

# ERRATA FOR "DYNAMICAL SYSTEMS AND ERGODIC THEORY"

## PRELIMINARIES

Theorem G should read:

**Theorem G (Radon-Nikodym).** *If  $\mu$  is absolutely continuous with respect to  $\nu$  then there exists a (unique) function  $f \in L^1(X, \mathcal{B}, d\nu)$  such that for any  $A \in \mathcal{B}$  we can write  $\mu(A) = \int_A f d\nu$ .*

## CHAPTER I

**page 1, Example 1:** It should read  $0 < \theta < 1$  (Tex error).

**page 2, line 2:**  $y = (y_n)_{n \in \mathbb{Z}}$  instead of  $x = (x_n)_{n \in \mathbb{Z}}$ , again.

**page 2, Proof of Lemma 1.1:** The superscript  $ks$  should become  $ms$ , say (Too many  $ks$ )

**page 3, Example 1:** It should read " $T^k x = \sum_{n=0}^{+\infty} \frac{x_{n+k-1}}{2^{n+1}} \pmod{1}$ ."

**page 7, Theorem 1.8:** The sets in  $\mathcal{E}$  should also be closed.

## CHAPTER 2

**page 12, Proof of Theorem 1.8:** We should define  $T_i := T^i = (\sigma \circ \dots \circ \sigma)$

**page 13, line 2:** It should read  $(T^b T_i^a x)_0 = x_{b+ia} = z_a \in \{1, \dots, k\}$

**page 13, bottom:** It should read "In particular,  $d_{\mathcal{X}_N}(S^{n_i} \underline{z}', z) \rightarrow 0$  as  $n_i \rightarrow +\infty$  where  $\underline{z} = (z, \dots, z)$ ,  $\underline{z}' = (T_N^{-n_i} z, \dots, T_N^{-n_i} z) \in \mathcal{D}_N$ ."

**page 14-15, Sublemma 2.2.4 (and proof):** Each  $\mathcal{D}$  is really  $\mathcal{D}_N$  (subscripts omitted)

**page 15, mid-page:** It should read "The proof of Theorem 2.3 is finished".

**page 16, after (2.3):** It should read:  $d_{\mathcal{D}_N}(S^n z, z') < \delta$  ( $S^n$  for  $T^n$ ).

**page 16, displayed equation (2.3):** It should read  $d_{\mathcal{X}_N}(\hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z, \hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z') < \frac{\epsilon}{4}$ . (no negative powers.)

**page 16, displayed equation (2.4):**  $d_{\mathcal{X}_N}(S^n(\hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z), \hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z') < \frac{\epsilon}{4}$ . ( $S^n$  for  $T^n$ )

**page 16, last displayed equation :**

$$\begin{aligned} d_{\mathcal{X}_N}(S^n y, x) &\leq d_{\mathcal{X}_N}(S^n y, \hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z') + d_{\mathcal{X}_N}(\hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z', x) \\ &\quad + d_{\mathcal{X}_N}(\hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z, \hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z') \\ &\quad + d_{\mathcal{X}_N}(\hat{T}^{n_{1j}} \circ \dots \circ \hat{T}^{n_{Nj}} z, x) \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{4} + \frac{\epsilon}{4} = \epsilon. \end{aligned}$$

(term missing)

**page 17, after (2.5):** Another  $\mathcal{D}$  which should be a  $\mathcal{D}_N$  (subscript omitted).

**Comments and references:** It should read "An account also appears in [5]."

### CHAPTER 3

**page 19, last displayed equation:** There is a  $\cap$  missing.

**page 22, first paragraph of proof:** It should read : This means that

$$N(\bigvee_{i=0}^{k-1} T^{-i}(\bigvee_{n=0}^{N-1} T^{-n} \alpha)) \geq N(\bigvee_{i=0}^{k-1} T^{-i} \beta), \quad \text{for } k \geq 1$$

**page 22, Definition:** It should read The matrix  $A$  is called *aperiodic* if  $\exists N > 0$ ,  $\forall 1 \leq i, j \leq k$ ,  $A^N(i, j) \geq 1$ . (Power of  $N$  missing)

**page 22, mid-page.** The displayed equation should read

$$\bigvee_{n=-N}^N \sigma^{-n} \alpha = \{[i_{-N}, \dots, i_0, \dots, i_N]_{-N}^N : i_{-N}, \dots, i_0, \dots, i_N \in \{1, \dots, k\}\}$$

and several  $\sigma^{-i} \alpha$  should change to  $\sigma^{-n} \alpha$ .

**page 23, mid-page.** It should read  $A(v_1, \dots, v_k)^T = (v'_1, \dots, v'_k)^T = (\sum_{i=1}^k A(i, 1)v_i, \dots, \sum_{i=1}^k A(i, k)v_i)^T$  (the transpose  $\cdot^T$  is omitted)

**page 24, end of proof.** It should read: However, elementary calculus shows that the supremum is realised where  $z = (\frac{bd}{ac})^{1/2}$  and thus

$$C < \frac{\nu^{1/2} - \nu^{-1/2}}{\nu^{1/2} + \nu^{-1/2}} < 1,$$

where  $\nu := \frac{ad}{bc} > 1$ . **page 24, bottom of page.** It should read "a contraction with respect to this metric". (The error seems to appear in the source article: G. Birkhoff, *Extensions of Jentzsch's theorem*, Trans. Amer. Math. Soc., 85 (1957) 219-227)

**page 25, second paragraph.** It should read "  $C < 1$  is independent of the choices  $x, y$ ."

**page 25, last displayed equation.** It should read

$$H(\bigvee_{n=0}^{N-1} \sigma^{-n} \alpha) = \log \text{Card}(\bigvee_{n=0}^{N-1} \sigma^{-n} \alpha) = \dots$$

**page 26, near top.** It should read " ... Jordan block matrices  $B_2, \dots, B_l, \dots$ "

**page 26, Lemma 3.6 (i)** It should read:  $s(n, \epsilon') \geq s(n, \epsilon)$  and  $r(n, \epsilon') \geq r(n, \epsilon)$ .

**page 27, near top** For  $r(n, \epsilon) \geq s(n, \epsilon)$  read  $s(n, \epsilon) \geq r(n, \epsilon)$ .

**page 28, near the bottom** It should read:  $\limsup_{n \rightarrow +\infty} \frac{1}{n} \log N(\bigvee_{i=0}^{n-1} T^{-i} \beta)$  (by Lemma 3.7 (ii))(  $H$  instead of  $N$ )

**page 30, near the top** It should read:  $r_{T^m}(n, \epsilon) \leq r_T(nm, \epsilon)$  ( $r$  missing).

**page 30, middle of page.** It should read  $R$  is also an  $(n, \epsilon)$ -spanning set for  $T^m$  ( $T^m$  not  $T^n$ ).

## CHAPTER 4

**page 33, Proof of Lemma 4.3** It should read "Let  $J_2 = [a, b]$ "

**page 35, Sublemma 4.4.1** It should read:

- (ii)  $T^{n-1}(I_{n-1}) = J'$ ; and
- (iii)  $T^n(I_{n-1}) \supset J''$ .

**page 36, second paragraph.** The two  $J'$  should be  $J''$ .

**page 37, Proof of Lemma 4.5.** The intervals of monotonicity for  $S_1 \circ S_2 : I \rightarrow I$  take the form  $J_j \cap S_2^{-1}(I_i)$ . Thus  $\mathcal{N}(S_1 \circ S_2) \leq \text{Card}\{(i, j) : J_j \cap S_2^{-1}(I_i) \neq \emptyset\} \leq N(S_1) \cdot N(S_2)$ .

**page 37, Remark.** The chain rule says:  $(T^n)'(x) = \prod_{i=0}^{n-1} T'(T^i x)$ .

**page 37, Proof of Theorem 4.6.** It should read:  $0 \leq r_i \leq n - 1$  and  $0 \leq r_i \leq n - 1$ .

**page 37, penultimate paragraph.** It is better to write:  $\{x_{i_0} < x_{i_1} < \dots < x_{i_{m-1}}\} \subset E_n$

**page 37, last paragraph.** It should read: there are at least  $\frac{m\epsilon}{C}$  intervals of monotonicity for  $T^r$ , i.e.  $\mathcal{N}(T^r) \geq \frac{m\epsilon}{C} \geq \frac{N}{Cn}\epsilon$ . Thus

$$\lim_{n \rightarrow +\infty} \frac{1}{n} \log \mathcal{N}(T^n) \geq \lim_{n \rightarrow +\infty} \frac{1}{n} (\log s(n, \epsilon) -$$

**page 38, after first displayed equation.**

It should read.

$$\begin{aligned} m \left( \lim_{n \rightarrow +\infty} \frac{1}{n} \log \mathcal{N}(T^n) \right) &= m \left( \limsup_{n \geq 1} \frac{1}{n} \log \mathcal{N}(T^n) \right) \\ &\geq \limsup_{k \geq 1} \frac{1}{k} \log \mathcal{N}(S^k) \\ &= \lim_{k \rightarrow \infty} \frac{1}{k} \log \mathcal{N}(S^k) \end{aligned}$$

(since  $\left( \frac{m \log \mathcal{N}(S^k)}{k} \right)_{k \in \mathbb{N}}$  is a sub-sequence of  $\left( \frac{\log \mathcal{N}(T^n)}{n} \right)_{n \in \mathbb{N}}$ . Moreover, we can estimate

$$\begin{aligned} &m \left( \lim_{n \rightarrow \infty} \frac{1}{n} \log \mathcal{N}(T^n) \right) \\ &\leq \lim_{n \rightarrow \infty} \frac{m}{n} \log \left( \mathcal{N}(S^{\lfloor \frac{n}{m} \rfloor}) \mathcal{N}(T^{n - \lfloor \frac{n}{m} \rfloor m}) \right) \quad (\text{by Lemma 4.5}) \\ &\leq \lim_{n \rightarrow \infty} \frac{m}{n} \log \left( \mathcal{N}(S^{\lfloor \frac{n}{m} \rfloor}) + \max_{0 \leq i \leq m-1} \{\log \mathcal{N}(T^i)\} \right) \\ &= \lim_{k \rightarrow \infty} \frac{1}{k} \log \mathcal{N}(S^k). \end{aligned}$$

**page 39, first displayed equation.** It should read

$$\geq \limsup_{k \rightarrow +\infty} \frac{1}{k} \log (\mathcal{N}(S^k)/3^k) \quad (\text{since } \mathcal{N}(S^k) \leq 3^k N(\vee_{r=0}^{k-1} S^{-r} \alpha))$$

**page 42, Proof of (i).** It should read for every point  $x$  in the complement of the dense set  $\cup_{n \in \mathbb{Z}^+} T^{-n} \{x_0, \dots, x_k\}$

**page 42, Proof of (ii).** It should read  $\pi(w) = z$  (twice) instead of  $\pi(w) = x$ .

**page 43, Proof of Theorem 4.9.** We actually use Proposition 3.11.

**page 43, Proof of sublemma 4.9.1.** We actually use Proposition 3.5.

**page 44, Proof of sublemma 4.10.1.** For  $\lambda_0$  read  $\lambda_1$ .

## CHAPTER 5

**page 47** We should use  $GL(2, \mathbb{Z})$  instead of  $SL(2, \mathbb{Z})$ .

**page 47** To give the broadest definition of hyperbolicity we should use

Let  $A \in GL(2, \mathbb{Z})$  have eigenvalues  $\lambda_1, \lambda_2$  then if  $|\lambda_1| > 1 > |\lambda_2| (= \frac{1}{|\lambda_1|})$  we call the matrix  $A$  *hyperbolic*

**page 49, below figure 5.2** It should read: (where  $(x_1, x_2) \in [0, 1) \times [0, 1)$ ).

**page 50, near the top** Delete the upper bound on  $k$  (For the "boxes" below to cover we should require  $k \leq \frac{4}{\epsilon^2 \sin \theta}$ , where  $\theta$  is the angle between the eigenvectors.)

**page 50, Next paragraph add** "We can choose boxes such that  $\text{Box}(x_1^i, x_2^i)$ ,  $i = 1, \dots, n$  cover the torus."

**page 50, Proof of Sublemma 5.3.1** Change the first line(s) to: "For any point  $(z_1, z_2) \in \mathbb{R}^2/\mathbb{Z}^2$  we can choose some  $i = 1, \dots, k$  such that  $(z_1, z_2) \in \text{Box}(x_1^i, x_2^i)$  and so ..."

**page 51, displayed equation (5.2)** Change to:  $\lim_{\epsilon \rightarrow 0} \lim_{n \rightarrow +\infty} \frac{1}{n} \log r(n, \epsilon) \leq \lim_{\epsilon \rightarrow 0} \lim_{n \rightarrow +\infty} \frac{1}{n} \log (k(2[|\lambda_1|^n] + 1))$  ■

**page 51, sublemma 5.3.2.** The statement should read: " $S$  is an  $(n, 2\epsilon/|\lambda_1|)$ -separated".

## CHAPTER 6

**page 59, proof of part (i).** In fact, it is the argument in part (ii).

**page 59, statement of Lemma 6.3.** We are assuming  $\rho$  is irrational.

**page 60, first line.** Assume  $n_1 > n_2$

**page 60, first paragraph.** There  $T$ s should be  $\hat{T}$  (i.e. hats missing).

**page 61, lower part of page.** It should read: "It is easy to see that if  $T: \mathbb{R}/\mathbb{Z} \rightarrow \mathbb{R}/\mathbb{Z}$  is  $C^2$  with  $|T''|/|T'|$  is bounded then  $\text{Var}(\log |T'|)$  is finite."

**page 62, Statement of Sublemma 6.5.1.** The displayed equation should be  $|(T^{q_n})'(x)| \cdot |(T^{-q_n})'(x)| \geq C$ .

**page 62, Displayed equation, mid-page.** It should read  $|I_{q_n}| + |I_{-q_n}|$  (misplaced minus sign).

**page 62, last line.** It should read  $|T^{q_n} x - x| = \delta_n$ .

**page 63, displayed equation (6.4).** The second line should read  $\geq$  instead of  $=$ .

## CHAPTER 7

**page 67, first displayed equation .** In the first line the upper limit in the sum should be  $N_n^{(n)} - 1$ ; the second line should be  $f_k(T^{N_n^{(n)}}x) + \sum_{n=0}^{N_n^{(n)}-1} f_k(T^n x) - f_k(x)$  (reverse signs).

**page 68, item (2).** It should read "for every non-empty open set  $U \subset X$ "

**page 68, mid-page.** It should read  $\mu([z_0, \dots, z_n]) = (\frac{1}{2})^{n+1}$  ( $n+1$  instead of  $n$ ).

**page 69, second line.** For  $(1, \dots, 1)$  read  $(1, \dots, 1)^T$  (i.e. the transpose).

## CHAPTER 8

**page 74, Lemma 8.1 .** Part (1) should read  $I(\alpha|\{\emptyset, X\})(x) = I(\alpha)(x)$ ;

Part(2) should read  $I(\alpha|\mathcal{A})(Tx) = I(T^{-1}\alpha|T^{-1}\mathcal{A})(x)$ .

**page 74, line before definition.** Delete the erroneous comment "(in particular,  $\hat{\alpha}$  is countable and consists of all unions of elements from  $\alpha$ )."

**page 75, last displayed equation.** It should read  $I(\alpha \vee \beta | \hat{\gamma})(x)$  (hat missing in  $\hat{\gamma}$ ).

**page 76, proof of lemma 8.3.** It should be corrected to:

"For part (4) we have

$$\begin{aligned} H(\alpha|\gamma) &= - \sum_{A \in \alpha, C \in \gamma} \mu(A \cap C) \log \left( \frac{\mu(A \cap C)}{\mu(C)} \right) \\ &= - \sum_{C \in \gamma} \mu(C) \left[ \sum_{A \in \alpha} \frac{\mu(A \cap C)}{\mu(C)} \log \left( \frac{\mu(A \cap C)}{\mu(C)} \right) \right] \\ &\leq - \sum_{A \in \alpha} \left[ \sum_{C \in \gamma} \mu(A \cap C) \right] \log \left[ \sum_{C \in \gamma} \mu(A \cap C) \right] \\ &\leq - \sum_{A \in \alpha} \mu(A) \log \mu(A) = H(\alpha) \end{aligned}$$

since for fixed  $A \in \alpha$  we can bound

$$- \sum_{C \in \gamma} \frac{\mu(A \cap C)}{\mu(C)} \log \left( \frac{\mu(A \cap C)}{\mu(C)} \right) \leq - \left[ \sum_{C \in \gamma} \mu(A \cap C) \right] \log \left[ \sum_{C \in \gamma} \mu(A \cap C) \right]$$

using concavity of  $t \mapsto -t \log t$ ."

## CHAPTER 9

**page 95, line 3 :** Replace  $X \times \mathbb{R}/\mathbb{Z}$  by  $X \times G$ .

**page 95, line 25 from below :**  $\mu(A) > 0$  should be  $0 < \mu(A) < \infty$ .

**page 96, line 5 from below :**  $\hat{T} : X \rightarrow X$  should be  $\hat{T} : \hat{X} \rightarrow \hat{X}$ .

**page 96, line 1 :** *natural extension* of  $X$  should be *natural extension* of  $T$ .

**page 96, line 2 :**  $\pi((x_n)_{n \in \mathbb{Z}^+}) = X$  should be  $\pi((x_n)_{n \in \mathbb{Z}^+}) = x_0$

## CHAPTER 10

**page 100, line 2 from below :**  $\int f(x)g(x)d\mu(x)$  should be removed.

**page 101, line 6 :**  $\epsilon \|g\|_2$  should be  $\sqrt{\epsilon} \|g\|_2$ .

**page 102, line 5 from below :** We assume  $\min(f_+, f_-) = 0$ .

**page 105, line 9 :** We assume ergodicity.

**page 106, lines 17-18 :** Replace  $2^k, r_k$  by  $2^n, r_n$  respectively.

**page 107 :** (ii) should be corrected as follows. (ii) there exists  $D > 0$  such that

$\sup_{x,y,z \in I_k} \frac{|T''(x)|}{|T'(y)T'(z)|} \leq D, \forall 0 < x < 1$ . Then  $|\frac{\psi'_{k_0 \dots k_n}(x)}{\psi'_{k_0 \dots k_n}(y)}|$  is bounded from above by

$$\prod_{i=0}^n \left( 1 + \frac{D}{4^{[(n-i-1)/2]}} |T^n x - T^n y| \right) \leq C.$$

We assume in (a) that  $l(E \cap I_{k_0} \cap T^{-1}I_{k_1} \cap \dots \cap T^{-n}I_{k_n}) > 0$ .

**page 109, line 18 :** Replace "Absolutely continuous" by "nonsingular".

**page 111, line 19 from below :** Replace "sublemmas 10.5.1 and 10.5.2" by "sublemmas 10.2.1 and 10.2.2."

## CHAPTER 11

**page 115 : Lemma 11.4 :** " a bounded sequence of real numbers  $\{a_n\}$  " should read " a bounded sequence of positive numbers  $\{a_n\}$ . "

**page 115, line 3 from below :**  $N_k, N_{k+1}$  should be  $n_k, n_{k+1}$  respectively.

**page 117, line 6 from below :** " (11-1), (11.2) and (11.3) " should be " (11-2), (11.3) and (11.4) "

**page 118, line 3 from below :** It should read  $n \in \mathbb{Z}$ .

**page 119, line 1 :** It should read  $n \in \mathbb{Z}$ .

**page 120** All  $C^\perp$  should be  $\mathbb{C}^\perp$ .

**120, Proof of Proposition 11.8 :**  $\bar{\mu}$  denotes the spectral measure.

**page 121,(2)  $\implies$  (3):** All  $n_i$  should be  $n_k$ .

**page 121, line 8 from below :**  $\mu(T^{-n_i} B_i \cap D_j)$  should be removed.

**page 123, line 9 :** It should read " (by writing  $P$  in terms of Jordan forms) "

**page 123, line 7 from below :** It should read  $\mu(A \cap T^{-(n+l)} B) \rightarrow \mu(A)\mu(B)$  as  $n \rightarrow +\infty$ .

## CHAPTER 12

**page 125, line 8 from below :** It should read "  $\{k_i \circ T^n\}_{i=0}^{j_n}$  for  $L^2(T^{-n}\mathcal{B}) \ominus L^2(T^{-(n+1)}\mathcal{B})$ . It follows that  $\{k_i \circ T^n\}_{i=0}^{j_n} \xrightarrow{\infty} \dots$  "

**page 126, line 4 :** Example 1 should be one-sided aperiodic Markov shifts (aperiodicity is missing).

**page 127, line 4 :**  $\int_A f(x)dx$  should be  $\int_{T^{-1}A} f(x)dx$ .

**page 128, line 6 :**  $K \exp\left(|x-y| \frac{D'}{1-\frac{1}{\beta}}\right)$  should be  $K\left(|x-y| \frac{D'}{1-\frac{1}{\beta}}\right)$

**page 129, line 15 :** It should read " we can choose a finite disjoint set of cylinders  $\{I_{\underline{j}} : \underline{j} = (j_1, \dots, j_l)\}$ , with  $\mu\left(\left(\cup_{\underline{j}} I_{\underline{j}}\right) \Delta A\right) < \epsilon$ . "

**page 129 line 2 from below :** It should read

$$\sup_{x,y \in T^n I_{\underline{i}}} \frac{|\psi'_{\underline{i}}(x)|}{|\psi'_{\underline{i}}(y)|} \leq C. \quad (12.2)$$

**page 130, Sublemma 12.5.1.** The statement should read "There exist  $S > 0$  and a subset  $I'$  of  $T^l(I_{\underline{j}})$  which is a finite disjoint union of elements of  $\bigvee_{i=0}^{S-1} T^{-i}\{I_1 \dots I_k\}$  and satisfies  $T^S(I') = I$ . " In the proof,  $T^{s_i} U_i \supset T^{s_i} I_{m_1, \dots, m_{s_j}}^{(i,j)}$  should be  $T^{s_j} U_i \supset T^{s_j} I_{m_1, \dots, m_{s_j}}^{(i,j)}$ .

**page 131, Proof of Proposition 12.6 :** All  $I_{tm_1 \dots m_{s_i}}^{(i)}$  should be  $I_{m_1 \dots m_{s_i}}^{(i)}$ . Relace  $\psi_{tm_1 \dots m_{s_i} h_1 \dots h_t}(x)$  by  $\psi_{m_1 \dots m_{s_i}}(x)$ . "Here we take  $t = S - s_i$  " on the line 17 should be put before " Let  $l = l(\epsilon)$  " on the line 15.

**page 132, lines 5-6 :** It should read "  $\mathcal{L}^*(\mu) = \mu$ , i.e. the dual operator  $\hat{\mathcal{L}}^*$  acting on measures (defined by  $(\mathcal{L}^* \mu)(A) = \int \mathcal{L} \chi_A d\mu$ ) fixes  $\mu$ . "

**page 124, line 7 :** it should read " Channon-McMillan- Breiman theorem ".

**page 134, line 14 :** It should read  $\alpha_n = \bigvee_{i=0}^{n-1} T^{-i} \alpha$ .

**page 135 line 1 from below :**  $I(\alpha | \bigvee_{j=1}^{n-i} T^{-j} \alpha) T^i$  should be replaced by  $I(\alpha | \bigvee_{j=1}^{n-1-i} T^{-j} \alpha) T^i$ .

**page 135-136 :**

Define

$$F_m(x) =: |I(\alpha | \bigvee_{j=1}^m T^{-j} \alpha)(x) - I(\alpha | \bigvee_{j=1}^{\infty} T^{-j} \alpha)(x)|.$$

Then we see that

$$\frac{1}{n} \left| I(\bigvee_{i=0}^{n-1} T^{-i} \alpha) - \sum_{i=0}^{n-1} f T^i \right| \leq \frac{1}{n} \sum_{j=0}^{n-1} F_{n-j} T^j(x).$$

It follows from Corollary 1.2 in page 96 in [Mane's book] that if  $F_n$  is a sequence that converges to 0 almost everywhere and in  $L^1$  then  $\frac{1}{n} \sum_{j=0}^{n-1} F_{n-j} T^j(x)$  converges to 0 almost everywhere and in  $L^1$ .

#### CHAPTER 14

**page 147, line 12 :** It should read " a compact metric space. "

**page 147, line 12 from below :** It should read " a finite Borel measurable partition "

**page 148, line 7 :** Sub-lemma 14.1 should be Sub-lemma 14.7.

**page 148, line 2 from below :** It should read

$$" h_{\mu}(T^k, \bigvee_{i=0}^{k-1} T^{-i} \alpha) = \lim_{n \rightarrow +\infty} \frac{1}{n} H_{\mu}(\bigvee_{i=0}^{n-1} T^{-ik} (\bigvee_{j=0}^{k-1} T^{-j} \alpha)) "$$

**page 149, line 8 :** It should read

$$" h_{\mu}(T^k) \geq h_{\mu}(T^k, \bigvee_{i=0}^{k-1} T^{-i} \alpha) \geq k h_{\mu}(T, \alpha) \geq k h_{\mu}(T) - k \epsilon. "$$

**page 151, line 2 :** It should read " $C \in T^{-l} \alpha^{(N)}$  with  $(T^l)^* \nu_{n_i}(D) := \nu_{n_i}(T^{-l} D) = \nu_{n_i}(C)$  and  $C = T^{-l} D$ ."

**page 151, line 5 :** It should read :

$$" N \log(s(n_i, \epsilon)) \leq \sum_{l=0}^{n_i-1} \left( - \sum_{D \in \alpha^{(N)}} (T^l)^* \nu_{n_i}(D) \log((T^l)^* \nu_{n_i}(D)) \right) + 2N^2 \log k "$$

**page 151, line 9 from below :** It should read

$$" \leq -\frac{1}{N} \sum_{C \in \alpha^{(N)}} \mu_{n_i}(C) \log \mu_{n_i}(C) + \frac{2N \log k}{n_i}, "$$

**page 151, line 5 to page 152, line 2 :** All  $H_{\nu}$  should be  $H_{\mu}$ .

#### CHAPTER 15

**page 154, (3) :** It should read

$$" \int f(x+a) d\mu = \lim_{n \rightarrow +\infty} \int E(f(x+a) | S^{-n} \mathcal{B}) d\mu = \lim_{n \rightarrow +\infty} \int E(f(x) | S^{-n} \mathcal{B}) d\mu = \int f(x) d\mu "$$

and thus we know that  $\mu$  is the Haar-Lebesgue measure. "

**page 154, (4) :** It should read

$$"T'(Sx) \cdot S'(x) = (TS)'(x) = (ST)'(x) = S'(Tx) \cdot T'(x)."$$

**page 155, line 6 :** it should read " $E(\cdot|T^{-1}\mathcal{B})(x) : L^2(X, \mathcal{B}, \mu) \rightarrow L^2(X, \mathcal{B}, \mu)$  is an orthogonal projection which is a contraction "

**page 155, line 7 from below :** It should read " $S^{n'}$  and  $S'$  are  $\mathcal{A}_n$ -measurable "

**Proof 157 :** It should read : " Thus by subadditivity the limit  $h(T|\mathcal{A}) := \lim_{n \rightarrow +\infty} \frac{1}{n} H(\bigvee_{i=0}^{n-1} T^{-i}\gamma|\mathcal{A})$  exists.

By the basic equalities for entropy we see that

$$\begin{aligned} h(T|\mathcal{A}) &= \lim_{n \rightarrow +\infty} \frac{1}{n} H(\bigvee_{i=0}^{n-1} T^{-i}\gamma|\mathcal{A}) \\ &= \lim_{n \rightarrow +\infty} \frac{1}{n} \left( H(\bigvee_{i=0}^{n-2} T^{-i}\gamma|\mathcal{A}) + H(T^{-(n-1)}\gamma|\mathcal{A} \vee (\bigvee_{i=0}^{n-2} T^{-i}\gamma)) \right) \\ &= h(T) \end{aligned}$$

since

$$h(T) = \lim_{n \rightarrow +\infty} \frac{1}{n} (H(\bigvee_{i=0}^{n-2} T^{-i}\gamma))$$

and

$$H(T^{-(n-1)}\gamma|\mathcal{A} \vee (\bigvee_{i=0}^{n-2} T^{-i}\gamma)) \leq H(T^{-(n-1)}\gamma|\mathcal{A}) = H(\gamma|\mathcal{A}) < +\infty.$$

It follows from Sub-lemma 15.1.3 that  $h(T|\mathcal{A}) = \frac{\log 3}{\log 2} h(S|\mathcal{A})$  and so we have that the following limit exists

$$h(S|\mathcal{A}) := \lim_{n \rightarrow +\infty} \frac{1}{n} H(\bigvee_{i=0}^{s_n-1} T^{-i}\gamma|\mathcal{A}).$$

Observe that if we replace  $\mathcal{A}$  by the trivial sigma-algebra then the same argument gives that  $h(T) = \frac{\log 3}{\log 2} h(S)$ . Comparing these identities we see that  $h(S) = h(S|\mathcal{A})$ . ■

"

**page 158 :** In Sub-lemma 15.1.5 and in the proof, all  $\mathcal{B}, \beta$  should be  $\gamma$ .