2 Assignment 2. Math Course Dynamical Systems 99-00

One of the following two quesitons will be marked for credit. They both have to be handed in by Thursday 26 October, 11am in the filing cabinet outside general office (in the Atrium).

Question 1: Prove that a Morse-Smale diffeomorphism which reverses orientation is structurally stable.

Question 2: Show that if f and g are conjugate then their rotation numbers are the same. Use this and the theorem above, to show that a circle diffeomorphism with a irrational rotation number cannot be structurally stable.

Question 3: In the course it is shown (Theorem 1.8) that if f is a Morse-Smale map f satisfies $\rho(f) = 0$ then f is conjugate to maps which are C^1 nearby. Extend this result to the case where $\rho(f)$ is an arbitrary rational number p/q. hint consider f^q, show that $\rho(f^q) = 0$ and match fixed points of f^q to corresponding fixed points of g^q when g is a map which sufficiently near f in the C^1 sense.

Question 4: Let M some metric (or topological) space and $f: M \to M$ be a continuous map. Define $\Omega(f)$ to be the set of points x such that for all neighbourhoods U of x there exists $n > 0$ such that $f^n(U) \cap U \neq \emptyset$. Show that if M is compact then $\Omega(f) \neq \emptyset$.

Question 5: Show that a diffeomorphism $f: S^1 \to S^1$ is Morse-Smale (in the sense we defined in class) if and only if the following three conditions are satisfied

- $\Omega(f)$ is equal to the closure of periodic points;
- $\Omega(f)$ is hyperbolic;
- $W^s(x)$ and $W^u(y)$ is transversal for each $x, y \in \Omega(f)$ (this assertion is void in dimension 1).

The usual definition of Morse-Smale is that these three conditions hold. So the definition we gave for circle maps, coincides with the usual definition.