Name: .... Supervisor: ....

Class Teacher: ....

MA131 - Analysis 1
Workbook 3 Assignments

Due in 20th Oct

Assignment 1 Prove the 'Uniqueness of Limits' theorem by assuming $(a_n) \to a$, $(a_n) \to b$ with $a < b$ and obtaining a contradiction.

Assignment 2 Prove that every convergent sequence is bounded.

Assignment 3 Use the Sum Rule for *null* sequences to prove the Sum Rule for sequences.

Assignment 4 Use the identity

$$(a_n - a)(b_n - b) + a(b_n - b) + b(a_n - a) = a_n b_n - ab$$

and the rules for *null* sequences to prove the Product Rule for sequences.

Assignment 5 Write a proof of the Quotient Rule.

Assignment 6 Find the limit of each of the sequences defined below.

1. $\frac{7n^2+8}{4n^2-3n}$
2. $\frac{2^n+1}{2^n-1}$
3. $\frac{\left(\sqrt{n}+3\right)\left(\sqrt{n}-2\right)}{4\sqrt{n}-5n}$
4. $\frac{1+2+\cdots+n}{n^2}$

Assignment 7 Prove the Sandwich Theorem for sequences.

Assignment 8 Prove that if a sequence is eventually bounded, then it is bounded.

Assignment 9 Suppose $(a_n) \to a$. Prove that if $a_n \geq 0$ for all n then $a \geq 0$.

Assignment 10 Prove or disprove the following statement:

"Suppose $(a_n) \to a$. If $a_n > 0$ for all n then $a > 0$."

Assignment 11 Suppose $(a_n) \to a$ and $(b_n) \to b$. Prove that if (eventually) $a_n \leq b_n$ then $a \leq b$.

Assignment 12 Let $(a_n) = (n^2)$. Write down the first four terms of the three subsequences (a_{n+4}), (a_{3n-1}) and (a_{2^n}).

Assignment 13 Suppose we have a sequence (a_n) and are trying to prove that it converges. Assume that we have have shown that the subsequences (a_{2n}) and (a_{2n+1}) both converge to the same limit a. Prove that $(a_n) \to a$ converges.

Assignment 14 Prove that every sequence has a monotonic subsequence.

Assignment 15 Consider a sequence (a_n) where $a_0 = 1$ and $a_{n+1} = \sqrt{a_n + 2}$. Show that $2 - a_{n+1} = \frac{2-a_n}{2+\sqrt{2+a_n}}$. Use this identity and induction to show that $(2 - a_n) \leq \frac{1}{(2+\sqrt{3})^n}$ for all n. How many iterations are needed so that a_n is within 10^{-100} is its limit 2?

Assignment 16 Use induction to show that $1 \leq a_n \leq 2$ for all n. Assuming that (a_n) converges, show that the limit must be $\sqrt{2}$.

Assignment 17

Show that $(a_{n+1} - \sqrt{2}) = \frac{(a_n - \sqrt{2})^2}{2a_n}$. Using this identity show by induction that $|a_n - \sqrt{2}| \leq \frac{1}{2^{2^n}}$. How many iterations do you need before you can guarentee to calculate $\sqrt{2}$ to within an error of 10^{-100}?