Black-hole & white-hole horizons for capillary-gravity waves in superfluids

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Warwick
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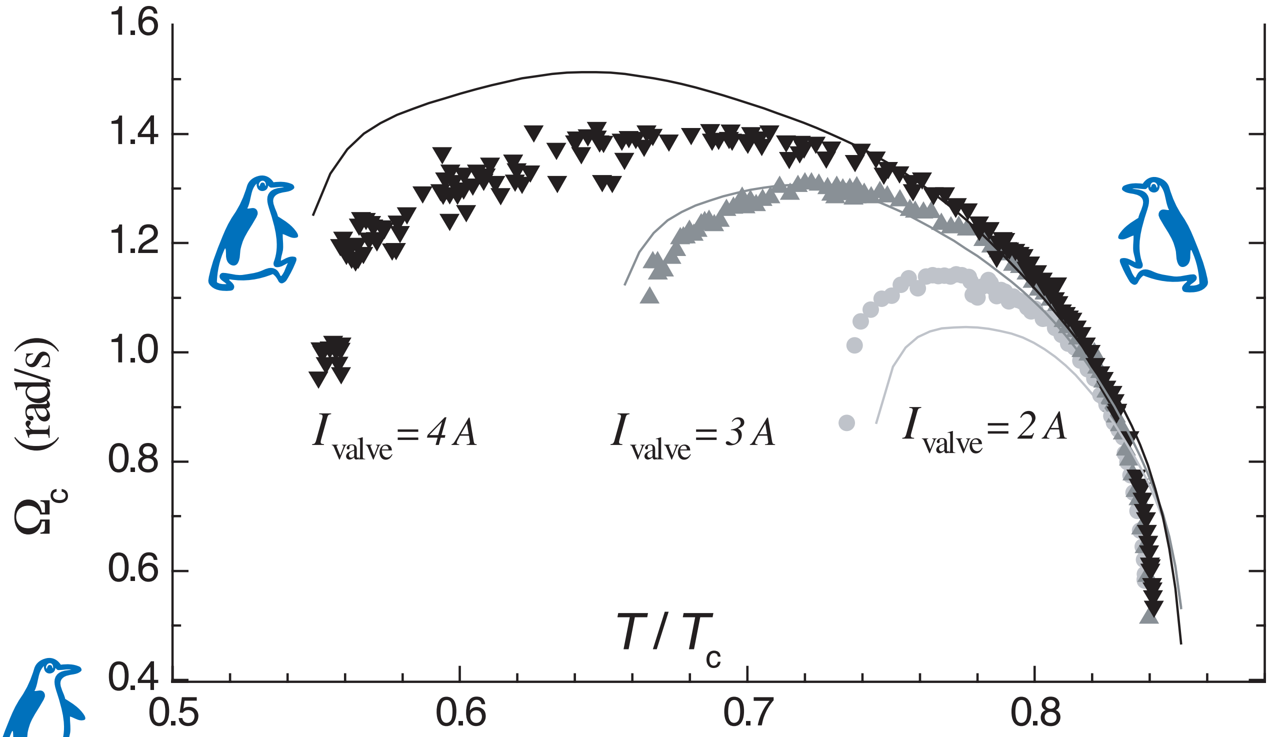
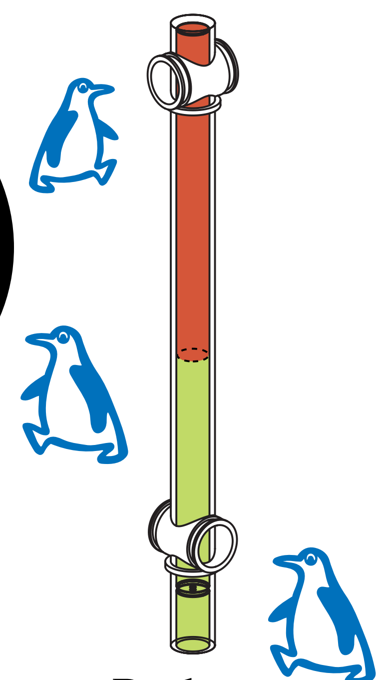




Effective Lorentzian metric is typical for condensed matter

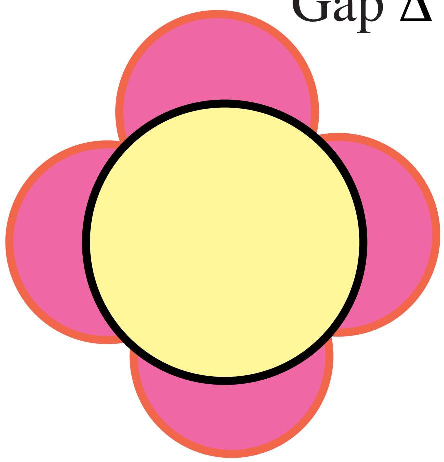
$$L= (-g)^{1/2} g^{\mu\nu} \partial_\mu \phi \partial_\mu \phi$$

* Phonons in moving superluids & sound waves in moving liquid

Landau-Khalatnikov two-fluid equations are analogs of Einstein equations for gravity and matter $T^{\mu\nu}{}_{;\mu} = 0$

* Spin waves in inhomogeneous medium
* Elasticity theory with dislocations and disclinations
* Light in moving dielectric
* Quasiparticles in superconductors with nodes

Gap Δ (θ)



* Quasiparticles in ^{3}He-A   This system is most instructive since gravity appears together with

chiral fermions, gauge fields,Lorentz invariance
gauge invariance, the same speed of light for fermions & bosons

* Ripplons on the surface of liquid or interface between liquids

best system for simulating event horizon of black hole

Acoustic metric in liquids (*Unruh, 1981*) & superfluids

review:
Barcelo, Liberati & Visser,
Analogue Gravity
gr-qc/0505065

Doppler shifted spectrum in moving liquid

$$E = cp + \mathbf{p}\cdot\mathbf{v}$$

c speed of sound

$$\downarrow$$

$$E - \mathbf{p}\cdot\mathbf{v} = cp$$

$$\downarrow$$

$$-(E - \mathbf{p}\cdot\mathbf{v})^2 + c^2 p^2 = 0$$

$$g^{\mu\upsilon} p_\mu p_\nu = 0 \qquad p_\nu = (-E, \mathbf{p})$$

$$g^{00} = -1 \qquad g^{0i} = -v^i \qquad g^{ij} = c^2\delta^{ij} - v^i v^j$$

$$\downarrow$$

inverse metric $g_{\mu\upsilon}$ determines effective spacetime
in which phonons move along geodesic curves

$$ds^2 = -\,dt^2\, \mathbf{c}^2 + (d\mathbf{r} - \mathbf{v}dt)^2 \longleftarrow ds^2 = g_{\mu\upsilon} dx^\mu dx^\nu$$

reference frame for phonon is dragged
by moving liquid

Acoustic gravity

Superfluids		*Gravity*
speed of sound	c	speed of light
Doppler shifted spectrum in moving liquid $E = cp + \mathbf{p}\cdot\mathbf{v}$	$g^{\mu\upsilon}$	$g^{\mu\upsilon}p_\mu p_\nu = 0$
acoustic metric	$g_{\mu\upsilon}$	geometry

$$ds^2 = -\,dt^2\,\mathbf{c}^2 + (d\mathbf{r} - \mathbf{v}dt)^2$$

reference frame for phonon is dragged by moving liquid

$$ds^2 = g_{\mu\upsilon}dx^\mu dx^\nu$$

Effective metric for phonons propagating in radial superflow $v(r)$

Painleve-Gulstrand metric

$$ds^2 = -\,dt^2\,(c^2-v^2) + 2\,v\,dr\,dt + dr^2 + r^2 d\Omega^2$$

g_{00} g_{0r}

after time transformation

$$dt = dt - vdr/(c^2-v^2)$$

Schwarzschild metric: $$ds^2 = -\,dt^2\,(c^2-v^2) + dr^2/(c^2-v^2) + r^2 d\Omega^2$$

g_{00} g_{rr}

$v^2(r)$ Kinetic energy of superflow = potential of gravitational field $v^2(r) = \dfrac{2GM}{r}$

03a

Acoustic Black Hole

Liquids & superfluids | *Gravity*

acoustic horizon

(Unruh, 1981)

black hole horizon

Painleve-Gulstrand metric

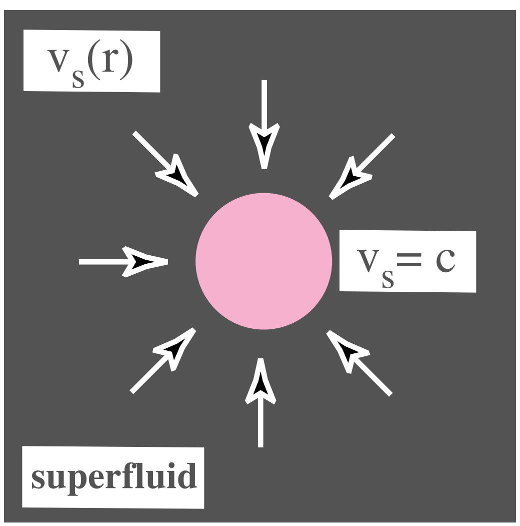
$$ds^2 = -\underbrace{dt^2\,(c^2-v^2)}_{g_{00}} + \underbrace{2\,v\,dr\,dt}_{g_{0r}} + dr^2 + r^2 d\Omega^2$$

Schwarzschild metric

$$ds^2 = -\underbrace{dt^2\,(c^2-v^2)}_{g_{00}} + \underbrace{dr^2/(c^2-v^2)}_{g_{rr}} + r^2 d\Omega^2$$

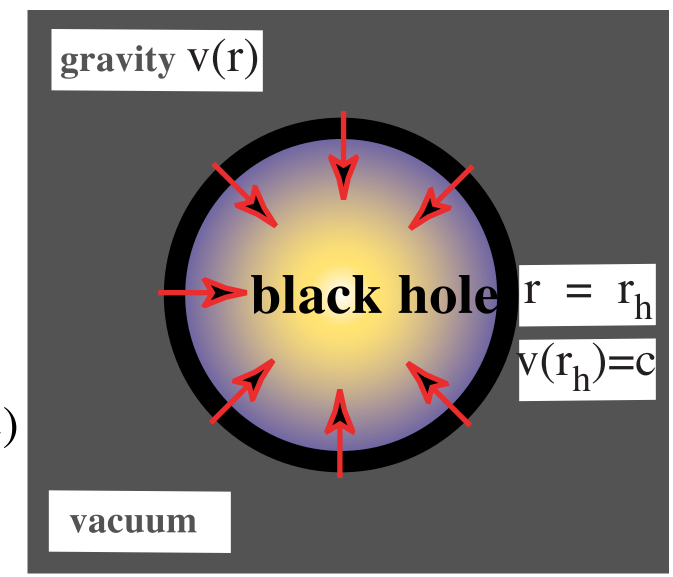
$$v^2(r) = c^2\frac{r_h}{r}$$

$$v^2(r) = \frac{2GM}{r} = c^2\frac{r_h}{r}$$



Information from interior region cannot be transferred by light or sound

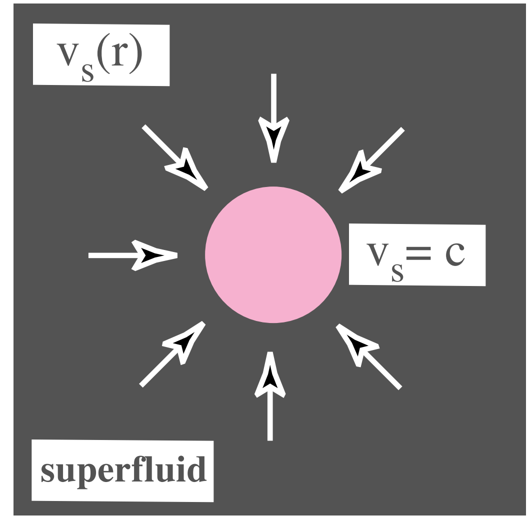
horizon at $g_{00} = 0$ (or $v(r_h) = c$)



03b

Vacuum resistance to formation of horizon

Hydrodynamic instability of spherical black hole

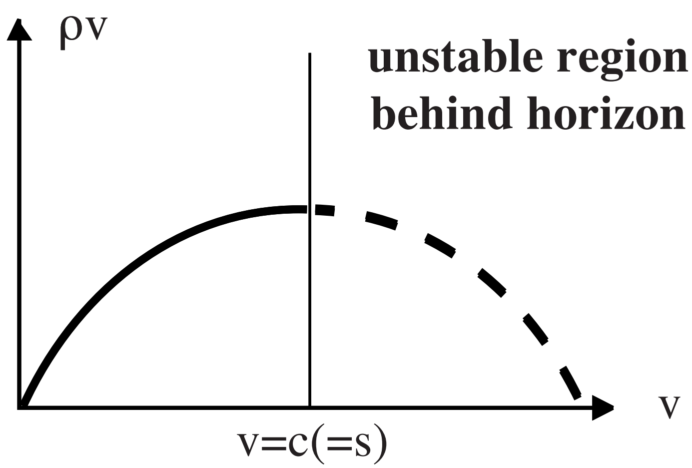


along the stream line of stationary flow:

$$\frac{d(\rho v)}{dv} = \rho(1 - v^2(r)/c^2)$$

continuity equation:

$$\rho v = \mathrm{Const} / r^2$$



horizon cannot be achieved because continuity equation requires

$$\frac{d(\rho v)}{dv} > 0$$

Such instability is absent if speed of "light" c < s speed of sound

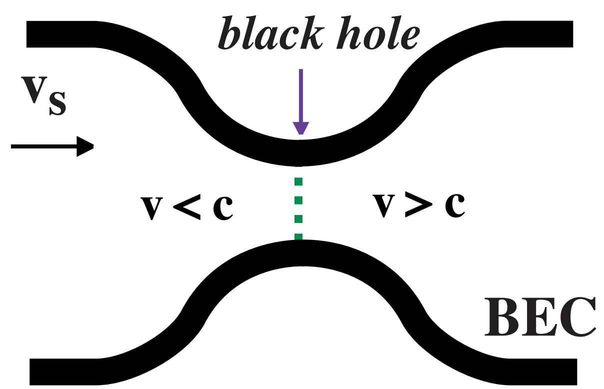
in Fermi superfluids $c << s$

Analog Black Holes

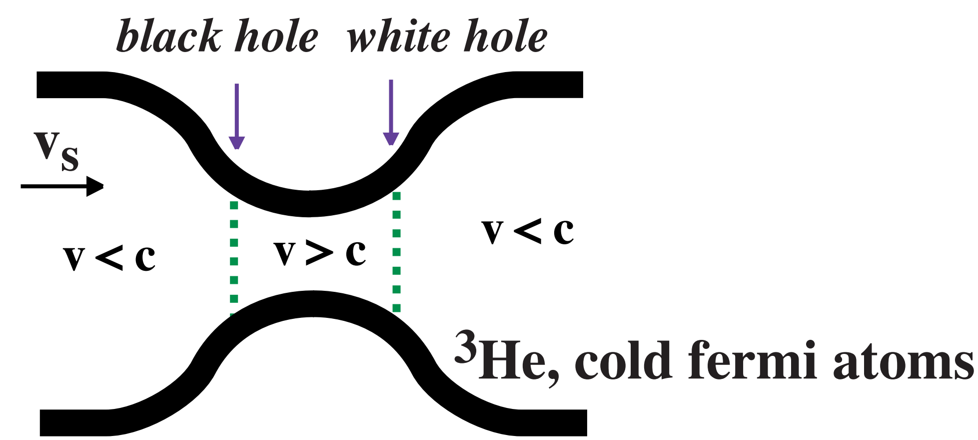
Painleve-Gulstrand metric

$$ds^2 = - dt^2\,(c^2-v^2) \; + \; 2\,v\,dr\,dt \; + dr^2 + \; r^2 d\Omega^2$$

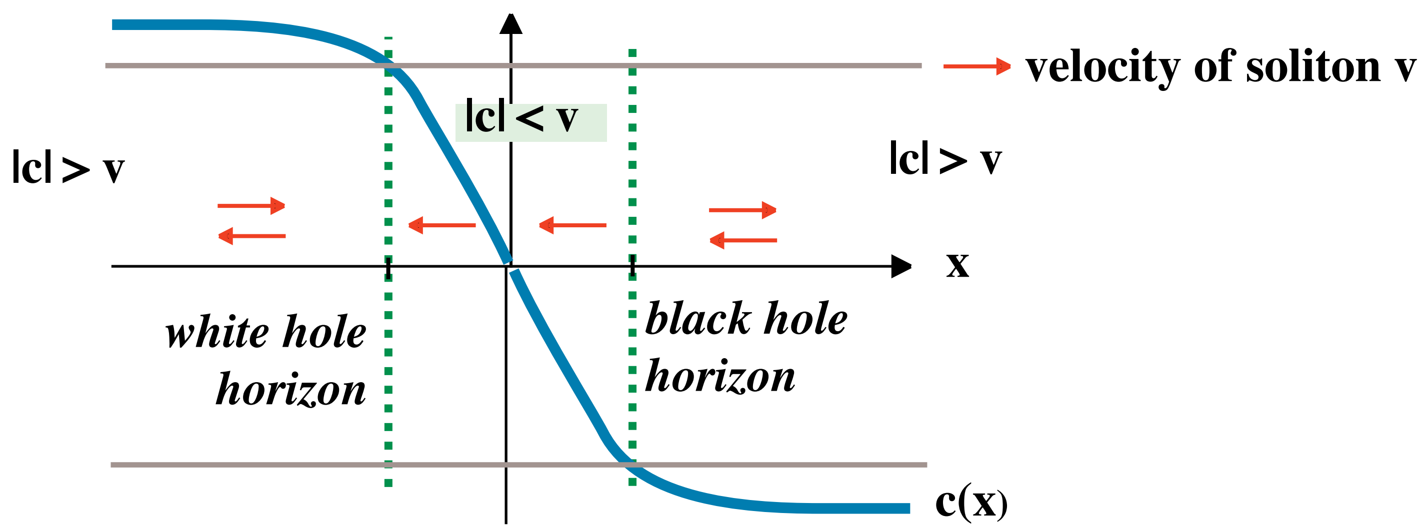
Acoustic horizon in Laval nozzle

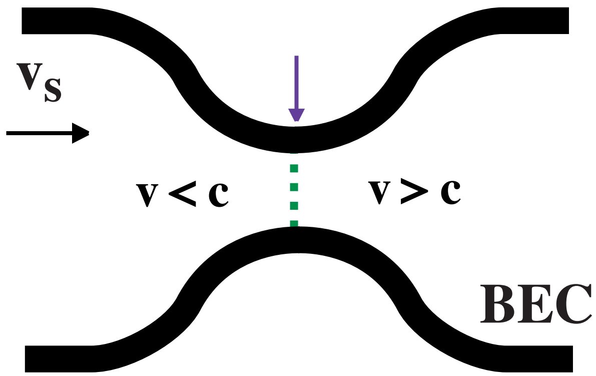


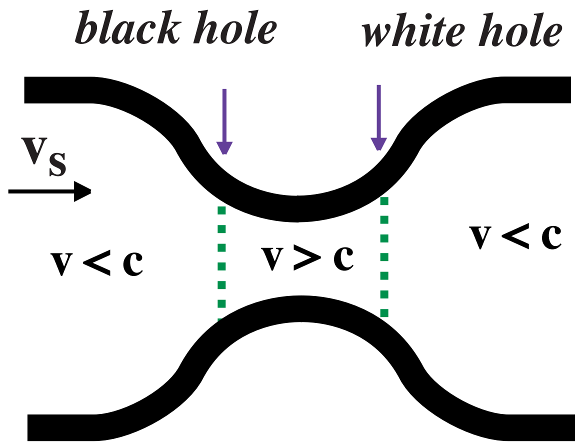
Horizons for quasiparticles in moving superfluids

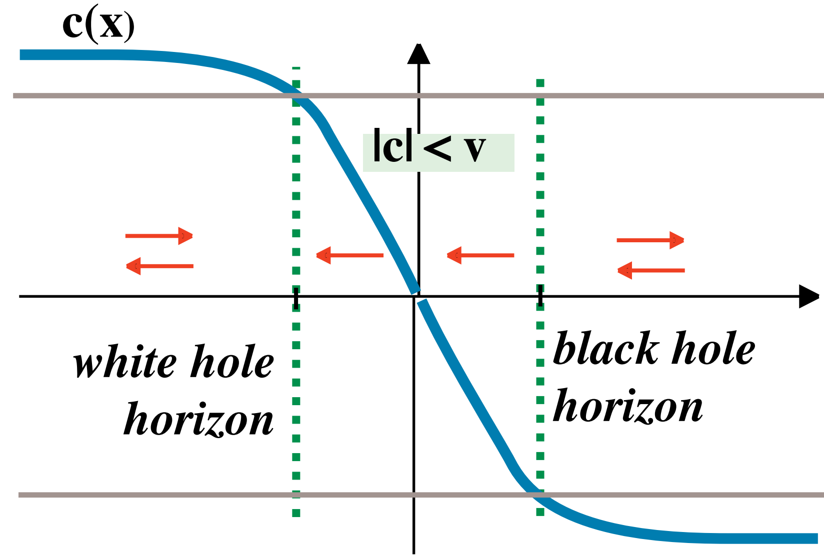


Horizons for quasiparticles in soliton









there are many principle difficulties with above schemes

Let us try surface waves

Schutzhold & Unruh 2002:
horizon for gravity waves in shallow water

Helsinki experiments 2002:
ergoregion instability for surface waves at the interface between superfluids

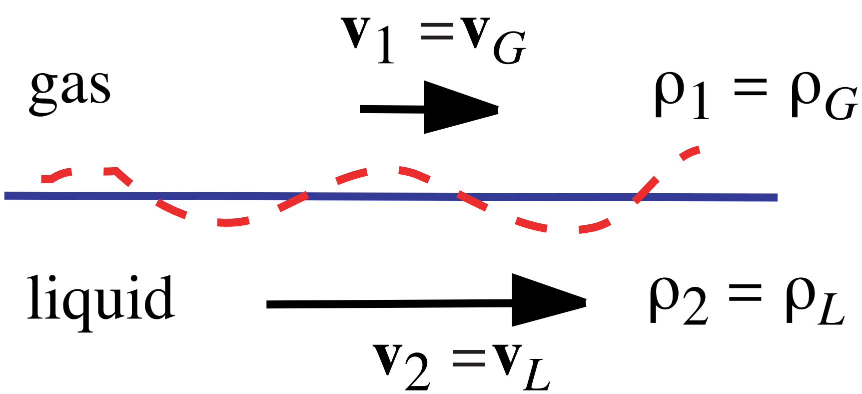
ENS experiments 2005 (Rolley et al. physics/0508200):
hydraulic jump in superfluids as white hole (physics/0508215)

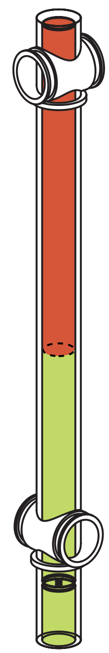
*** Kelvin-Helmholtz criterion** (dynamic instability of interface under shear flow)

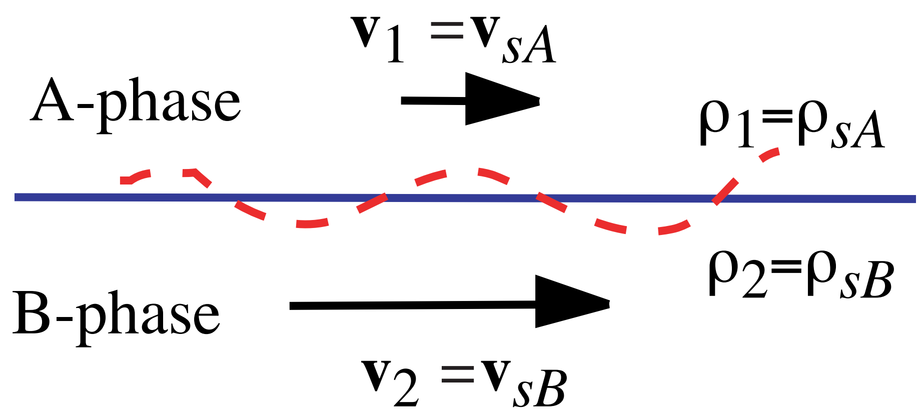
$$\tilde{\rho}(\mathbf{v}_1 - \mathbf{v}_2)^2 = 4\sqrt{F\sigma}$$

$$k_c = \sqrt{F/\sigma}$$

$$\tilde{\rho} = 2\rho_2\rho_1/(\rho_2+\rho_1)$$







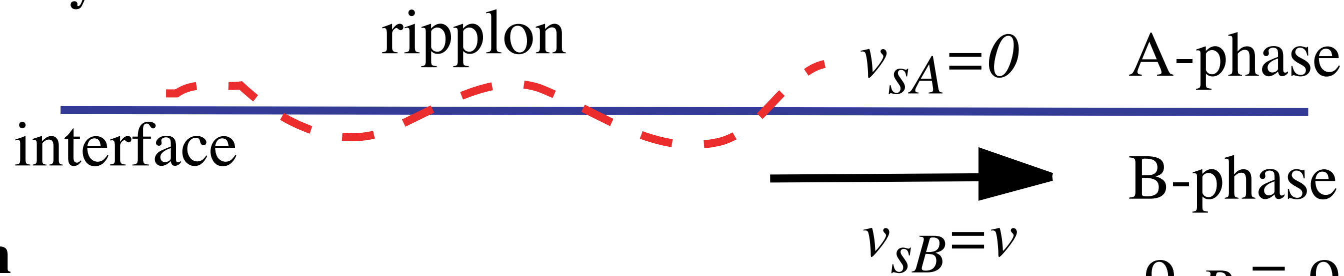
$$F = g(\rho_2 - \rho_1)$$

gravity in liquids

$$F = \nabla(\chi_B H^2 - \chi_A H^2)$$

magnetic forces in ^3He

3 criteria for interface instability at $T=0$



$\rho_{sB} = \rho$
$\rho_{sA} = \rho$

*** Kelvin-Helmholtz criterion**

(dynamic instability of interface under shear flow)

$$\boxed{\rho v^2 = 4\sqrt{F\sigma}}$$

$$\boxed{k_c = \sqrt{F/\sigma}}$$

*** Landau criterion**

(excitation of quasiparticles --

ripplons = capillary-gravity waves)

$$\boxed{v = min_k \frac{\omega(k)}{k}}$$

$$\boxed{\omega(k)=\frac{\sqrt{kF+k^3\sigma}}{\sqrt{2\rho}}}$$

ripplon spectrum in deep water

$$\boxed{\rho v^2 = \sqrt{F\sigma}}$$

$$\boxed{k_c = \sqrt{F/\sigma}}$$

*** Thermodynamic (ergoregion) instability criterion**

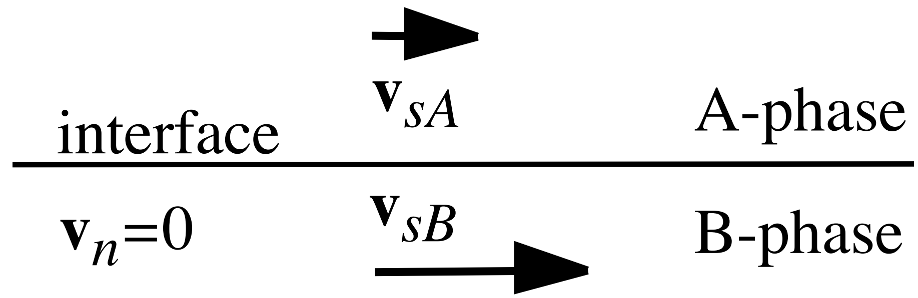
(negative free energy = ergoregion at T=0)

$$\boxed{\rho v^2 = 2\sqrt{F\sigma}}$$

$$\boxed{k_c = \sqrt{F/\sigma}}$$

*** Thermodynamic instability criterion**

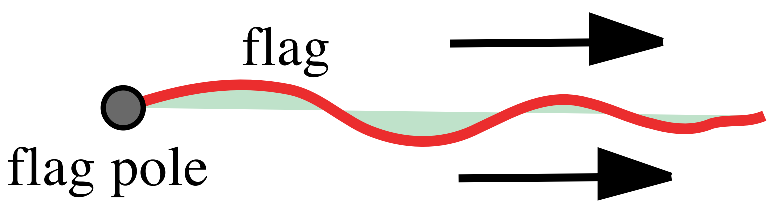
(negative free energy of perturbations)



$$\boxed{\rho_{sB}(\mathbf{v}_{sB}-\mathbf{v}_n)^2+\rho_{sA}(\mathbf{v}_{sA}-\mathbf{v}_n)^2=2\sqrt{F\sigma}}$$

instability is caused by winds of superfluid components
with respect to the normal component (or with respect to the container wall at T=0).
It occurs even if $\mathbf{v}_{sB}=\mathbf{v}_{sA}$

flapping of sails and flags (Rayleigh)



wind velocity with respect to flag pole
is the same on both sides of flexible membrane

Landau criterion (T=0)

interface v_{sA} A-phase $\rho_{sA} = \rho$

$v_n=0$ v_{sB} B-phase $\rho_{sB} = \rho$

* **Conventional Landau criterion** (applicable for single superfluid liquid only)

$$\boxed{\omega(\mathbf{k},\mathbf{v}_s)=\omega(k)+\mathbf{k}\bullet\mathbf{v}_s<0} \longrightarrow \boxed{v = \min \frac{\omega(k)}{k}}$$

* **Generalized Landau criterion** for two superfluid liquids

$$\boxed{\omega(\mathbf{k},\mathbf{v}_{sA},\mathbf{v}_{sB})<0}$$

ripplon spectrum

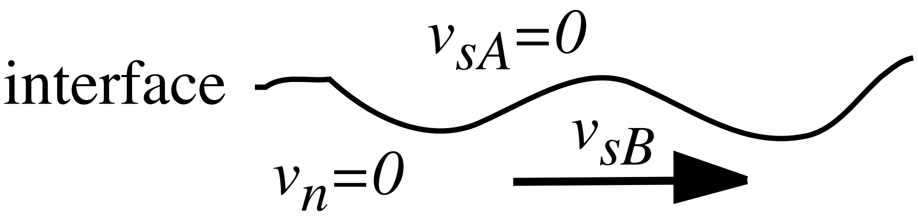
$$\boxed{\omega(\mathbf{k},\mathbf{v}_{sA},\mathbf{v}_{sB}) = \mathbf{k}\bullet(\mathbf{v}_{sA}+\mathbf{v}_{sB})/2 + \frac{\sqrt{kF+k^3\sigma - k^2\rho(\mathbf{v}_{sA}-\mathbf{v}_{sB})^2/2}}{\sqrt{2\rho}}}$$

$$\downarrow$$

$$\boxed{\rho(v_{sB}-v_n)^2+\rho(v_{sA}-v_n)^2=2\sqrt{F\sigma}}$$

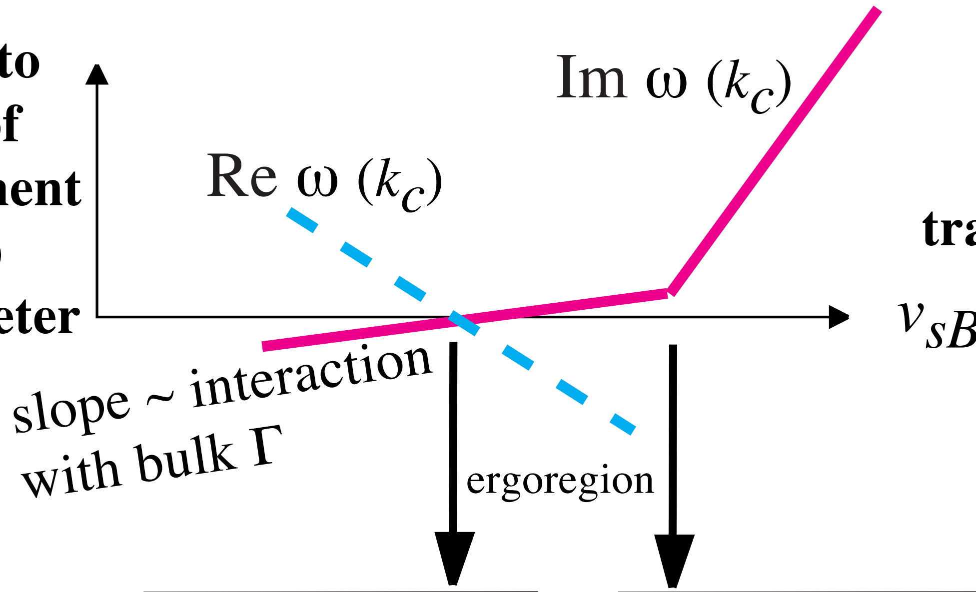
Coincides with thermodynamic instability criterion

Landau-Ergoregion instability vs Kelvin-Helmholtz instability
(effect of environment)



amplitude $a(t) \sim \exp(\, t \,\mathrm{Im}\, \omega\, (k_c)\,)$
increment of growth
of critical ripplon $k_c = \sqrt{F/\sigma}$

KH analysis taking into account interaction of interface with environment (normal component)
Γ – interaction parameter



Attenuation of ripplon due to environment transforms to amplification in the ergoregion

Thermodynamic & Landau ergoregion instability condition

$$\rho v_{sB}^2 = 2\sqrt{F\sigma}$$

$$\rho v_{sB}^2 = 4\sqrt{F\sigma}$$

KH instability condition at $\Gamma=0$

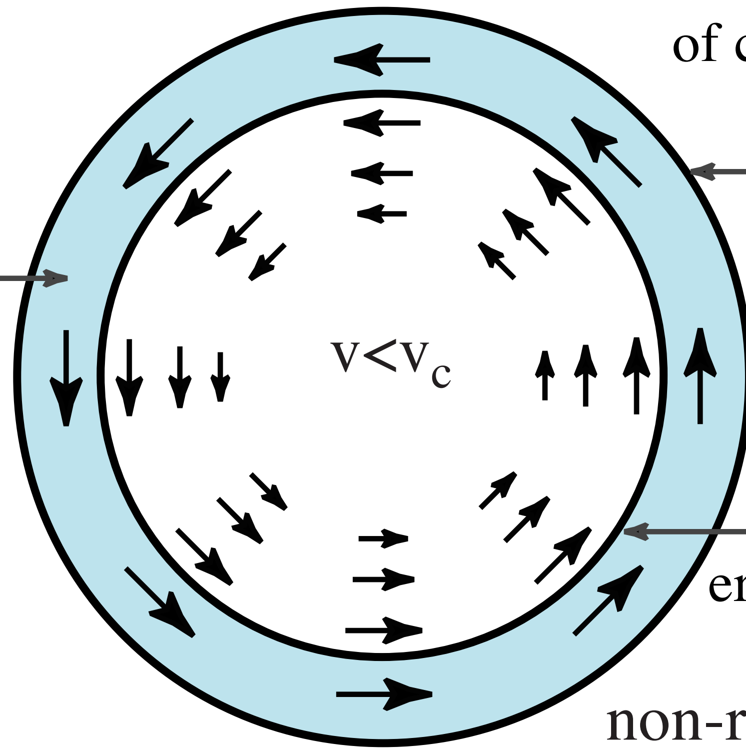
$\Gamma \sim T^3$ (Kopnin, 1987)

Ergoregion for ripplons living at the AB-brane in Helsinki experiments

interface between static B-phase and A phase circulating with solid-body velocity $v=\Omega r$
(velocity is shown by arrows)

boundary of the AB
interface
at side wall
of cylindrical
container

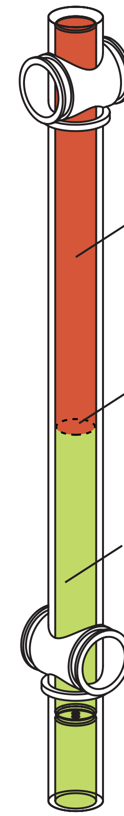
ergoregion:
region where
the energy
of some quasiparticles
is negative
in rotating frame



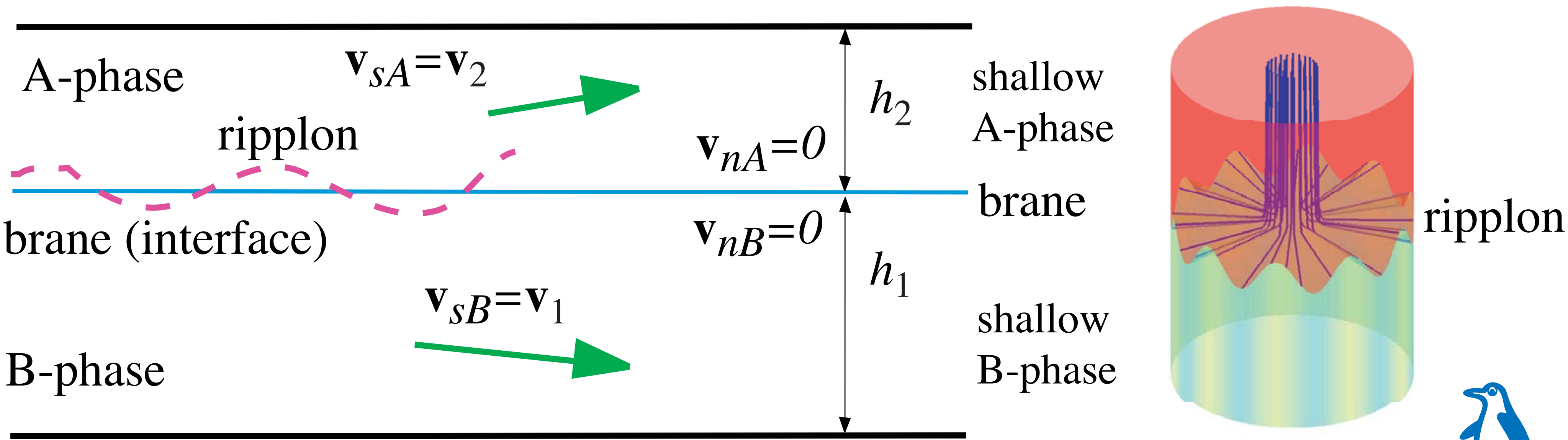
ergosurface

non-relativistic
analog of $g_{00}>0$

non-relativistic analog of $g_{00}=0$



Relativistic ripplons -- quasiparticles living on brane between two **shallow** superfluids (future experiments, see "The Universe in a Helium Droplet" Oxford 2003)



Spectrum of ripplons

$$\boxed{\alpha_1(\omega-\mathbf{k}\cdot\mathbf{v}_{sA})^2+\alpha_2(\omega-\mathbf{k}\cdot\mathbf{v}_{sB})^2 = c^2k^2}$$

Effective metric for ripplons

$$\boxed{ds^2 = -dt^2\,\frac{c^2-W^2-U^2}{c^2-U^2} + dr^2\,\frac{1}{c^2-W^2-U^2} + r^2\,d\phi^2}$$

Speed of light

$$\boxed{c^2=(F/\rho_s)h_1h_2/(h_1+h_2)}$$

$$U^2 = \alpha_1\alpha_2(\mathbf{v}_1-\mathbf{v}_2)^2 \qquad \mathbf{W} = \alpha_1\mathbf{v}_1+\alpha_2\mathbf{v}_2$$

Artificial black hole for ripplons at AB-brane (radial flow)

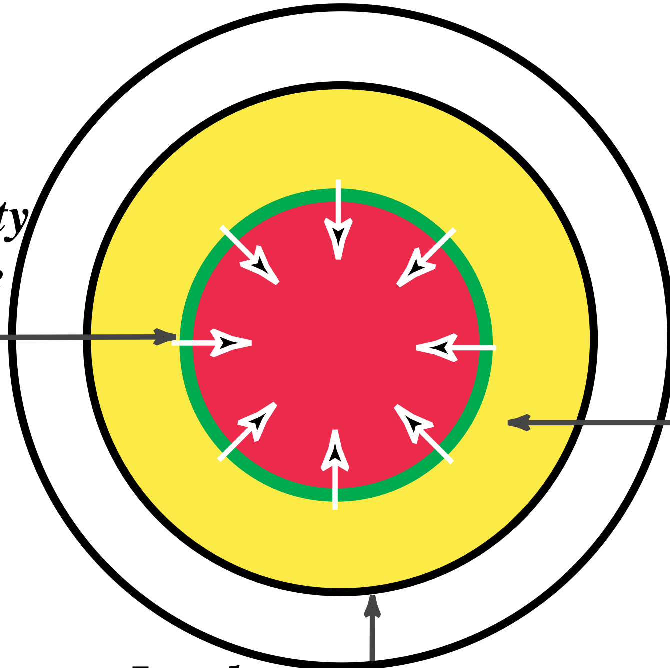
$g_{00}=\infty$

physical singularity within black hole

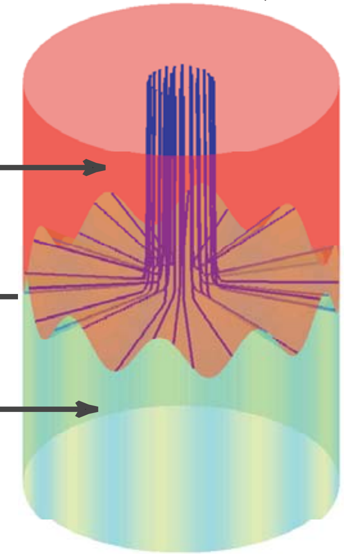
$\equiv$

Kelvin-Helmholtz instability

$U^2=c^2$



bulk A-vacuum



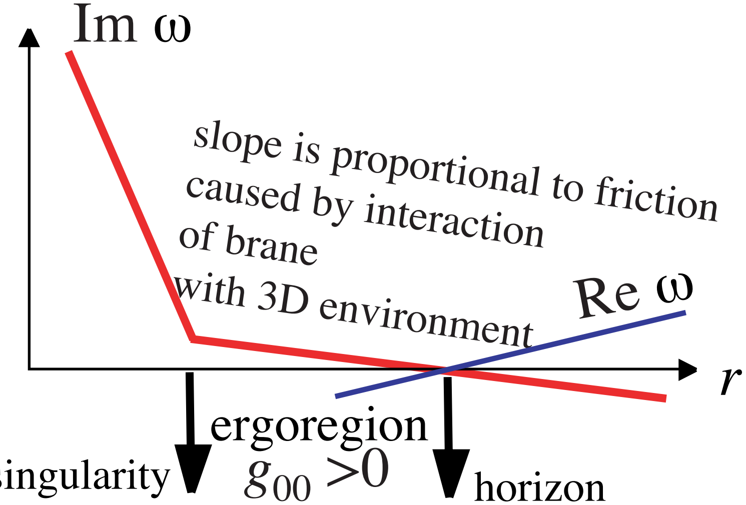
bulk B-vacuum

ergoregion

$g_{00}>0$

$W^2+U^2=c^2$ ***Landau instability*** $\equiv$ ***horizon*** $g_{00}=0$

$$ds^2 = -dt^2\ \frac{c^2-W^2-U^2}{c^2-U^2} + dr^2\ \frac{1}{c^2-W^2-U^2} + r^2\ d\phi^2$$



Im ω & Re ω

cross 0 simultaneously: behind horizon attenuation transforms to amplification

lesson from AB-brane:

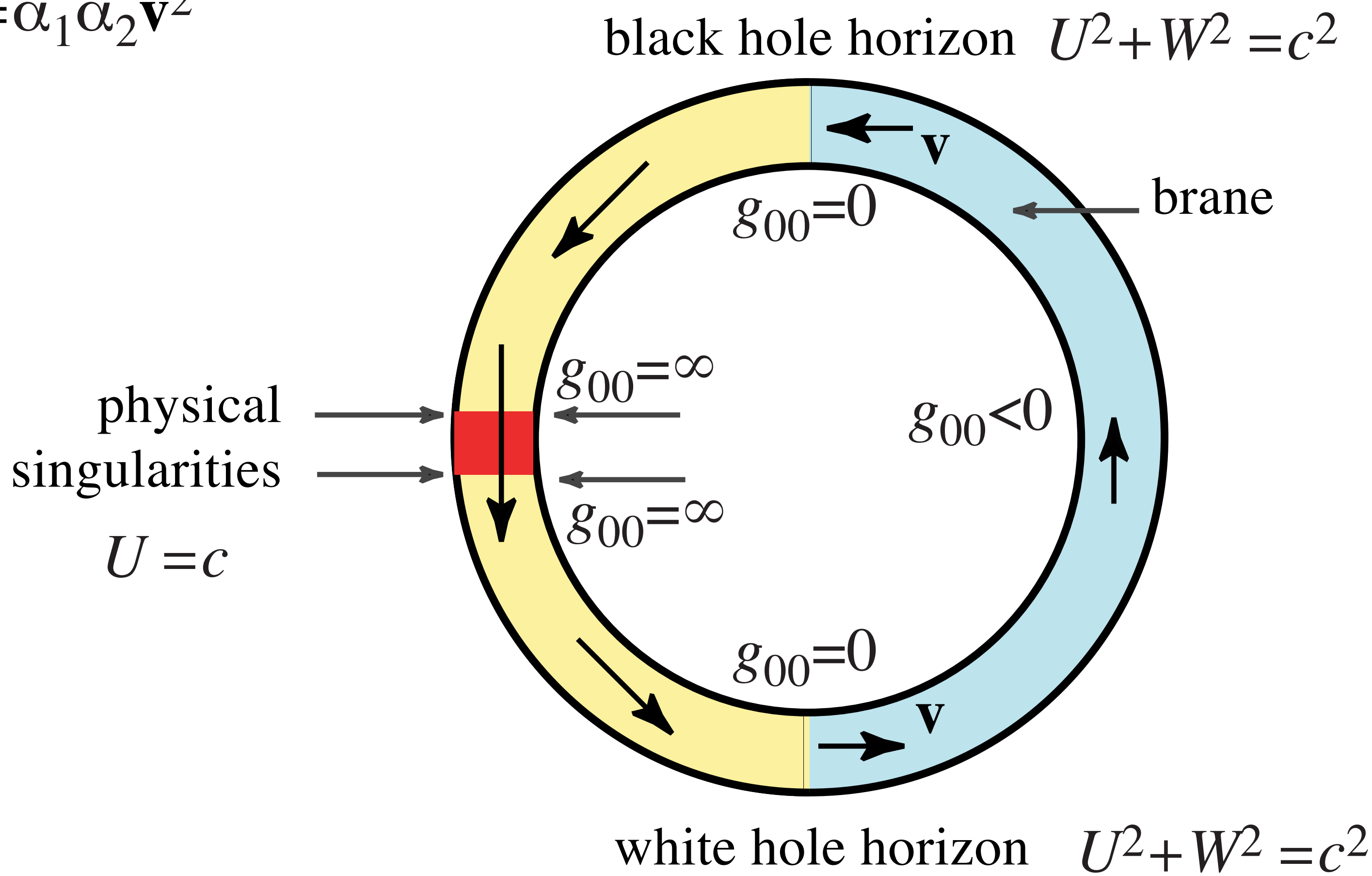
Black hole may collapse due to vacuum instability

Artificial black hole for ripplons within AB-brane (azimuthal flow)

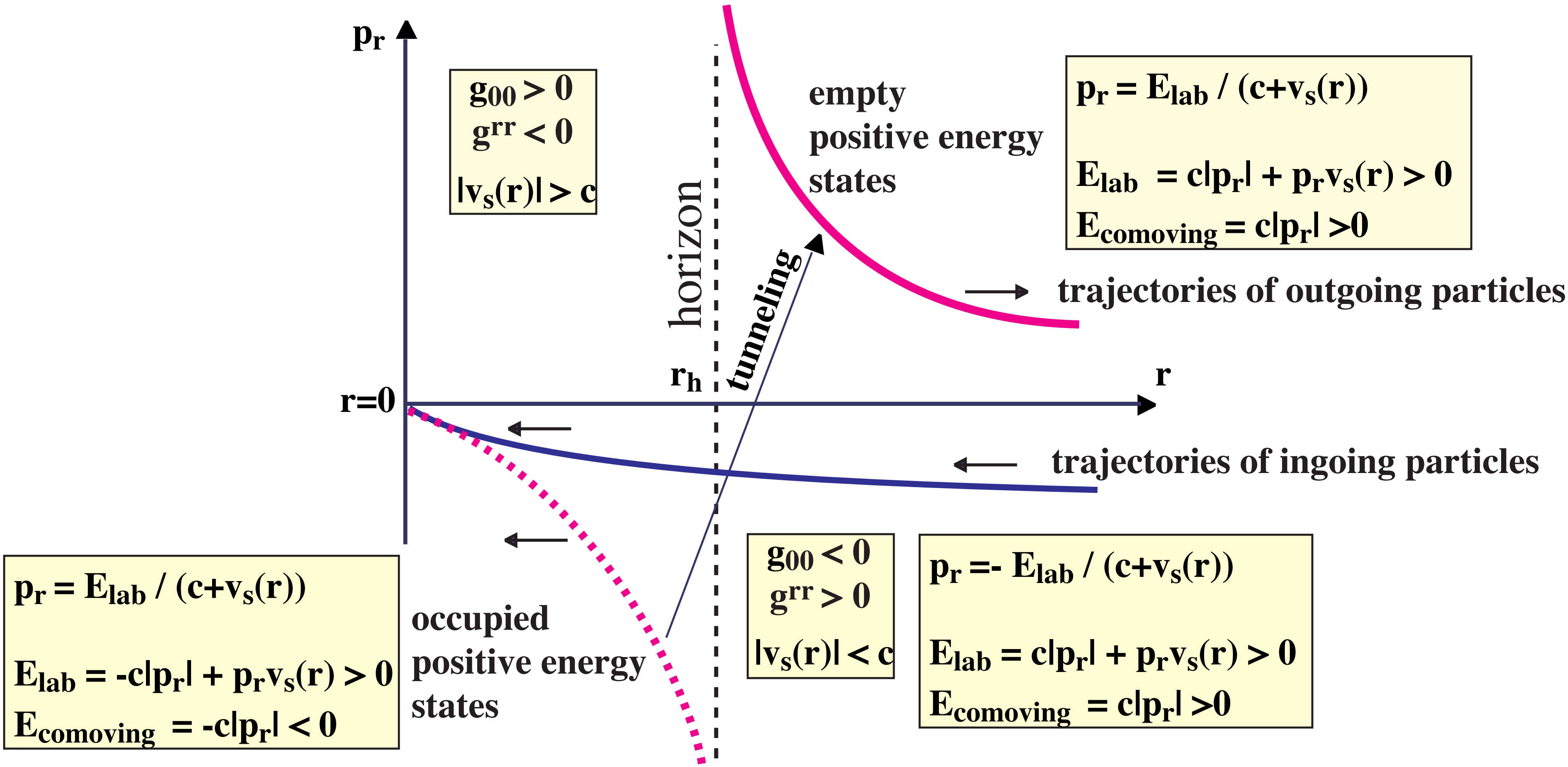
$$ds^2 = -dt^2 \frac{c^2 - W^2 - U^2}{c^2 - U^2} + r^2\, d\phi^2 \frac{1}{c^2 - W^2 - U^2} + dr^2$$

$W^2 = \alpha_1{}^2 \mathbf{v}^2$

$U^2 = \alpha_1 \alpha_2 \mathbf{v}^2$



Hawking radiation as tunneling



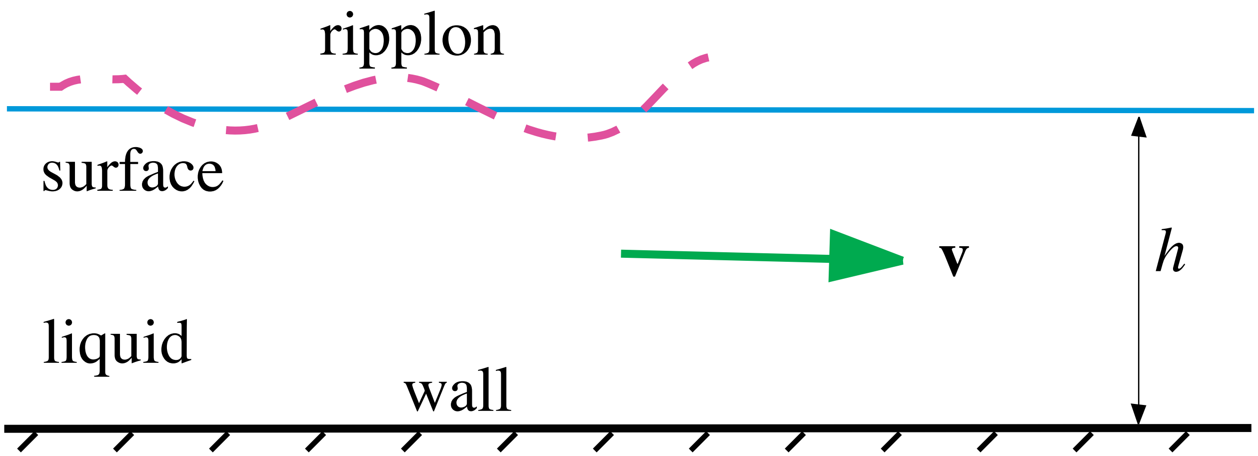
$$W = we^{-S} = we^{-E/T_H} \qquad S = 2\,\mathrm{Im}\int dr\, p_r\,(E_{lab}) = E_{lab}/T_H$$

tunneling exponent and Hawking temperature

$$T_H = v'/2\pi$$

Hydraulic jump

1. Relativistic ripplons in shallow water



Spectrum of ripplons

$$\boxed{(\omega-\mathbf{k}\cdot\mathbf{v})^2 = c^2k^2} + c^2k^2\,(k^2/k_c^2 - k^2h^2/3)$$

$$k << k_c = \sqrt{F/\sigma}$$

$$kh << 1$$

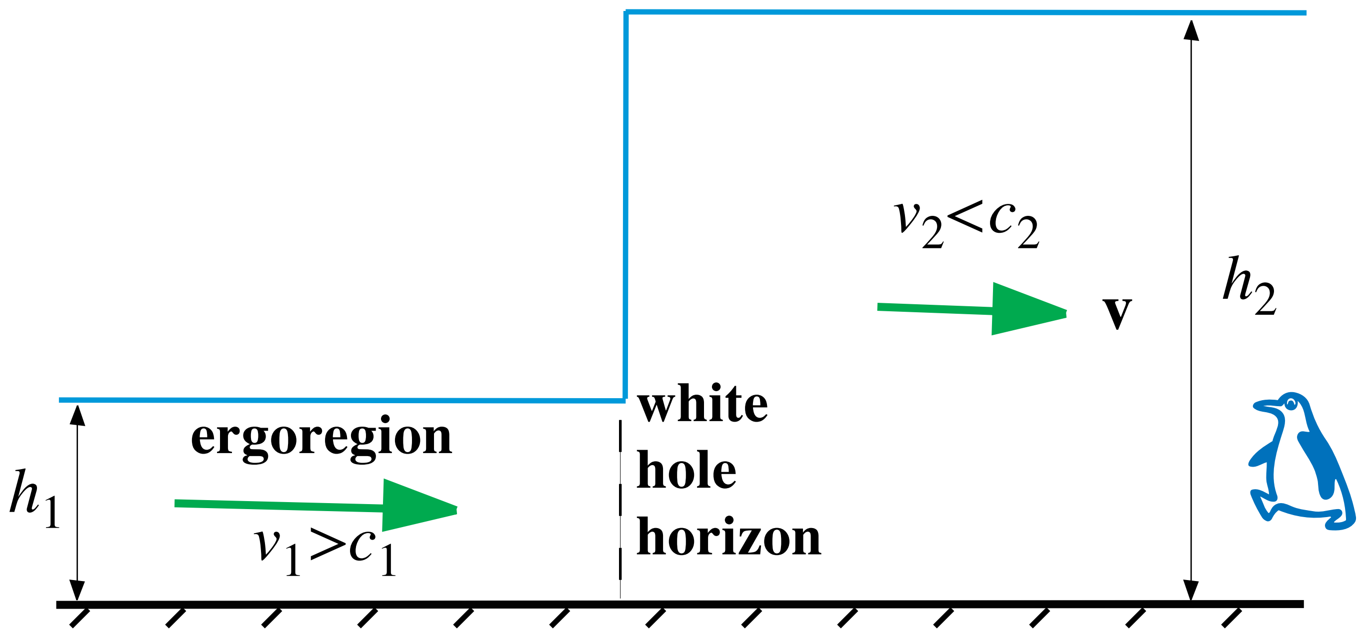
Effective metric for ripplons

$$ds^2 = -dt^2\,(1-v^2/c^2) + dr^2\,\frac{1}{c^2-v^2} + r^2\,d\phi^2$$

Speed of "light"

$$\boxed{c^2=(F/\rho_s)h}$$

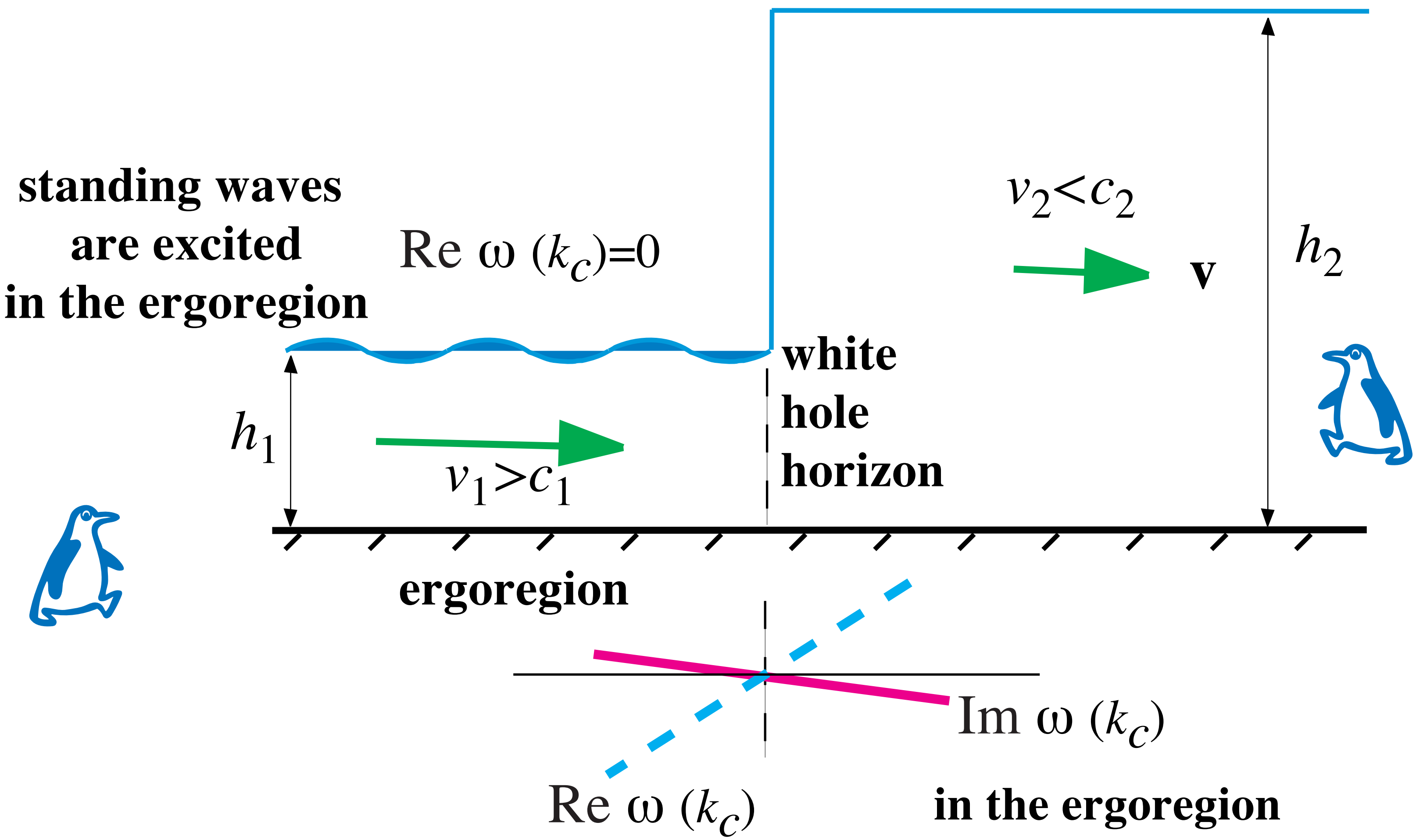
2. Hydraulic jump as white hole



$$ds^2 = -dt^2\,(1-v^2/c^2) + dr^2\,\frac{1}{c^2-v^2} + r^2\,d\phi^2$$

3. Observation of instability in the ergoregion

E. Rolley et al. physics/0508200



in the ergoregion
attenuation of ripplon
transforms to amplification

$$ds^2 = -dt^2\,(1-v^2/c^2) + dr^2\,\frac{1}{c^2-v^2} + r^2\,d\phi^2$$

Conclusion

* Ripplons on the surface of liquid or interface between liquids:
  best system for simulating event horizon of black & white holes

* Thermodynamic instability of interface:
  analog of vacuum instability in the ergoregion

* Kelvin-Helmholtz instability of interface:
  analog of black-hole singularity

* Lesson for gravity:
  **vacuum instability in the ergoregion may be
  the main mechanism of decay of black hole**

* Hydraulic jump in superfluids:
  first realization of instability of relativistic ergoregion