Solution sheet 3

Dynamical Systems (MA424)

1. For $x, y \in [\alpha_0, \ldots, \alpha_{n-1}]$ we have that $x_i = y_i \,(= \alpha_i)$ for $i = 0, \ldots, n-1$. In particular, $d'_\lambda(x,y) \le (N-1)\sum_{i=n}^{\infty} \lambda^{-i} = \frac{(N-1)\lambda^{-n}}{1-\lambda^{-1}} = \lambda^{1-n}\left(\frac{N-1}{\lambda-1}\right) < \lambda^{1-n}$. Conversely, if $d'_\lambda(x,y) < \lambda^{1-n}$ then $x_i = y_i$ for $i = 0, \ldots, n-1$, i.e., they lie in the same n-cylinder.

2. By definition, $x, y \in [\alpha_0, \ldots, \alpha_{n-1}] \iff d''_\lambda(x,y) = \lambda^{-n}$. Thus $y \in B_{d''_\lambda}(x, \lambda^{-n})$.

3. Since $|d'_\lambda| \le \frac{N-1}{1-\lambda^{-1}}$ and $|d''_\lambda| \le 1$ we can choose the corresponding sensitivity constant Δ to be any smaller value (i.e., given a sequence x we can always choose a nearby sequence whose distance eventually differs by this constant.)

4. We can explicitly $\pi : \Sigma_n \to \Sigma_m$ by $\pi(x_i) = (y_i)$ where
$$y_i = \begin{cases} x_i & \text{if } x_i \in \{1, \ldots, m\} \\ m & \text{if } x_i \in \{m+1, \ldots, n\} \end{cases}$$
This is easily seen to be a semi-conjugacy.

5. We explicitly define $\pi : \Sigma_A \to \Sigma_{A'}$ by $\pi(x_i) = (y_i)$ where
$$y_i = \begin{matrix} 1 & \text{if } x_i = 1 \text{ and } x_{i+1} = 2 \\ 2 & \text{if } x_i = 2 \text{ and } x_{i+1} = 1 \\ 3 & \text{if } x_i = 1 \text{ and } x_{i+1} = 1. \end{matrix}$$