Lectures: Mon 16-17 (MS.04)
Tue 13-14 (MS.04)
Thu 11-12 (MS.05)
(Ex: Fri 9-10 (MS.03))

Dynamical Systems

Question: What do we mean by a "dynamical system"?

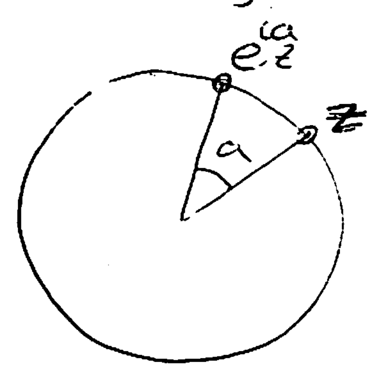
- Basically, something that evolves with time. It might be a solution $\omega(t)$ ($t \in \mathbb{R}$) to a differential equation, e.g.,
$$\omega''(t) + c\,\omega(t) = 0$$
- More generally, consider maps $T : X \to X$ ("snapshots") on a set X, e.g.,

(i) Rotations on a circle K

Write $K = \{ z \in \mathbb{C} \mid |z| = 1 \}$
$$= \{ e^{i\theta} \mid 0 \le \theta < 2\pi \}$$

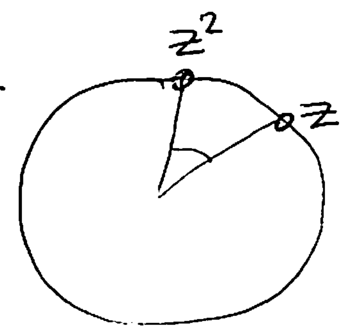
Fix $0 \le a < 2\pi$

Consider $\begin{cases} T : K \to K \\ T : z \mapsto e^{ia} z \end{cases}$



(ii) Doubling map on a circle K

Consider $\begin{cases} S : K \to K \\ S : z \mapsto z^2 \end{cases}$



Question: What can we say about typical dynamical systems?

We would like to understand what happens as we continue to iterate the transformation

$$\begin{cases} T : X \to X \\ T^2 = T \circ T : X \to X \\ T^3 = T \circ T \circ T : X \to X \\ \vdots \\ T^n = T^{n-1} \circ T : X \to X \end{cases}$$

Can we describe either:

- The behaviour of individual orbits; or
- The global behaviour of the dynamical system?

For example, rotations of the circle are more "deterministic": T^n rotates every point by e^{ina}. However the doubling map is more "chaotic": nearby points move apart at an exponential rate (i.e. 'sensitive dependence on initial conditions').

Broadly speaking, we have 2 aims

Aim I: To understand "complicated" examples by using simpler models;

Aim II: To describe general features of the dynamical systems, and to understan[d] them both quantatively and qualatively.

Important names in area.

H. Poincaré (1854 - 1912)

Denjoy (1884 - 1974)

D. Anosov (1936 -)

S. Smale (1930 -)

Sinai (1935 -)

Useful Books.

B. Hasselblatt + A. Katok, "A first course in Dynami[cs]" (Cambridge University Press).

Devaney, "Chaotic Dyncemical Systems" (Benjamin/Cummings).

In addition: we would like to use the ideas from the area to study problems in other areas: eg

a) Number Theory

Using dynamical systems one can give a proof of:

Van der Waerden's Theorem: Let $B_1 \cup \ldots \cup B_N = \mathbb{N}$ be any partition of the natural numbers. (ie, $B_i \cap B_j = \phi$ for $i \neq j$). Then one of these sets B_i contains arithmetic progressions of arbitrary length

(ie, $\forall k \geq 1, \exists l, m$ such that $l + am \in B_i$ for $a = 0, \ldots, k$)

b) Geometry: Consider a linkage, ie, a collection of rods in the plane which have either fixed or moving pivots at each end.

Hunt-Mackay: There exist configurations with "chaotic" motions.