Name: .... Supervisor: ....

Class Teacher: ....

MA131 - Analysis 1
Workbook 2 Assignments

Due in 13th Oct

Assignment 1 Test whether each of the sequences defined below has any of the following properties: increasing; strictly increasing; decreasing; strictly decreasing; non-monotonic.

1. $a_n = -\frac{1}{n}$	2. $a_{2n-1} = n, a_{2n} = n$	3. $a_n = 1$
4. $a_n = 2^{-n}$	5. $a_n = \sqrt{n+1} - \sqrt{n}$	6. $a_n = \sin n$

Assignment 2 Decide whether each of the sequences defined below is bounded above, bounded below, bounded. If it is none of these things then explain why. Identify upper and lower bounds in the cases where they exist.

1. $\frac{(-1)^n}{n}$	2. $\sqrt{n}$	3. $a_n = 1$
4. $\sin n$	5. $\sqrt{n+1} - \sqrt{n}$	6. $(-1)^n n$

Assignment 3 When does the sequence $(\sqrt{n})$ eventually exceed 2, 12 and 1000? Then prove that $(\sqrt{n})$ tends to infinity.

Assignment 4 Think of examples to show that:

1. an increasing sequence need not tend to infinity;
2. a sequence that tends to infinity need not be increasing;
3. a sequence with no upper bound need not tend to infinity.

Assignment 5 Prove that the sequence $\left(\frac{1}{\sqrt{n}}\right)$ tends to zero.

Assignment 6 Prove that the sequence $(1, 1, 1, 1, 1, 1, 1, \dots)$ does *not* tend to zero. (Find a value of ε for which there is no corresponding N.)

Assignment 7 Prove that if $(a_n) \to \infty$ then $\left(\frac{1}{a_n}\right) \to 0$.

Assignment 8 Think of an example to show that the statement,

$$\text{if } (a_n) \to 0 \text{ then } \left(\frac{1}{a_n}\right) \to \infty,$$

is *false*, even if $a_n \neq 0$ for all n.

Assignment 9 Prove that if (a_n) is a null sequence and $0 \leq b_n \leq a_n$ then (b_n) is a null sequence. Now combine this with the Absolute Value Rule to construct a proof of the Sandwich Theorem, assuming that $0 \leq |b_n| \leq a_n$ for all n.

Assignment 10 Prove that the following sequences are null. Indicate what null sequence you are using to make your Sandwich.

$$1. \ \left(\frac{\sin n}{n}\right) \qquad 2. \ \left(\sqrt{n+1} - \sqrt{n}\right)$$

Assignment 11 Write a proof of the Sum Rule.

Assignment 12 Write a proof of the Product Rule.

Assignment 13 Let a_n and A_n be such that $a_4 = 2$, $A_4 = 4$, and

$$a_{2n} = \sqrt{a_n A_n} \qquad A_{2n} = \frac{2A_n a_{2n}}{A_n + a_{2n}} .$$

Why is the sequence $a_4, a_8, a_{16}, a_{32}, \ldots$ increasing? Why are all the values between 2 and π? What similar statements can you make about the sequence $A_4, A_8, A_{16}, A_{32}, \ldots$?

Assignment 14 Explain why $\left(\frac{a_{2n}\sqrt{A_n}}{(A_n+a_{2n})(\sqrt{A_n}+\sqrt{a_n})}\right)$ is never larger than 0.4. Hence show that the error $(A_n - a_n)$ in calculating π reduces by at least 0.4 when replacing n by $2n$. Show that by calculating $A_{2^{10}}$ and $a_{2^{10}}$ we can estimate π to within 0.0014.