Example Sheet 4

Dynamical Systems (MA424)

1. Construct a Markov partition for the linear hyperbolic toral automorphism associated to the matrix $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$.

2. Show that any hyperbolic toral has a Markov Partition (Hint: Extend the lines given by the eigenvectors and use these to construct the edges of the Markov Partition)

3. Let Σ be a subshift of finite type, and let $\pi : \Sigma \to \mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ be the associated semi-conjugacy, associated to a hyperbolic toral automorphism.
   i) Find an upper bound on the number of preimages $\pi^{-1}(x)$.
   ii) Describe the set of points $x \in \mathbb{T}^2$ such that $\pi^{-1}(x)$ is a single sequence in Σ. Is it open? Is it residual? Is it dense?

4. Let $T : \mathbb{T}^2 \to \mathbb{T}^2$ be a linear hyperbolic toral automorphism and let $\pi : \Sigma \to \mathbb{T}^2$ be the semiconjugacy in Question 2. How many of the Periodic points for T have more than one Preimage under π.

5. Let $T : \mathbb{T}^n \to \mathbb{T}^2$ be a hyperbolic toral automorphism (on $\mathbb{T}^n = \mathbb{R}^n/\mathbb{Z}^n$). Describe the periodic points.