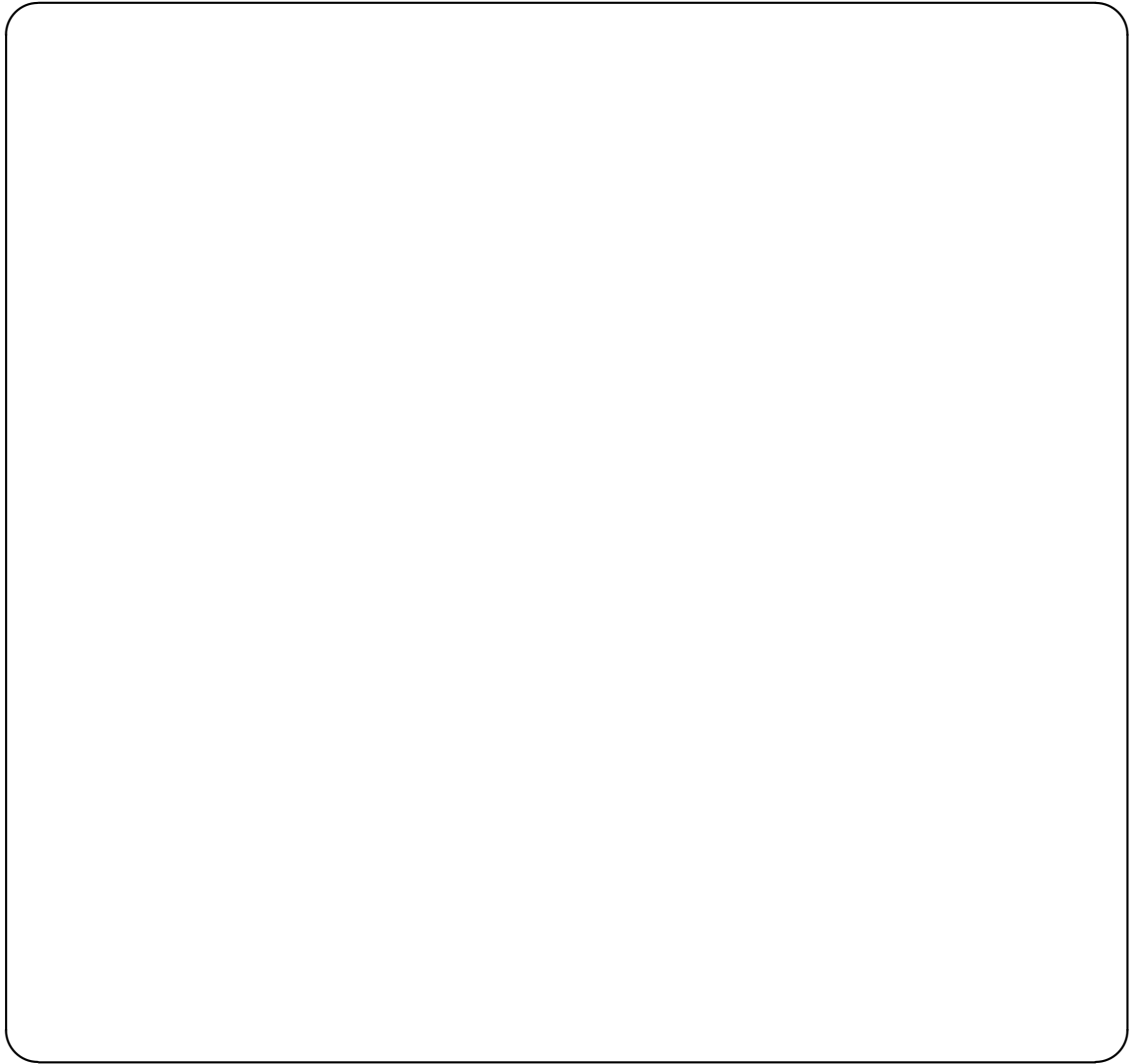
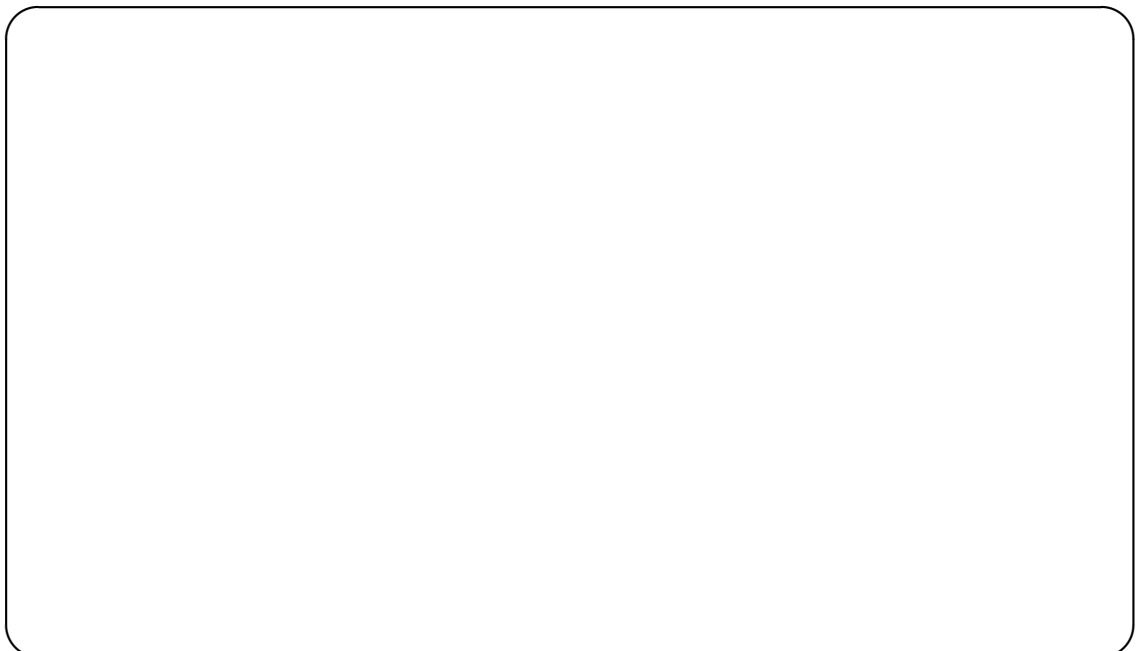
Find the limits of the following sequences. Give reasons.

1. $\left(\frac{n^4 11^n + n^9 9^n}{7^{2n} + 1}\right)$

2. $\left((4^{10} + 2^n)^{1/n}\right)$

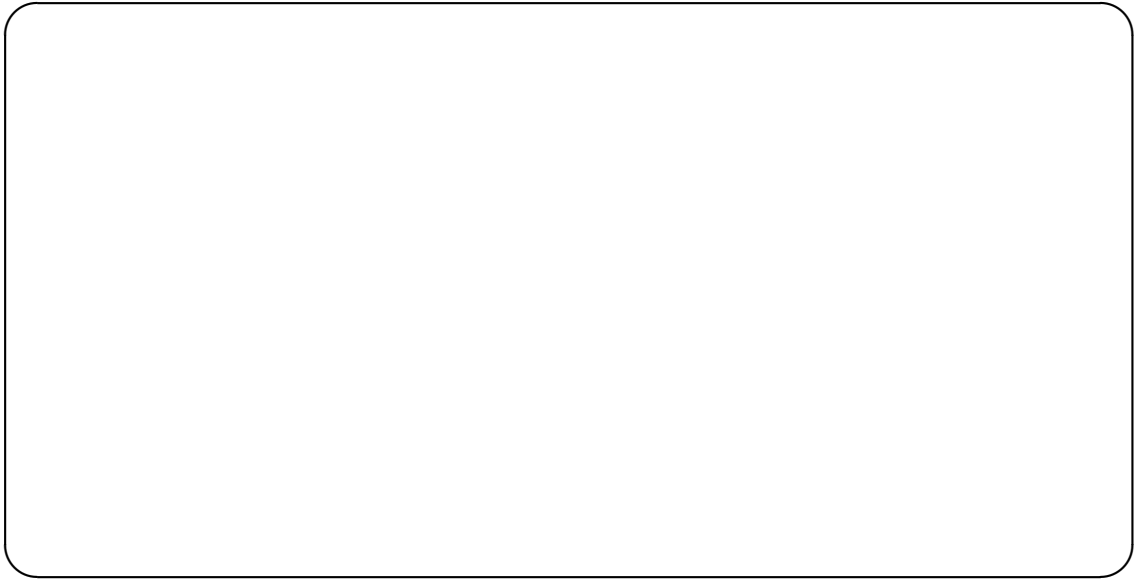
3. $\left(\frac{3n^3 + n \cos^2 n}{n^2 + \sin^2 n}\right)$

4. $\left((3n^2 + n)^{1/n}\right)$

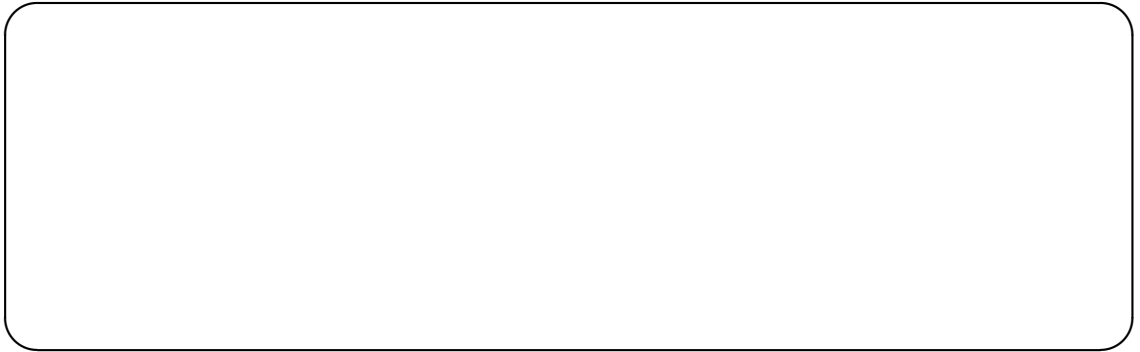
**Assignment 12**Obtain a lower bound on $n!$.

Assignment 13

Use your upper and lower bounds on $n!$ to find the following limits: (i) $\left(\frac{n!2^n}{n^n}\right)$ (ii) $\left(\frac{n!4^n}{n^n}\right)$

**Assignment 14**

Show that $\lim p_n = 2\pi$.

**Assignment 15**

Consider the sequence of shapes in figure 5. Each is produced from the former by replacing each large step by two half-sized ones. Draw the “limiting” shape. Given that the original shape is a square of area 1, what is the perimeter and area of the n^{th} shape? Compare the limits of these sequences with the perimeter and area of the limiting shape.



Assignment 16

Consider the construction of the Koch curve, let p_n be its perimeter at the n^{th} stage and A_n be its area.

1. What is the number of sides of the shape at the n^{th} stage (for $n = 1$ the answer is 3).
2. Show that $p_{n+1} = \frac{4}{3}p_n$.
3. Prove that $(p_n) \rightarrow \infty$.
4. Show that, in making the $(n + 1)^{\text{th}}$ shape, each little triangle being added has area $\frac{1}{9^n}A_1$.
5. Show that $A_{n+1} = A_n + \frac{3}{9} \left(\frac{4}{9}\right)^{n-1} A_1$.
6. Prove that $(A_n) \rightarrow \frac{8}{5}A_1$.

