Canonical 3-folds complete intersections with only Veronese cone singularities

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(notes dating back to 1978)

Assume X is a canonical 3-folds having N Veronese cone singularities $\frac{1}{2}(1,1,1)$ as its only singularities. Write p_g, K^3 as usual. RR says

$$\chi(\mathcal{O}_X(nK_X)) = \frac{(2n-1)n(n-1)}{6}\frac{K^3}{2} - (2n-1)\chi(\mathcal{O}_X) + \begin{cases} +\frac{N}{8}.n & \text{if } n \text{ is even,} \\ +\frac{N}{8}.(n-1) & \text{if } n \text{ is odd.} \end{cases}$$

Therefore $K_X^3 \in \frac{1}{2}\mathbb{Z}$ and $\frac{K^3}{2} + \frac{N}{4} \in \mathbb{Z}$. Vanishing gives

$$h^0(nK_X) = \chi(nK_X) \quad \text{for } n \geq 2.$$

This written as a conjecture in 1978 and is standard now; the same notes conjectured falsely and repeatedly that the index of 3-fold canonical singularities is always ≤ 2.

Because I'm looking for complete intersections, I assume that $H^i(\mathcal{O}_X) = 0$ for $i = 1, 2$, so that $\chi(\mathcal{O}_X) = 1 - p_g$.

$p_g = 0$	$K^3 = 1/2$	$N = 27$	$V_{6^3} \subset \mathbb{P}(2^4, 3^3)$
$p_g = 1$	$K^3 = 1/2$	$N = 15$	$V_{6,10} \subset \mathbb{P}(1, 2^3, 3, 5)$
$p_g = 1$	$K^3 = 1$	$N = 18$	$V_{4,6,6} \subset \mathbb{P}(1, 2^4, 3, 3)$
$p_g = 2$	$K^3 = 1/2$	$N = 3$	$V_{12} \subset \mathbb{P}(1, 1, 2, 3, 4)$
$p_g = 2$	$K^3 = 1/2$	$N = 7$	$V_{14} \subset \mathbb{P}(1, 1, 2, 2, 7)$
$p_g = 2$	$K^3 = 1$	$N = 10$	$V_{4,10} \subset \mathbb{P}(1, 1, 2^3, 5)$
$p_g = 2$	$K^3 = 3/2$	$N = 9$	$V_{6,6} \subset \mathbb{P}(1, 1, 2^3, 3)$
$p_g = 2$	$K^3 = 2$	$N = 12$	$V_{4,4,6} \subset \mathbb{P}(1, 1, 2^4, 3)$
$p_g = 3$	$K^3 = 1$	$N = 2$	$V_{12} \subset \mathbb{P}(1^3, 2, 6)$
$p_g = 3$	$K^3 = 3/2$	$N = 1$	$V_9 \subset \mathbb{P}(1^3, 2, 3)$
$p_g = 3$	$K^3 = 3/2$	$N = 5$	$V_{3,10} \subset \mathbb{P}(1^3, 2, 2, 5)$
$p_g = 3$	$K^3 = 2$	$N = 0$	$V_{6,6} \subset \mathbb{P}(1^3, 2, 3, 3)$
$p_g = 3$	$K^3 = 2$	$N = 4$	$V_8 \subset \mathbb{P}(1^3, 2, 2)$
$p_g = 3$	$K^3 = 5/2$	$N = 3$	$V_{5,6} \subset \mathbb{P}(1^3, 2, 2, 3)$
$p_g = 3$	$K^3 = 3$	$N = 6$	$V_{4,6} \subset \mathbb{P}(1^3, 2^3)$
$p_g = 3$	$K^3 = 4$	$N = 8$	$V_{4,4,4} \subset \mathbb{P}(1^3, 2^4)$
$p_g = 4$	$K^3 = 2$	$N = 0$	$V_{10} \subset \mathbb{P}(1^4, 5)$
$p_g = 4$	$K^3 = 7/2$	$N = 1$	$V_7 \subset \mathbb{P}(1^4, 2)$
$p_g = 4$	$K^3 = 4$	$N = 0$	$V_{4,6} \subset \mathbb{P}(1^4, 2, 3)$
$p_g = 4$	$K^3 = 9/2$	$N = 3$	$V_{3,6} \subset \mathbb{P}(1^4, 2, 2)$
$p_g = 4$	$K^3 = 5$	$N = 2$	$V_{4,5} \subset \mathbb{P}(1^4, 2, 2)$
$p_g = 4$	$K^3 = 6$	$N = 4$	$V_{3,4,4} \subset \mathbb{P}(1^4, 2^3)$
$p_g = 5$	$K^3 = 4$	$N = 0$	$V_{2,8} \subset \mathbb{P}(1^5, 2)$
$p_g = 5$	$K^3 = 6$	$N = 0$	$V_6 \subset \mathbb{P}(1^5)$
$p_g = 5$	$K^3 = 15/2$	$N = 1$	$V_{3,5} \subset \mathbb{P}(1^5, 2)$
$p_g = 5$	$K^3 = 8$	$N = 0$	$V_{4,4} \subset \mathbb{P}(1^5, 2)$
$p_g = 5$	$K^3 = 9$	$N = 2$	$V_{3,3,4} \subset \mathbb{P}(1^5, 2, 2)$

Table 1: Canonical 3-folds with at worst Veronese cone singularities