

# Dynamical Systems (MA424) Example Sheet 2

1. Is  $F(x) = x + \frac{1}{2} \sin(x)$  the lift of a circle homeomorphism?
2. Is  $F(x) = x + \frac{1}{4\pi} \sin(2\pi x)$  the lift of a circle homeomorphism?
3. Let  $F(x) = x + c + b \sin(2\pi x)$ . Show that if  $|2\pi b| < 1$  then this map is orientation preserving. If  $|c| < |b|$  show that  $\rho(F) = 0$  for the corresponding map  $f: K \rightarrow K$ .
4. Let  $c_1, c_2 \in (0, \frac{1}{2\pi})$ , then show that  $F_1(x) = x + c_1 \sin^2(2\pi x)$  and  $F_2(x) = x - c_2 \cos^2(2\pi x)$  are lifts of circle maps with the same rotation number.
5. Show that  $w: [0, 1] \rightarrow \mathbb{R}$  defined by
$$w(x) = \begin{cases} 0 & \text{if } x = 0 \\ x \sin(1/x) & \text{if } x \neq 0 \end{cases}$$
is not a function of bounded variation.
6. Show that  $w: [0, 1] \rightarrow \mathbb{R}$  defined by
$$w(x) = \begin{cases} 0 & \text{if } x = 0 \\ x^2 \sin(1/x) & \text{if } x \neq 0 \end{cases}$$
is a function of bounded variation.