Nonlinear dynamics in the semitransparent equatorial waveguide. Resonant excitation of Rossby and Yanai waves and their interactions

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Plan:

- The model and the linear wave spectrum.
- Removal of resonances.
- Solutions to the synchronism conditions.
- Resonant growth of equatorial waves.
- Nonlinear saturation.
- Effects of spatial modulation.

1 The model and the linear wave spectrum.

2-layer rotating shallow water on the equatorial tangent plane (non-dissipative):

$$\partial_t \mathbf{u}_i + \mathbf{u}_i \cdot \nabla \mathbf{u}_i + \beta y \hat{\mathbf{z}} \times \mathbf{u}_i + \frac{1}{\rho_i} \nabla \pi_i = 0 \, , i = 1, 2; \tag{1}$$

$$\partial_t h_i + \nabla \cdot (\mathbf{u}_i h_i) = 0 \, , i = 1, 2, \tag{2}$$

where βy is the Coriolis parameter in the vicinity of the equator. Coordinates: x - "zonal"(west-est); y - meridional (south-north), z - vertical. Dynamical variables: $\mathbf{u}_i = (u_i(x, y, t), v_i(x, y, t))$ - velocity fields, h_i - depths of the layers, $h_1 + h_2 = H$ - rigid lid upper b.c..

Barotropic and baroclinic velocities:

$$\mathbf{u}_{bt} = \frac{h_1\mathbf{u}_1 + h_2\mathbf{u}_2}{H}, \ \mathbf{u}_{bc} = \mathbf{u}_1 - \mathbf{u}_2. \tag{3}$$

In terms of barotropic streamfunction ψ, the baroclinic velocity $\mathbf{u} = (u, v)$ and the depth of the upper layer $h_1 = h$:

$$\begin{aligned}
\nabla^2\psi_t + \psi_x &= \epsilon\left[-J(\psi, \nabla^2\psi) - s(\partial_{xx} - \partial_{yy})\left[(1+\epsilon q h)(uv)\right]\right. \\
&+ \left. s\partial_{xy}\left[(1+\epsilon q h)(u^2 - v^2)\right]\right)\Big]
\end{aligned}$$

$$\begin{aligned}
\mathbf{u}_t + \nabla h + y\hat{\mathbf{z}} \times \mathbf{u} &= \epsilon\left[-J(\psi, \mathbf{u}) + \mathbf{u}\cdot\nabla(\hat{\mathbf{z}} \times \nabla\psi) - q\mathbf{u}\cdot\nabla\mathbf{u}\right. \\
&+ \left. \epsilon s\left(2h\mathbf{u}\cdot\nabla\mathbf{u} + \mathbf{u}\,\mathbf{u}\cdot\nabla h\right)\right],
\end{aligned}$$

$$h_t + \nabla\cdot\mathbf{u} = \epsilon\left[-J(\psi, h) - q\nabla\cdot(\mathbf{u}h) + \epsilon s\nabla\cdot\left(h^2\mathbf{u}\right)\right].$$

Parameters and characteristic scales.

$$q = \frac{H - 2H_1}{H}, \quad s = \frac{H_s}{H}, \quad \epsilon = \frac{\Delta H}{H_s}, \tag{4}$$

ΔH - typical variation of the interface, and $H_s = \frac{H_1(H-H_1)}{H}$.
Equatorial scaling:

$$L = \frac{(g' H_s)^{\frac{1}{4}}}{\sqrt{\beta}}; \; T = \frac{1}{\beta L}; \quad U = \frac{g' \Delta H}{\beta L^2}. \tag{5}$$

Reduced gravity: $g' = g(\rho_2 - \rho_1)/\rho_1$

Linear wave spectrum

Barotropic Rossby waves propagating at any angle:

$$\psi_0 = A_\psi e^{i(\theta + ly)} + c.c.;\ \theta = kx - \sigma t; \tag{6}$$

with the dispersion relation

$$\sigma = -k/(k^2 + l^2), \tag{7}$$

and the trapped **baroclinic** waves

$$(u, v, h) = (iU_m, \phi_m, iH_m)\, Ae^{i\theta_m} + c.c.;\ \theta_m = \hat{k}x - \sigma_m t \tag{8}$$

with the dispersion relation

$$\sigma_m^3 - (\hat{k}^2 + 2m + 1)\sigma_m - \hat{k} = 0;\ m = 0, 1, 2, \ldots\,. \tag{9}$$

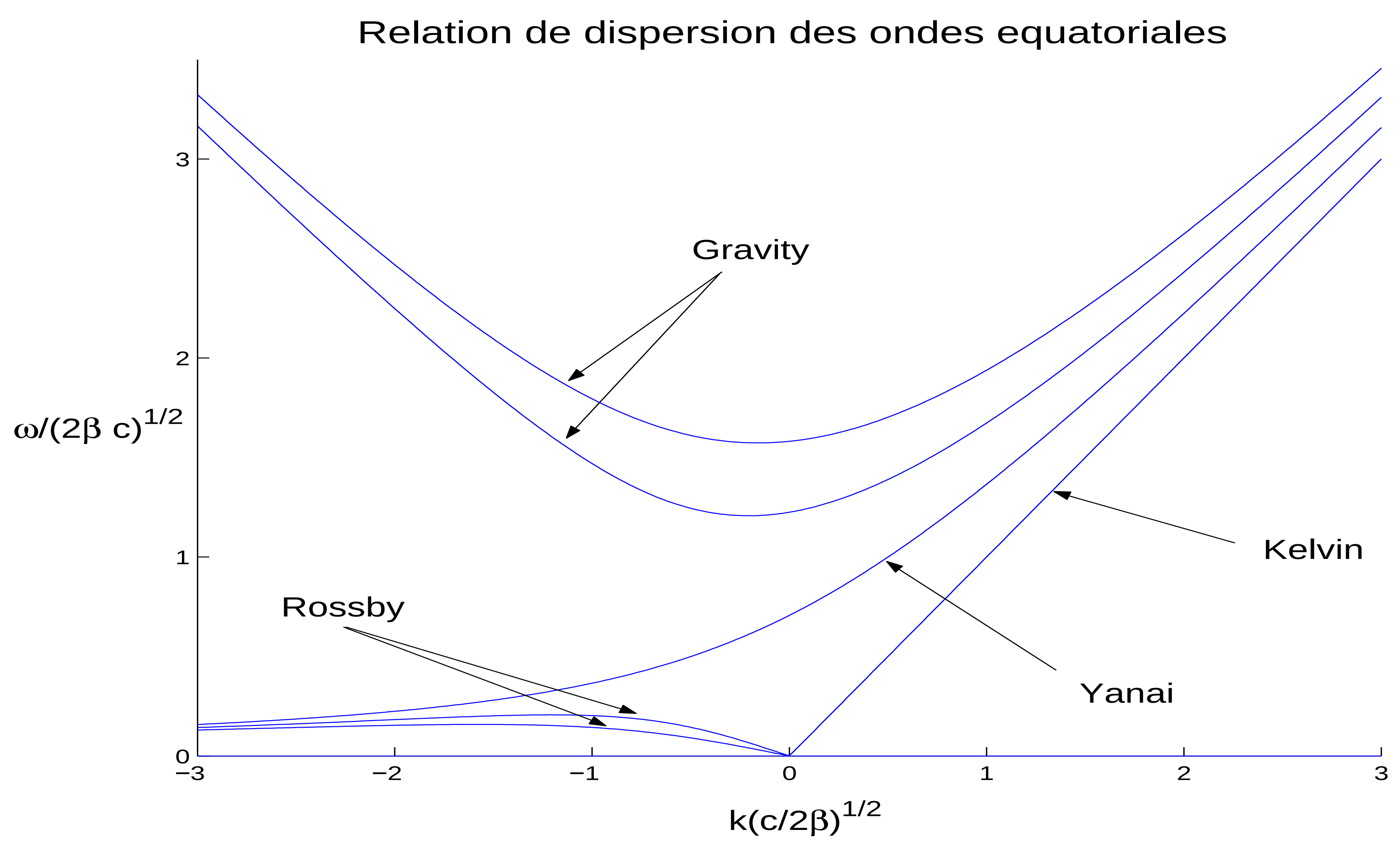


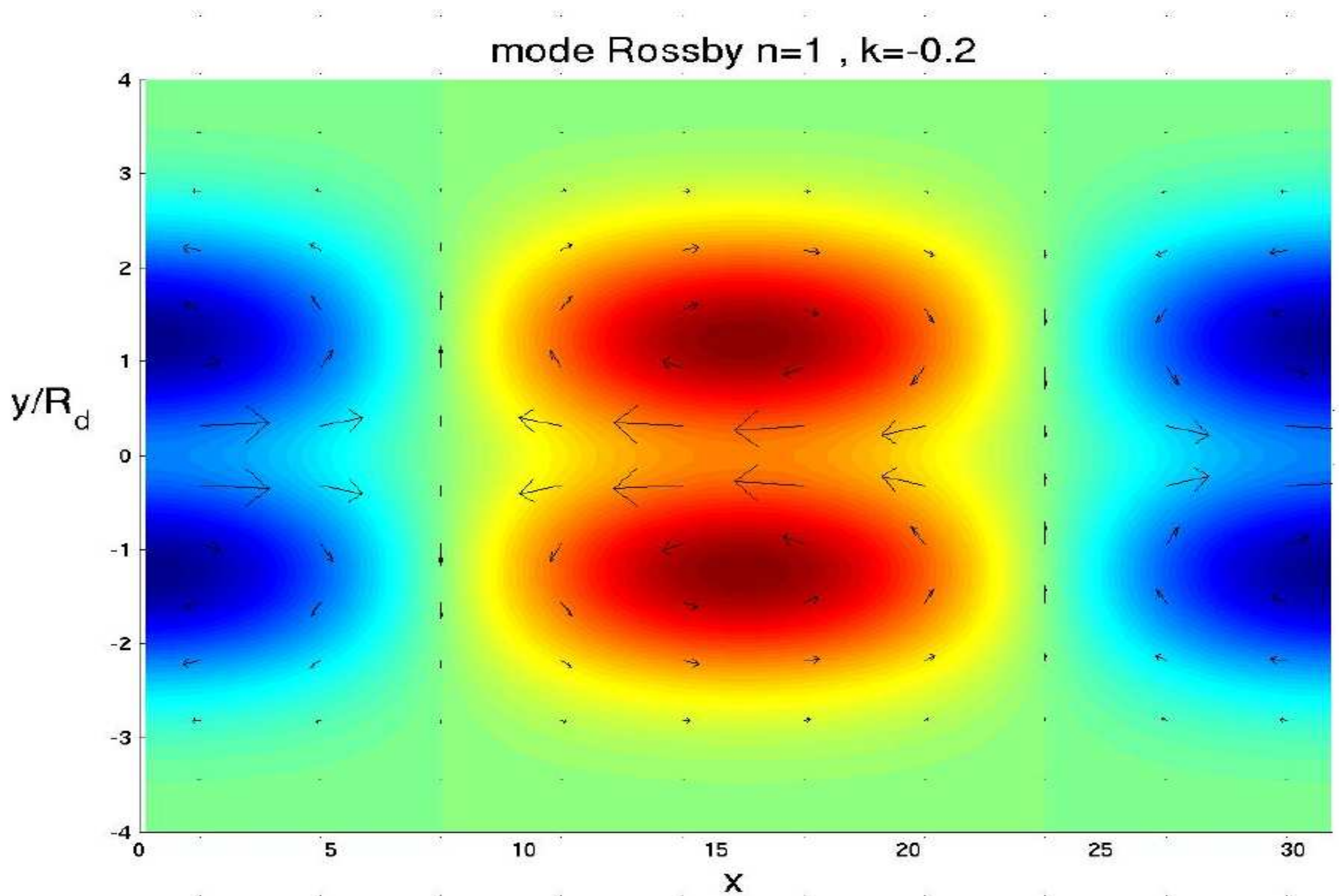
Figure 1: Dispersion curves for trapped equatorial waves.

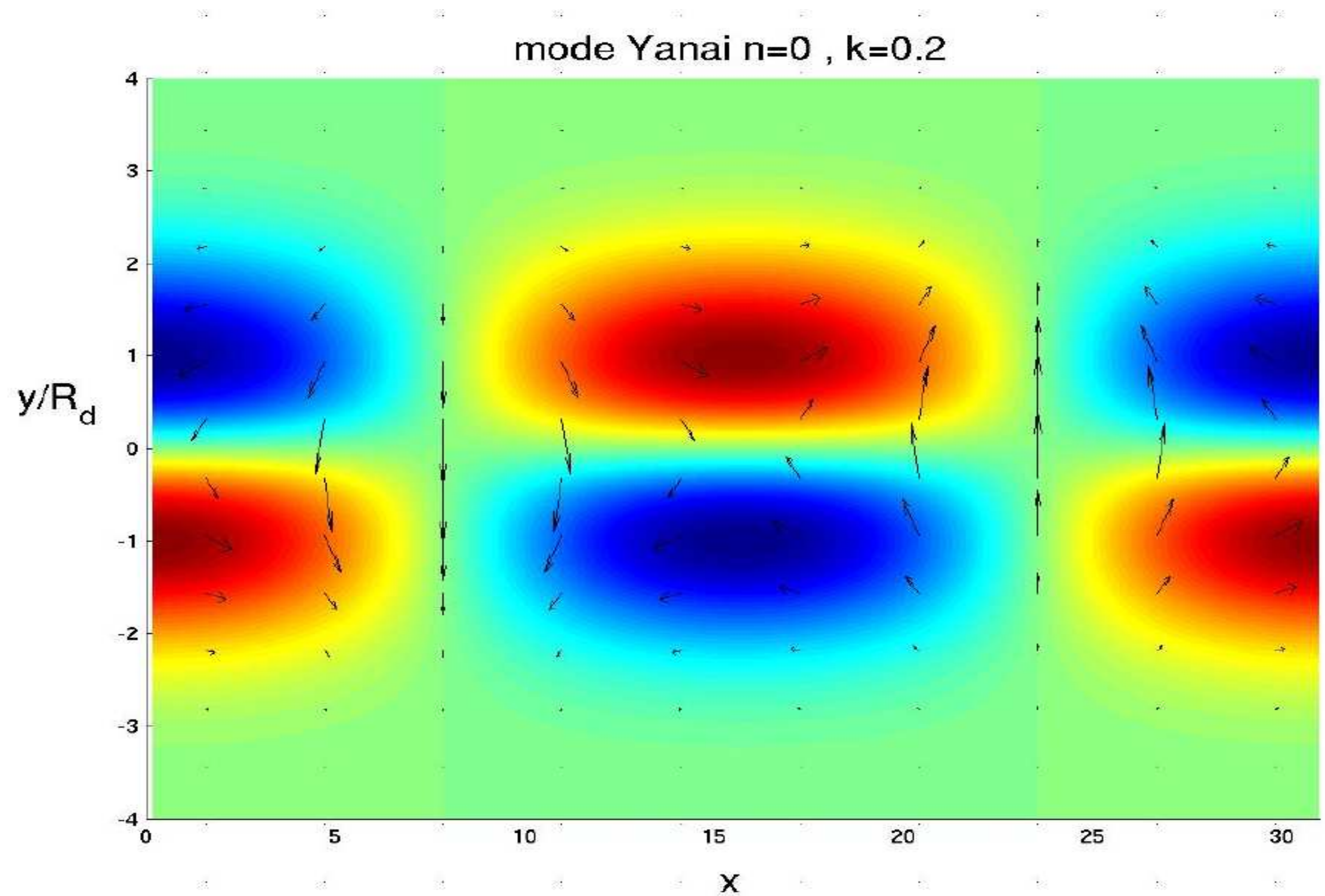
The functions $\mathbf{U}_m = (U_m, V_m), H_m$ are strongly localized near the equator $(y = 0)$. They are expressed in terms of the parabolic cylinder functions:

$$V_m(y) = \phi_m(y) = \frac{\mathcal{H}_m(y)e^{-\frac{y^2}{2}}}{\sqrt{2^m m! \sqrt{\pi}}},\ U_m(y) = \frac{\sigma_m y \phi_m - \hat{k}\phi_m'}{\sigma_m^2 - \hat{k}^2},$$

$$H_m(y) = \frac{\hat{k} y \phi_m - \sigma_m \phi_m'}{\sigma_m^2 - \hat{k}^2},$$

where $\mathcal{H}_m(y)$ are the Hermite polynomials and prime means y-differentiation.


mode Rossby n=1 , k=-0.2
y/R_d
x
4
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30


mode Yanai n=0 , k=0.2
y/R_d
x
4
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30

2 Removal of secular terms.

The forced linearized system has the form:

$$\nabla^2\psi_t + \psi_x = Q_\psi \tag{10}$$

$$u_t - yv + h_x = Q_u,\ v_t + yu + h_y = Q_v,\ h_t + u_x + v_y = Q_h. \tag{11}$$

Solution is bounded provided the orthogonality conditions are satisfied:

$$\langle\hat{\psi} Q_\psi\rangle_{x,y,t} = 0, \tag{12}$$

$$\int_{-\infty}^{\infty} dy\,\langle\hat{u}Q_u + \hat{v}Q_v + \hat{h}Q_h\rangle_{x,t} = 0. \tag{13}$$

$\hat{\psi}, \hat{u}, \hat{v}, \hat{h}$ - arbitrary bounded solution of the homogeneous equations, angles denote averaging:

$$\langle ... \rangle_x = \lim_{L_x \to \infty} \frac{1}{2L_x} \int_{-L_x}^{L_x} dx \, ..., etc \tag{14}$$

In what follows, the source terms are of the form:

$$Q_{\psi,...,h} = \sum_q Q^q_{\psi,...,h}(y) e^{i(k_q x - \sigma_q t)}, \tag{15}$$

with $Q^q_{\psi,...,h}(y)$ rapidly decaying at $y \to \pm\infty$. For such $Q_{\psi,...,h}(y)$ the conditions (12), (13) are not only necessary, but also sufficient if $k_q \neq 0$, $\sigma_q \neq 0$

3 Solutions to the synchronism conditions.

Resonant triads:

$$k_1 \pm k_2 = k; \quad \sigma_1 \pm \sigma_2 = \sigma. \tag{16}$$

$k_{1,2},\ \sigma_{1,2}$ - baroclinic trapped waves, k, σ - barotropic wave.
Parametric resonance: upper sign - **most interesting**. Allowed domains for the parametric resonance in the phase-space (k, l) of the barotropic waves:

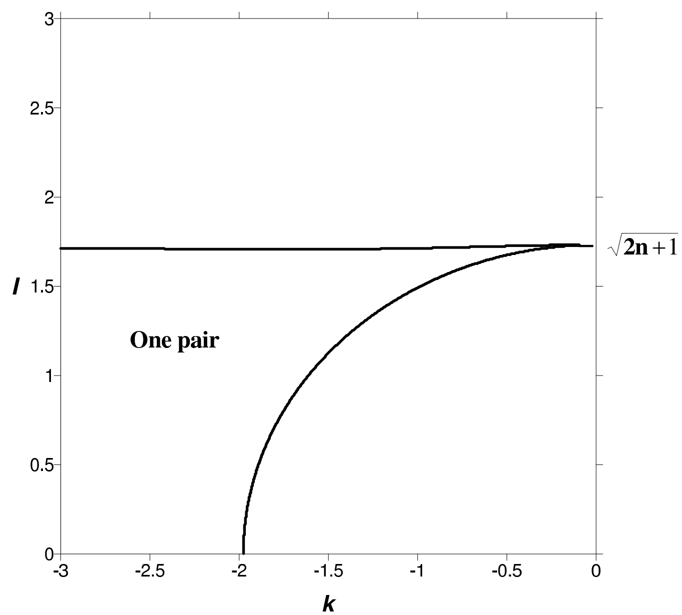


Figure 2: Two baroclinic Rossby waves with equal meridional wavenumbers.

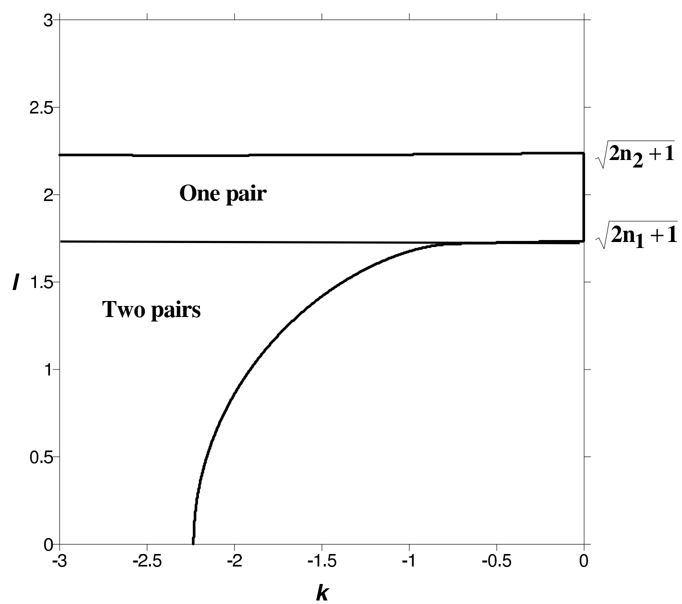


Figure 3: Two baroclinic Rossby waves with different meridional wavenumbers.

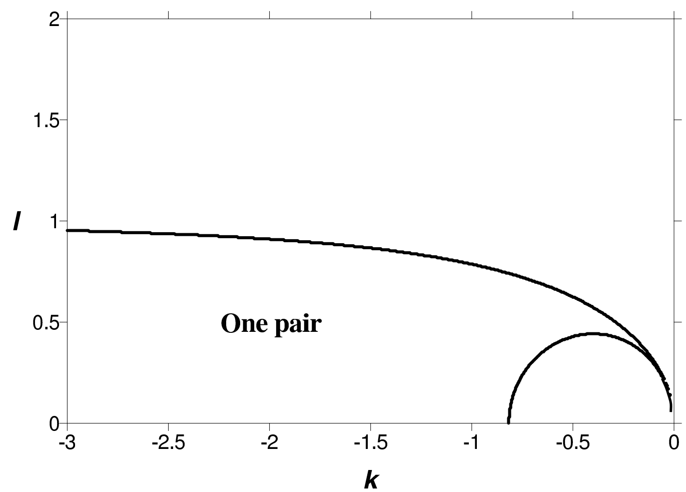


Figure 4: One baroclinic Rossby and one Yanai wave.

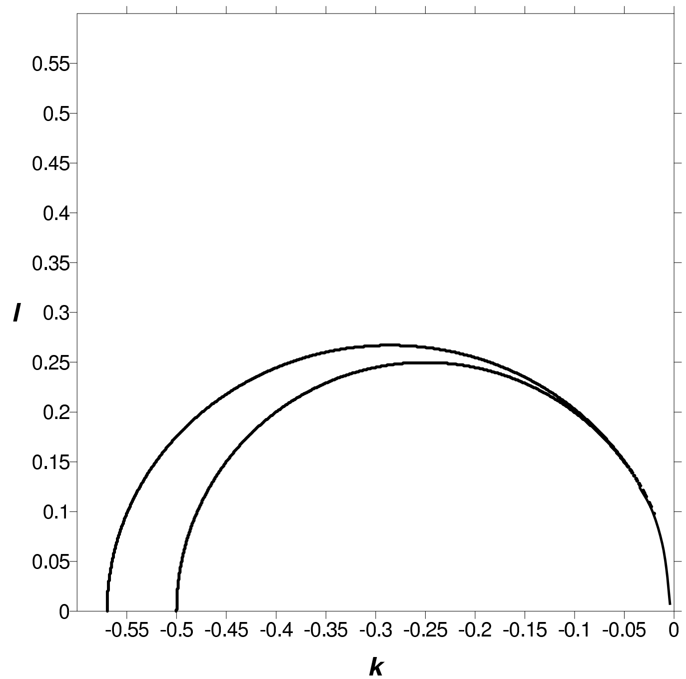


Figure 5: Two Yanai waves.

4 Resonant growth of equatorial waves.

Asymptotic multi time-scale expansion for small ϵ starting from a resonant wave triad:

$$(\psi, u, v, h) = (\psi^{(0)},\ u^{(0)}, v^{(0)}, h^{(0)})(x, y, t, T) + \epsilon(\psi^{(1)}, u^{(1)}, v^{(1)}, h^{(1)})(x, y, t, T) + \ldots,$$

$$\psi^{(0)} = A_\psi(T)e^{i(\theta+ly)} + c.c.,\ (u^{(0)}, v^{(0)}, h^{(0)}) = \sum_{\alpha=1,2} (u_\alpha^{(0)}, v_\alpha^{(0)}, h_\alpha^{(0)}),$$

$$(u_\alpha^{(0)}, v_\alpha^{(0)}, h_\alpha^{(0)}) = \frac{1}{\sqrt{a_\alpha}} \left(iU_\alpha(y), \phi_\alpha(y), iH_\alpha(y)\right) A_\alpha e^{i\theta_\alpha} + c.c.$$

$$\alpha = 1, 2;\ \theta = kx - \sigma t,\ \theta_{1,2} = k_{1,2}x - \sigma_{1,2}t,\ T = \epsilon t.$$

The equations for the first correction are

$$
\begin{aligned}
\nabla^2\psi_t^{(1)} + \psi_x^{(1)} &= -\nabla^2\psi_T^{(0)} + N_\psi, \\
u_t^{(1)} - yv^{(1)} + h_x^{(1)} &= -u_T^{(0)} + N_u, \\
v_t^{(1)} + yu^{(1)} + h_y^{(1)} &= -v_T^{(0)} + N_v, \\
h_t^{(1)} + u_x^{(1)} + v_y^{(1)} &= -h_T^{(0)} + N_h,
\end{aligned}
$$

Nonlinear interaction terms are:

$$
\begin{aligned}
N_\psi &= -J(\psi^{(0)}, \nabla^2\psi^{(0)}) - s(\partial_{xx} - \partial_{yy})(u^{(0)}v^{(0)}) + s\partial_{xy}(u^{(0)2} - v^{(0)2}), \\
N_u &= \underline{-J(\psi^{(0)}, u^{(0)}) + \mathbf{u}^{(0)} \cdot \nabla\psi_y^{(0)}} - q\mathbf{u}^{(0)} \cdot \nabla u^{(0)} \\
N_v &= \underline{-J(\psi^{(0)}, v^{(0)}) - \mathbf{u}^{(0)} \cdot \nabla\psi_x^{(0)}} - q\mathbf{u}^{(0)} \cdot \nabla v^{(0)} \\
N_h &= \underline{-J(\psi^{(0)}, h^{(0)})} - q\nabla \cdot (\mathbf{u}^{(0)}h^{(0)}).
\end{aligned}
$$

Only underlined terms may contain resonances.

The resonance removal condition with $(\hat{u}, \hat{v}, \hat{h} = (\overline{u^{(0)}}_\alpha, \overline{v^{(0)}}_\alpha, \overline{h^{(0)}}_\alpha)$ gives the following equations for the slow evolution of the amplitudes of the baroclinic waves:

$$A_{1_T} = L_1^+ A_\psi \bar{A}_2, \;\; \bar{A}_{2_T} = \bar{L}_2^+ \bar{A}_\psi A_1,$$

or

$$A_{1_T} = L_1^- A_\psi \bar{A}_2, \;\; \bar{A}_{2_T} = L_2^- \bar{A}_\psi A_1,$$

where $\pm$ signs correspond to the signs in the synchronism conditions. The equations (17), or (17), are reduced to a single one:

$$A_{\alpha_{TT}} = C^\pm \left|A_\psi\right|^2 A_\alpha, \;\; C^+ = L_1^+ \bar{L}_2^+, \, C^- = L_1^- L_2^- .$$

$C^\pm$ are real. An important property is

$$C^+ > 0, \, C^- < 0.$$

$\Rightarrow$ **exponential growth in the + case.**

Energy balance

$$E = E_{bt} + E_{bc} = const,$$

where E_{bt}, E_{bc} are the barotropic and the baroclinic energies:

$$E_{bt} = \int_{-\infty}^{\infty} dy\,\langle(\nabla\psi)^2\rangle_x,\ E_{bc} = \frac{s}{2}\int_{-\infty}^{\infty} dy\,\langle(1+\epsilon qh)(u^2+v^2+h^2)\rangle_x.$$

Quadratic energy form conserved in the lowest order:

$$E_0 = \frac{s}{2}\int_{-\infty}^{\infty} dy\,\langle(u^{(0)2}+v^{(0)2}+h^{(0)2})\rangle_x + \int_{-\infty}^{\infty} dy\,\langle\nabla\psi^{(0)}\cdot\nabla\psi^{(1)}\rangle_x,\ ,$$

$\Rightarrow$ **First barotropic correction (the barotropic response of the equator) is crucial**

5 Nonlinear saturation.

Rearrangement of the asymptotic expansion:

$$
\begin{aligned}
\psi &= \psi^{(0)}(x, y, t, T_1, T_2, ...) + \epsilon^{\frac{1}{2}} \psi^{(1)}(x, y, t, T_1, T_2, ...) + ..., \\
(u, v, h) &= \epsilon^{-\frac{1}{2}} (u^{(0)}, v^{(0)}, h^{(0)})(x, y, t, T_1, T_2, ...) \\
&+ (u^{(1)}, v^{(1)}, h^{(1)})(x, y, t, T_1, T_2, ...) + ...,
\end{aligned}
$$

5.1 "Pure" parametric resonance $\sigma = 2\hat{\sigma},\ k = 2\hat{k}$.

Amplitude (Landau) equation:

$$A_{T_2} + LA_{\psi}\bar{A} + (P + iQ)\left|A\right|^2 A = 0.$$

$Q = Q_0 + qQ_1 + sQ_2,\ L, P, Q$ are real, $P \geq 0$.

The term $\propto L$ is due to the interaction of the primary harmonic barotropic wave with the zero-order baroclinic one, the term $\propto P + iQ_0$ is due to the interaction between the secondary barotropic mode and the zero-order baroclinic mode, the term containing Q_1 is due to the interaction between zero- and first-order baroclinic fields, and the term containing Q_2 is due to the cubic interaction of the zero-order baroclinic mode.

Stationary solutions:

$$A_0 = 0,\ A_\pm^2 = -\frac{LA_\psi}{P + iQ}.$$

A_0 is unstable, $A_\pm$ are stable if $P > 0$ (neutrally stable if $P = 0$). $\Rightarrow$ **nonlinear saturation always takes place**.

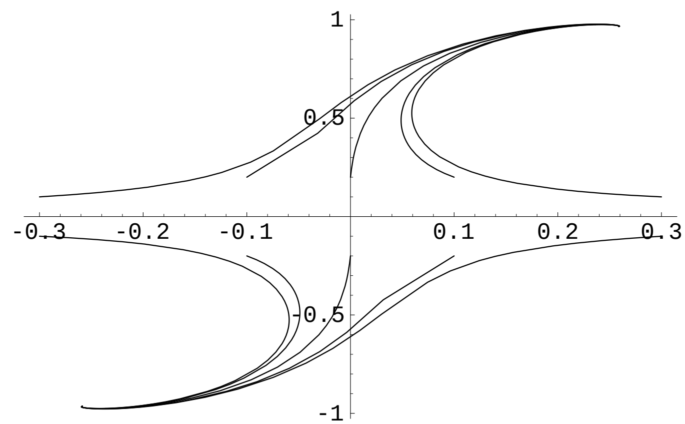


Figure 6: Typical behaviour of the trajectories of solutions of the Landau equation in the phase-space $ReA - ImA$. The trajectories in the upper (lower) half-plane are attracted to $A_{-(+)}$.

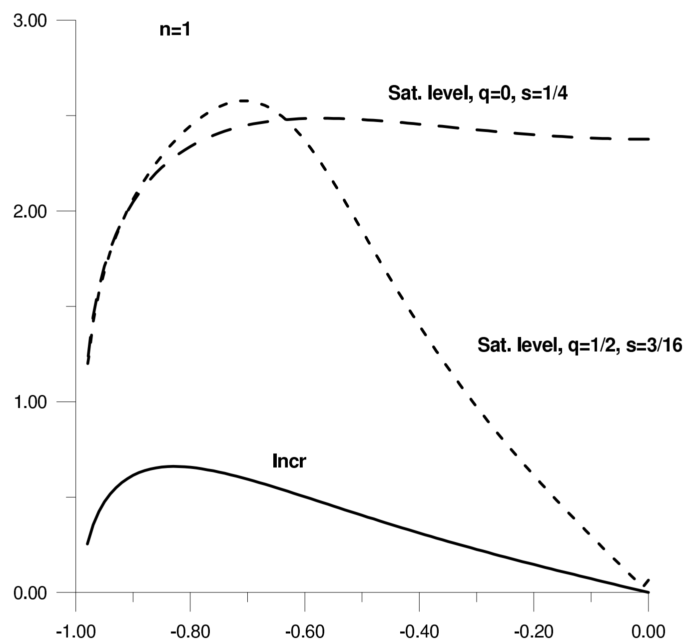


Figure 7: Increment and saturation amplitude of the baroclinic Rossby wave with $m = 1$ as functions of zonal wavenumber. The parametric resonance case.

Back influence of the trapped baroclinic mode on the barotropic field: primary vs secondary barotropic modes. If the baroclinic mode is saturated then A^2 is proportional to A_ψ and the ratio between primary and secondarymodes does not depend on θ, A_ψ.

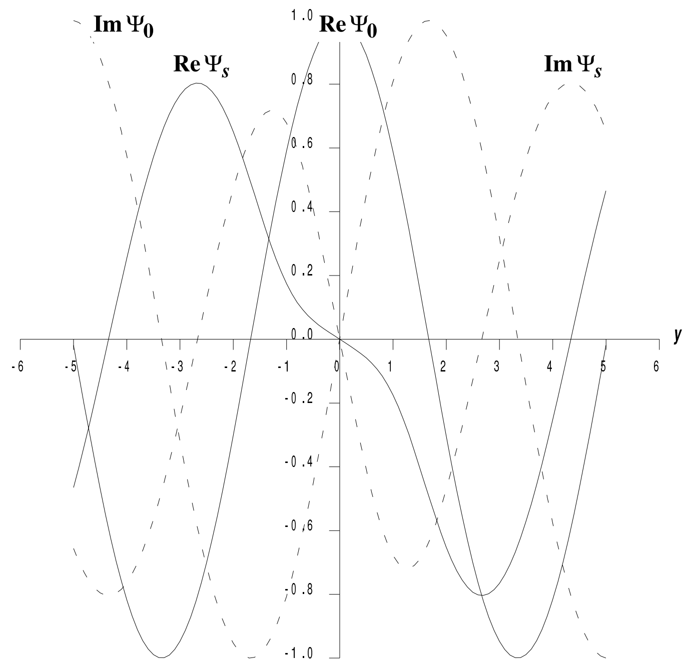


Figure 8: Comparison of the primary harmonic barotropic wave with the secondary barotropic mode generated by the saturated trapped Rossby wave with $m = 1$, $\hat{k} = .83$. The amplitudes of $Re\psi^{(s)}$ and $Im\psi^{(s)}$ reach ~ 0.8, i.e. the back influence of the saturated trapped mode can be very strong.

5.2 Two different baroclinic waves.

$$
\begin{aligned}
A_{1_T} + \alpha_1 \bar{A}_2 + \beta_1 \left| A_2 \right|^2 A_1 + \gamma_1 \left| A_1 \right|^2 A_1 &= 0 \\
A_{2_T} + \alpha_2 \bar{A}_1 + \beta_2 \left| A_1 \right|^2 A_2 + \gamma_2 \left| A_2 \right|^2 A_2 &= 0.
\end{aligned}
$$

The following properties of the coefficients may be established:

$$\alpha_1 \bar{\alpha}_2 > 0,\ Re(\beta_1 + \beta_2) \leq 0,\ Re(\gamma_1) \leq 0,\ Re(\gamma_2) \leq 0.$$

$\alpha_{1,2}$ are either both real or both imaginary, and at least one of $Re(\gamma_1), Re(\gamma_2)$ is < 0. $\Rightarrow$ **saturation.**

In terms of real amplitudes and phases: $A_{1,2} = |A_{1,2}|\, e^{i\Phi_{1,2}}$:

$$\begin{aligned}
|\dot{A}_1| - |\alpha_1||A_2| \cos(\phi_\alpha - \Phi_1 - \Phi_2) + Re\gamma_1 |A_1|^3 + Re\beta_1 |A_2|^2 |A_1| &= 0, \\
|\dot{A}_2| - |\alpha_2||A_1| \cos(\phi_\alpha - \Phi_1 - \Phi_2) + Re\gamma_2 |A_2|^3 + Re\beta_2 |A_1|^2 |A_2| &= 0,
\end{aligned}$$

$$\begin{aligned}
\dot{\Phi}_1 - |\alpha_1| \frac{|A_2|}{|A_1|} \sin(\phi_\alpha - \Phi_1 - \Phi_2) + Im\gamma_1 |A_1|^2 + Im\beta_1 |A_2|^2 &= 0, \\
\dot{\Phi}_2 - |\alpha_2| \frac{|A_1|}{|A_2|} \sin(\phi_\alpha - \Phi_1 - \Phi_2) + Im\gamma_2 |A_2|^2 + Im\beta_2 |A_1|^2 &= 0,
\end{aligned}$$

$$\alpha_1 = |\alpha_1|\, e^{i\phi_\alpha},\ \alpha_2 = |\alpha_2| e^{-i\phi_\alpha}.$$

Remarkably, $\Phi_- = \Phi_1 - \Phi_2$ splits out! No nontrivial time - independent solutions in the general case . Solution $|A_{1,2}| = const, \Phi_+ = \Phi_1 + \Phi_2 = const, \Phi_- \propto T_2$ may be found, leading to:

$$A_{1,2} = B_{1,2} e^{\pm i\omega T_2}$$

with constant complex $B_{1,2}$, and real ω.

Saturation:
direct numerical simulations show that such quasi-stationary solutions are stable and attractive for small initial values of the amplitudes.

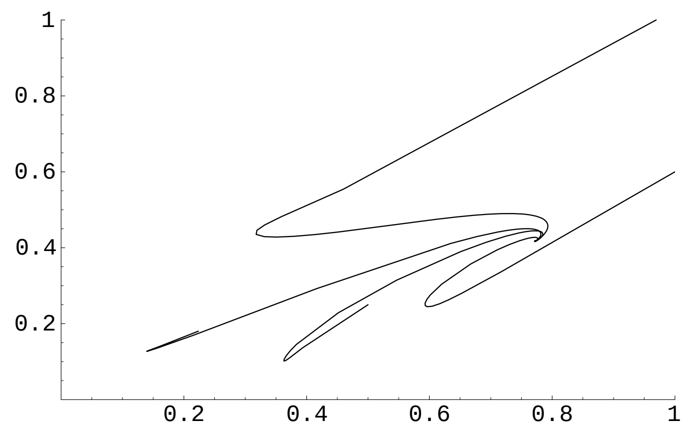


Figure 9: Typical behaviour of the trajectories of the solutions in the phase-space $|A_1| - |A_2|$. The trajectories are attracted to the fixed point.

At each k there is a longer and a shorter baroclinic wave. With increasing $|k|$ the longer (shorter) baroclinic wave is becoming even longer (shorter). No selectivity in k: increment gradually grows from $\approx .65$ to $\approx .74$. Peculiarity: there is a considerable difference in the values of two saturated amplitudes, the saturation level of the longer wave $>$ that of the shorter wave $\Rightarrow$ dominance of longer waves. Calculations with different $m_{1,2}$ confirm this tendency

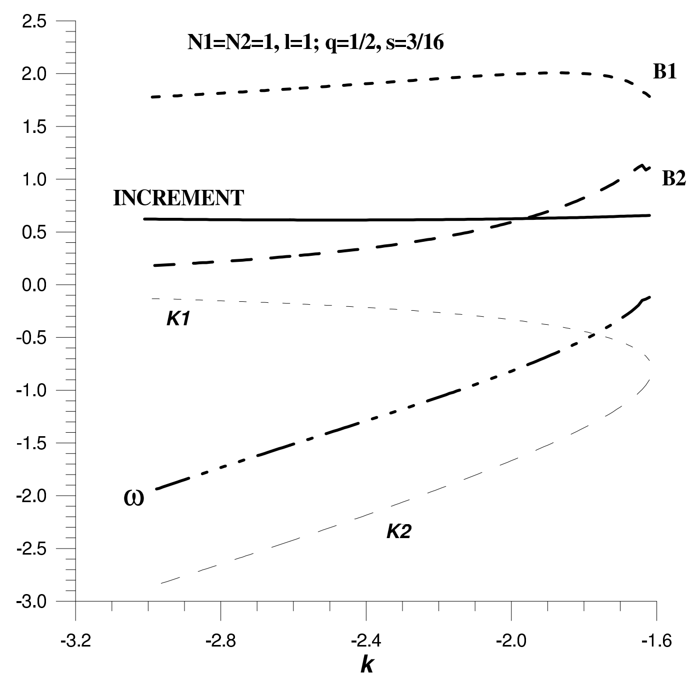


Figure 10: Increment, saturation amplitudes $|B_{1,2}|$, and frequency ω of slow oscillations as functions of the barotropic zonal wavenumber k for a pair of baroclinic Rossby waves with $m = 1$; $k_{1,2}$ are the zonal wavenumbers of the baroclinic waves, $l = 1$.

6 The effects of spatial modulation

Multiple scales both in space- and time:

$$
\begin{aligned}
\psi &= \psi^{(0)}(\mathbf{x}, X_1, X_2, ..., t, T_1, T_2, ...) \\
&+ \epsilon^{\frac{1}{2}}\psi^{(1)}(\mathbf{x}, X_1, X_2, ..., t, T_1, T_2, ...) + ..., \\
(u, v, h) &= \epsilon^{-\frac{1}{2}}(u^{(0)}, v^{(0)}, h^{(0)})(\mathbf{x}, X_1, X_2, ..., t, T_1, T_2, ...) \\
&+ (u^{(1)}, v^{(1)}, h^{(1)})(\mathbf{x}, X_1, X_2, ..., t, T_1, T_2, ...) + ...
\end{aligned}
$$

6.1 "Pure" parametric resonance: $\sigma = 2\hat{\sigma},\ k = 2\hat{k}$

Amplitude equation in the reference frame moving with the trapped wave:

$$A_{T_2} - \frac{i}{2}\hat{\sigma}''(\hat{k})A_{X_1X_1} - LA_\psi\bar{A} + (P + iQ)\left|A\right|^2 A = 0.$$

$\hat{\sigma}''(\hat{k})$ is the derivative of the group velocity with respect to $\hat{k}$.

Equation of the Ginzburg - Landau (GL) type: resonantly forced GL equations known in optics and in the Faraday effect. The mechanism of excitation and saturation in our case is *different* from the standard one:

- barotropic wave: not a pure external forcing, changed by the secondary wave,
- standard resonantly driven GL has the term μA with $Re\mu \neq 0$.

The saturated stationary solutions are still solutions, stable for $P > 0$. Two different stationary states - domain wall type solutions, as it is the case for similar GL equations? - A characteristic Bloch-type domain wall structures appear, forming a bound state (a so-called bubble, or "dark soliton")

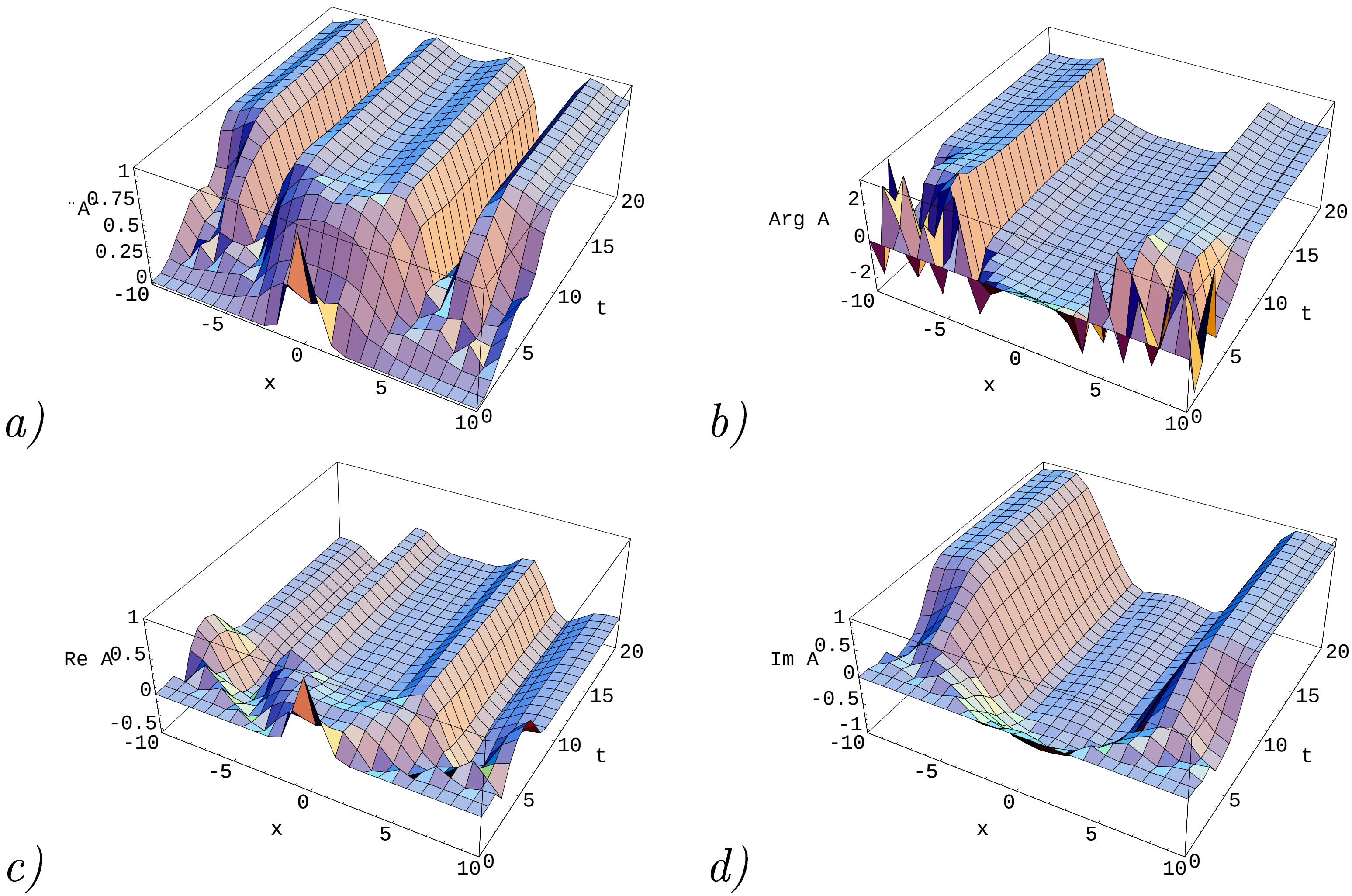


Figure 11: Space-time evolution of a localised wave-packet.

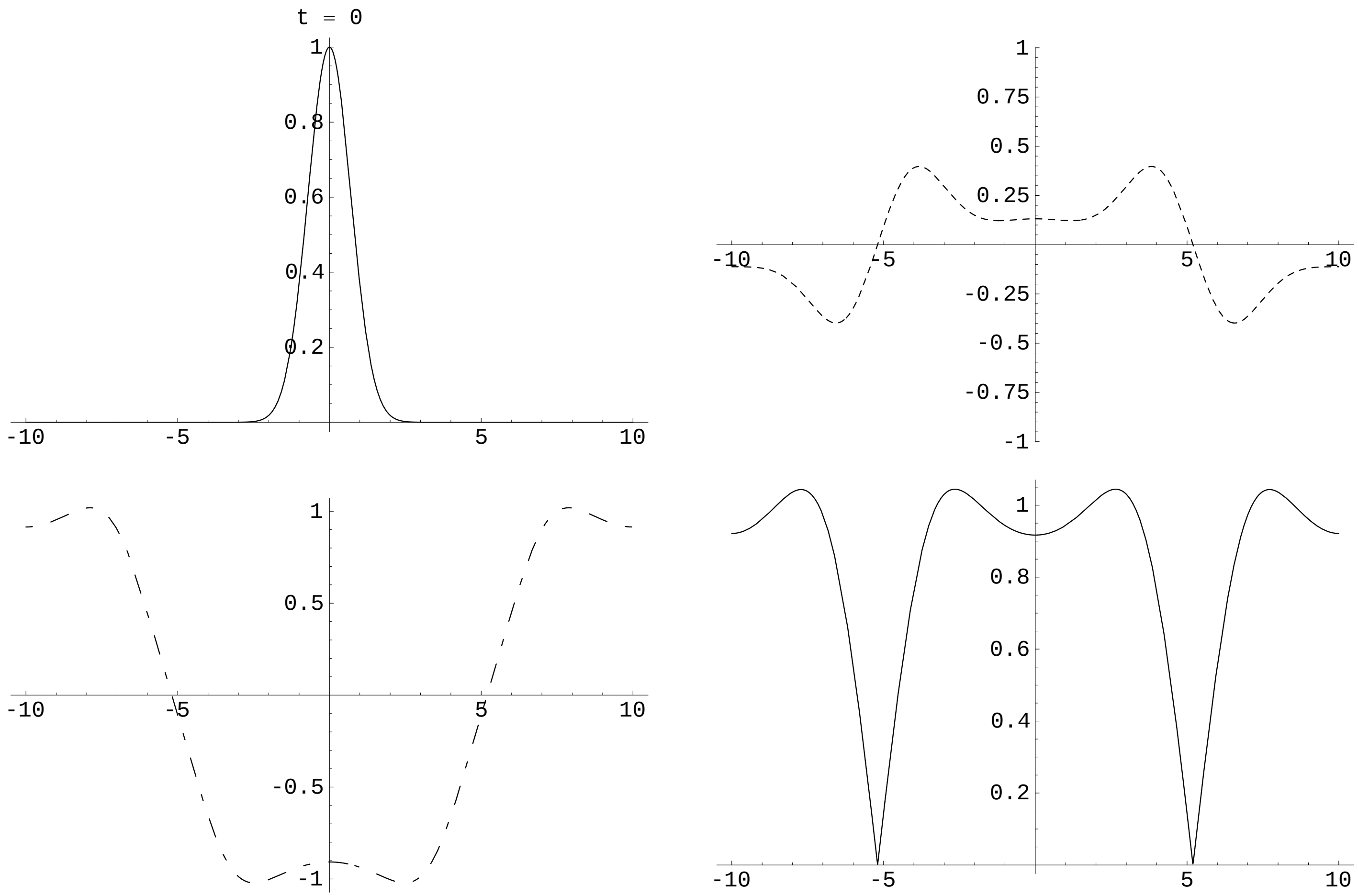


Figure 12: Initial (top left), and final profiles of the real (dashed) and imaginary (dash-dotted) parts of A, and its amplitude (solid)

6.2 A pair of baroclinic waves.

The amplitude equations in the lowest order:

$$A_{i_{T_1}} + c_{g_i} A_{i_{X_1}} = 0,\ i = 1, 2,$$

c_{g_i} - group velocities of the respective waves. Next order:

$$\begin{aligned} A_{1_{T_2}} + c_{g_1} A_{1_{X_2}} - \frac{i}{2}\sigma_1''(k_1) A_{1_{X_1 X_1}} + \alpha_1 \bar{A}_2 + \beta_1 \left|A_2\right|^2 A_1 + \gamma_1 \left|A_1\right|^2 A_1 &= 0 \\ A_{2_{T_2}} + c_{g_2} A_{2_{X_2}} - \frac{i}{2}\sigma_2''(k_2) A_{2_{X_1 X_1}} + \alpha_2 \bar{A}_1 + \beta_2 \left|A_1\right|^2 A_2 + \gamma_2 \left|A_2\right|^2 A_2 &= 0. \end{aligned}$$

"Synthetic" equations:

$$A_{1_{T_1}} + c_{g_1} A_{1_{X_1}} + \epsilon^{\frac{1}{2}} \left(-\frac{i}{2}\sigma_1''(k_1) A_{1_{X_1 X_1}} + \alpha_1 \bar{A}_2 + \beta_1 |A_2|^2 A_1 + \gamma_1 |A_1|^2 A_1 \right) = 0$$

$$A_{2_{T_1}} + c_{g_2} A_{2_{X_1}} + \epsilon^{\frac{1}{2}} \left(-\frac{i}{2}\sigma_2''(k_2) A_{2_{X_1 X_1}} + \alpha_2 \bar{A}_1 + \beta_2 |A_1|^2 A_2 + \gamma_2 |A_2|^2 A_2 \right) = 0.$$

$c_{g_1} \neq c_{g_2} \Rightarrow$ impossible to change a reference frame and remove the propagative effects.

If the wave-amplitudes do not depend on T_1, X_1 but only on T_2, X_2:

$$\begin{aligned} A_{1_{T_2}} + c_{g_1} A_{1_{X_2}} + \alpha_1 \bar{A}_2 + \beta_1 \left|A_2\right|^2 A_1 + \gamma_1 \left|A_1\right|^2 A_1 &= 0 \\ A_{2_{T_2}} + c_{g_2} A_{2_{X_2}} + \alpha_2 \bar{A}_1 + \beta_2 \left|A_1\right|^2 A_2 + \gamma_2 \left|A_2\right|^2 A_2 &= 0. \end{aligned}$$

The terms with second spatial derivatives, thus, appear as the next order corrections.

Hyperbolic system with straight characteristics $\frac{dX}{dT} = c_g(\hat{k}_{1,2})$: angle between the characteristics determines the zone of influence of the initial conditions. Expect formation of sharp fronts ("shocks"), as usual for non-dispersive hyperbolic systems, which is confirmed by preliminary direct numerical simulations. Resemblance to the dynamics of counterpropagating waves in the Faraday effect, with an important difference of absence of linear in $A_{1,2}$ terms, and different structure of the coefficients.

7 Summary/Perspective

7.1 Summary

- resonant excitation of trapped waves by non-trapped ones in *semitransparent* waveguides (generic!) = parametric instability of the barotropic wave with respect to baroclinic perturbations; teleconnections midlatitudes-tropics.
- new type of GL equation: spatio-temporal organization ("dark solitons")
- coupled GL-type equations: slow variability, fronts.

7.2 Perspective

- Whole sphere with discrete spectrum (resonant excitation of tides).
- Other semitransparent waveguides (coasts/topography)