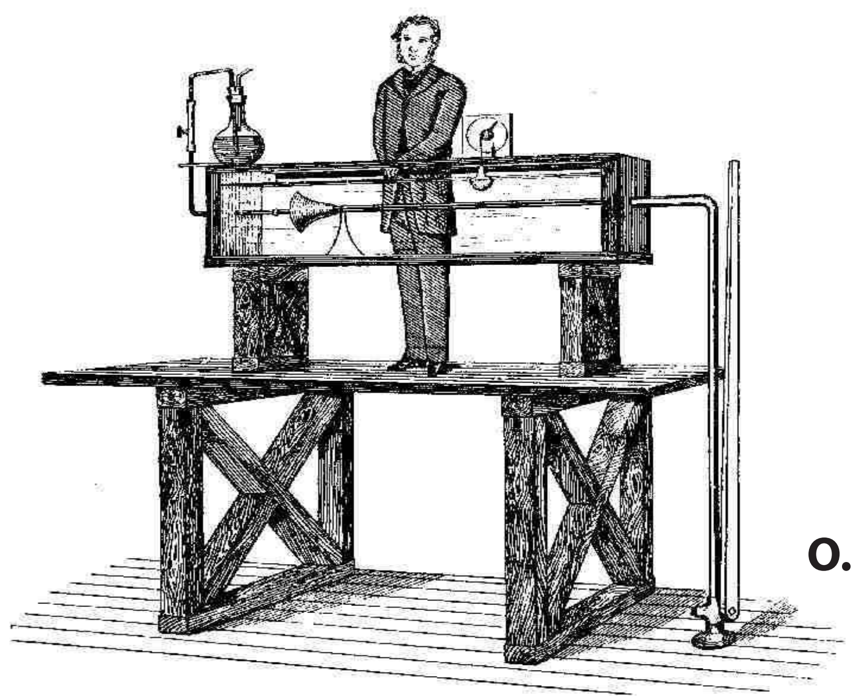
Transition to Turbulence in Pipe Flow



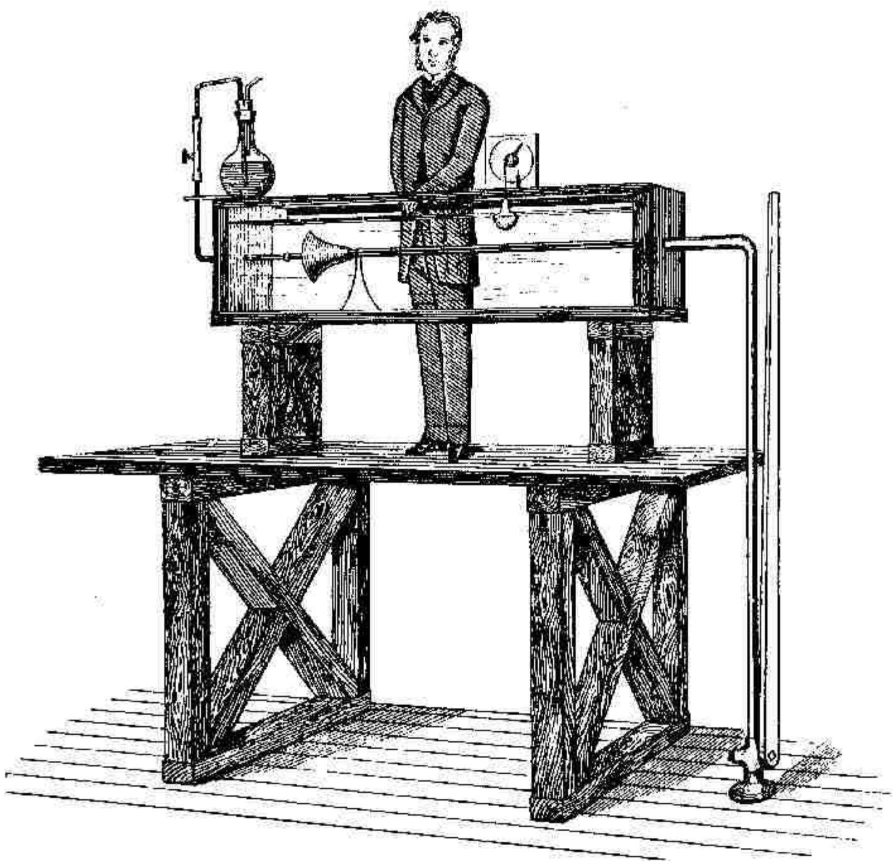
O. Reynolds 1883

T Mullin
Manchester Centre for Nonlinear Dynamics
with
J. Peixinho

Support from EPSRC

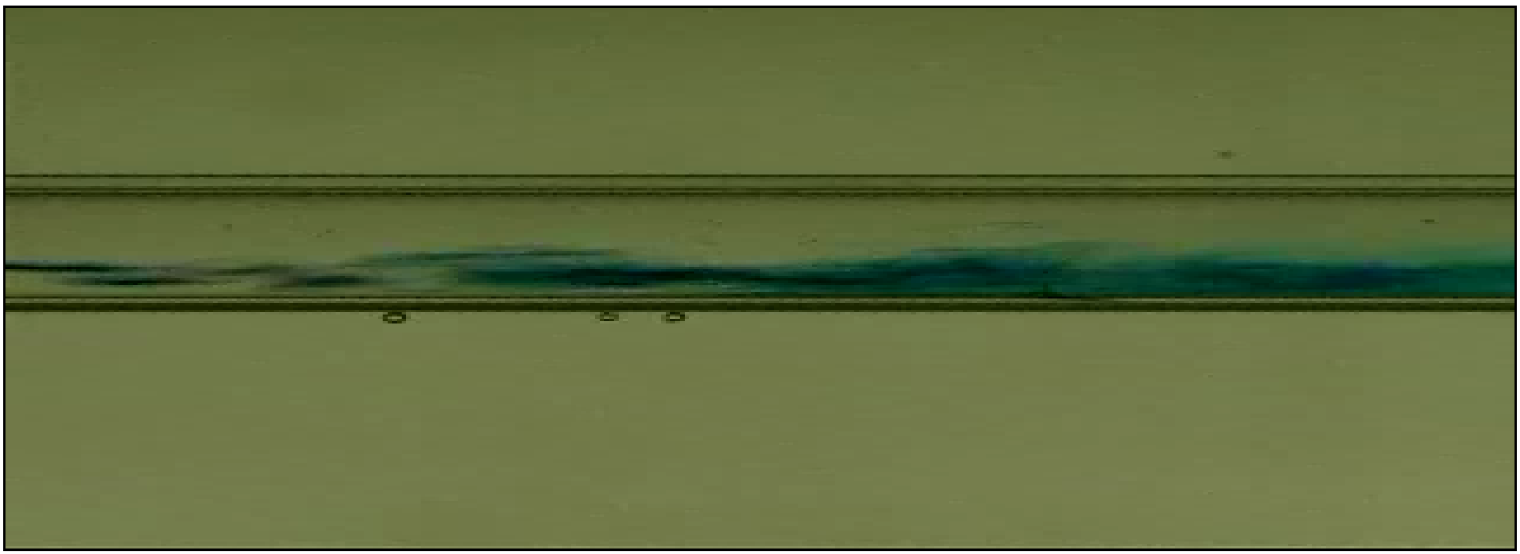
Warwick Dec.'05

Reynolds' experiment

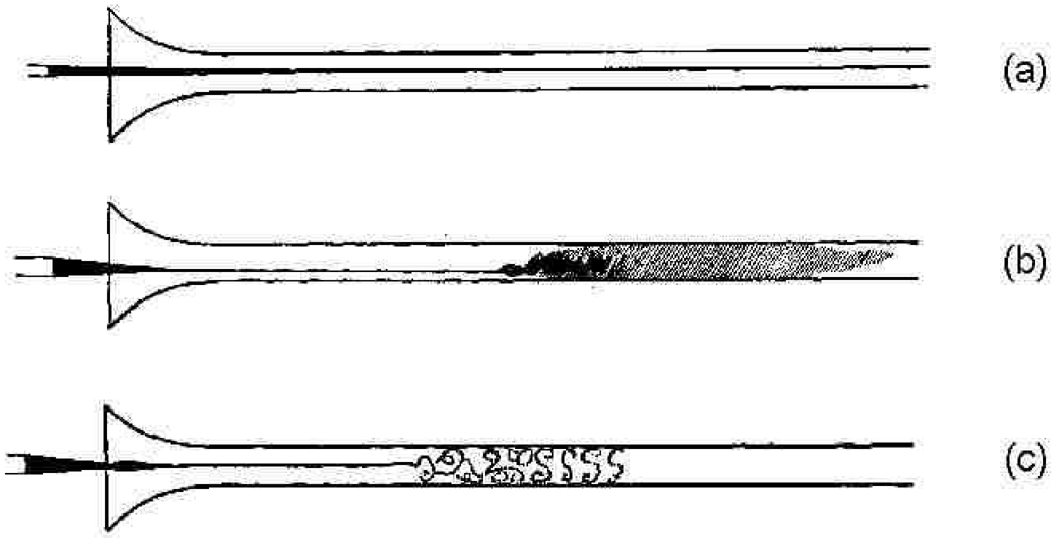


Royal Society, Phil. Trans., 1883

- Reynolds number: $Re = \frac{Ud}{\nu}$,
- Transition to turbulence for $Re > 2000$.



Stability of Reynolds' pipe flow



Reynolds number: $Re = \frac{Ud}{\nu}$,

with U mean velocity, d pipe diameter and ν kinematic viscosity.

In most experiments to date, natural or induced disturbances at the inlet

(not applied to the fully developed Hagen-Poiseuille flow profile, which is generally considered in theoretical studies).

Wygnanski & Champagne (1973):

(i) $2000 < Re < 2700$**:** turbulent puffs,

(ii) $Re > 3500$**:** turbulent slugs.

Entrance Flow

- Flat velocity profile, ‘inviscid’ core, viscous effects confined near wall.

- Shown to be linearly unstable above Re_c by Tatsumi (1952).

- Parabolic Hagen–Poiseulle flow takes $\sim Re/30$ diameters to develop in pipe
(eg ≈ 650 pipe diameters for $Re = 20,000$).

Our investigations are concerned with the stability of fully devloped Poiseuille flow.

Introduction

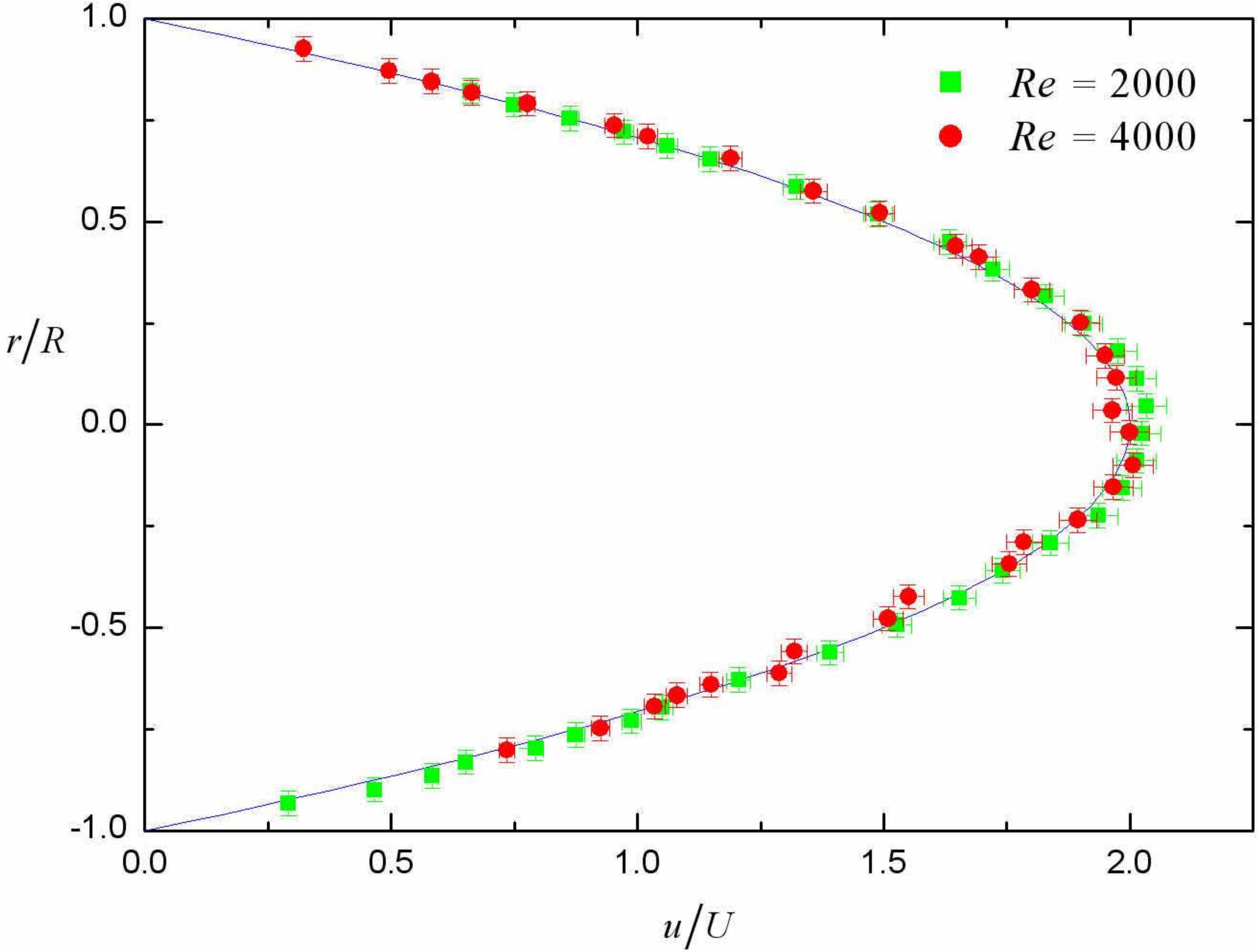
- Pipe flow is linearly stable
- Finite amplitude perturbation problem
- Laminar flow observed at Re =10^6 Pfenniger(1961) (Re =Ud/ν)

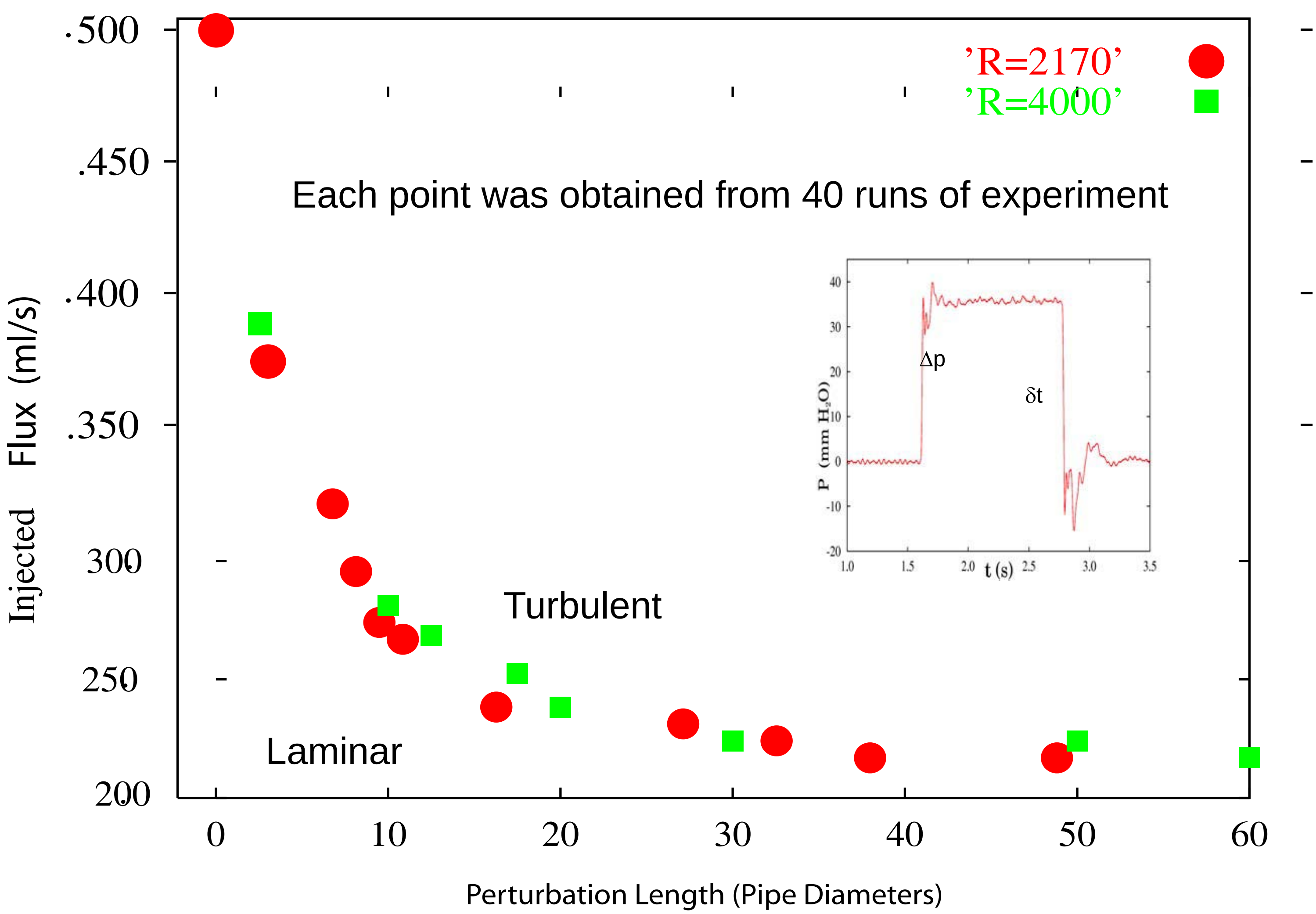
Experimental Setup

- Constant mass flux:
  Re can be varied in a controlled way
- Accurate in *Re* better than 1%
- Very long pipe: $D = 20\ \text{mm}$ and 800 D long
- Laminar flow can be achieved at $Re \sim 23\,000$

Picture of the Long Pipe

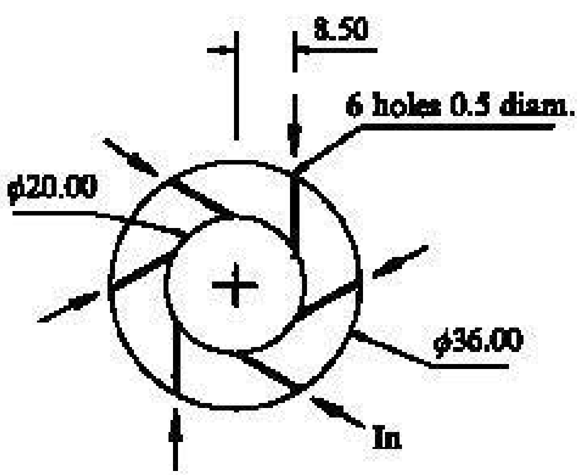
Laminar Profiles Measured In Pipe Using LDV



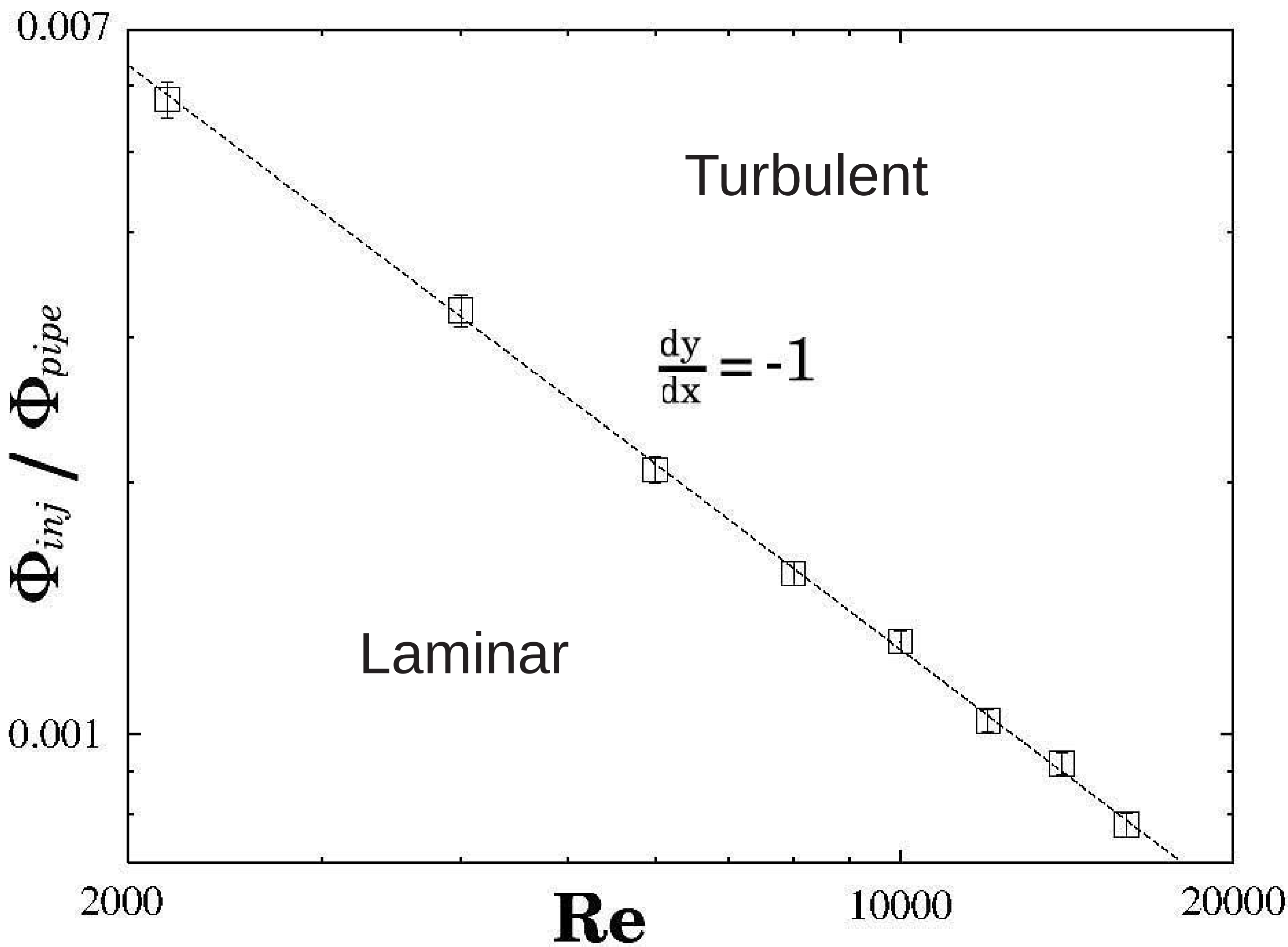


Amplitude versus length of perturbation in pipe diameters

Perturbation injected into six holes arranged azimuthally.



Stability in Long Pipe



Log-Log plot --> $\gamma \sim -1$

What happens for Re < 2000 ?

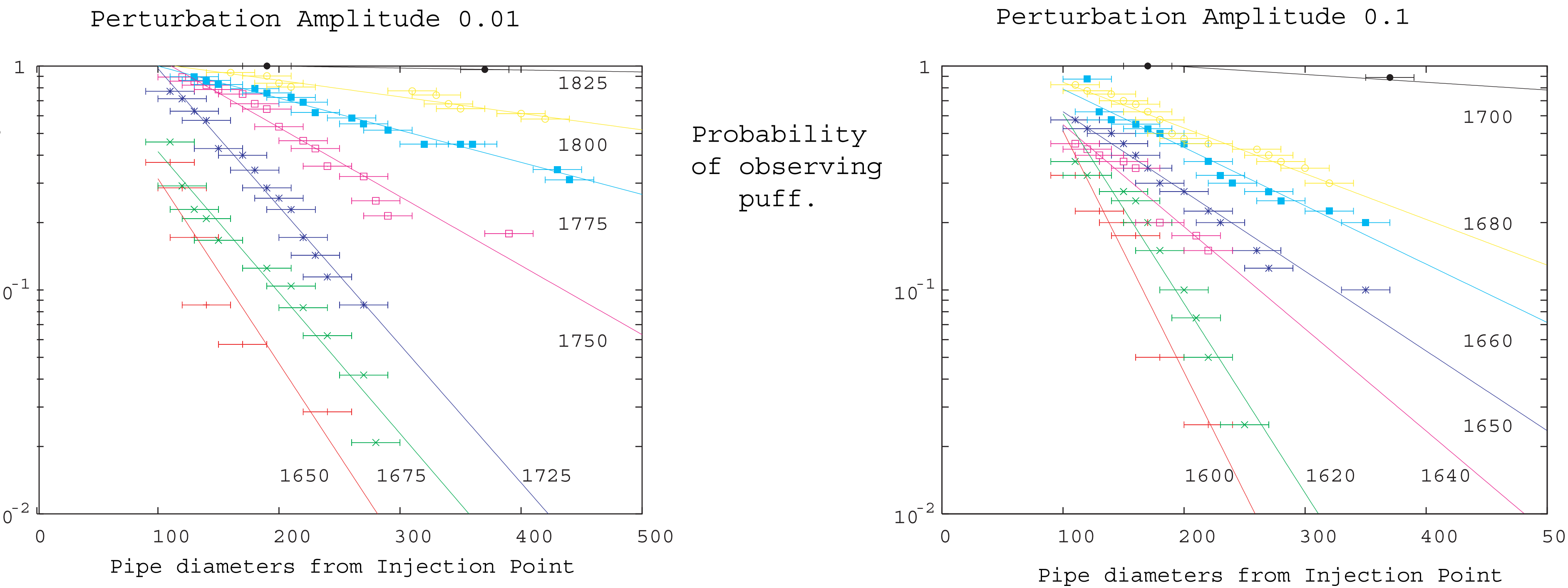
Lower Threshold for Sustained Turbulence

Observed that global stability as Re --> 0

Two investigations:

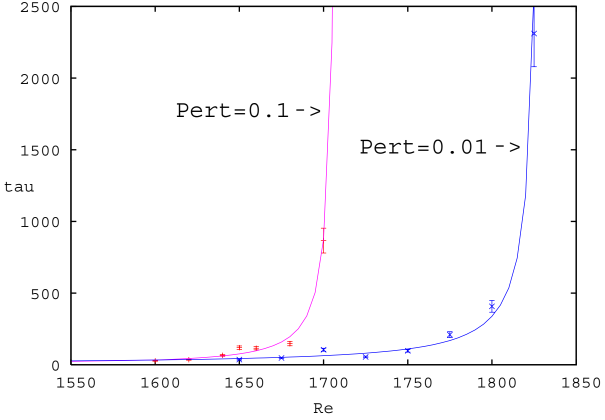
(a) seek boundary for zero growth of perturbation.

(b) seek boundary for zero decay of turbulence.

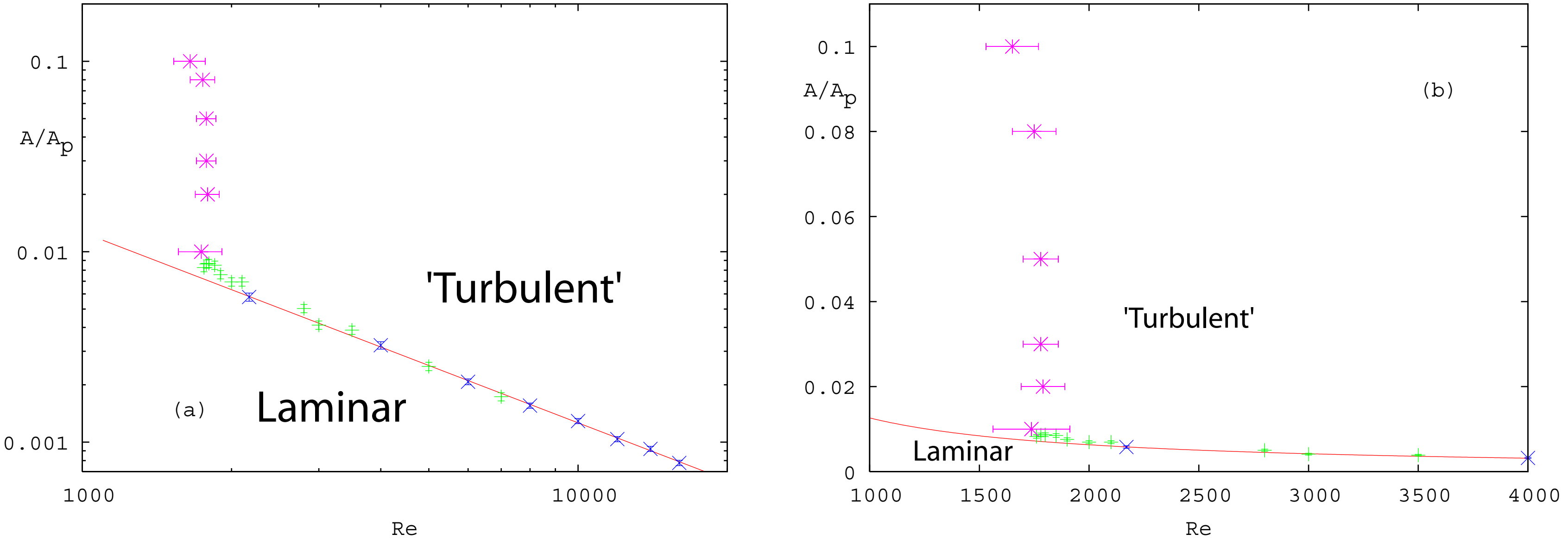


Decay of Injected Perturbations Below Threshold:
Exponential Decay Observed -> Critical Behaviour
(40 to 100 runs for each Re)

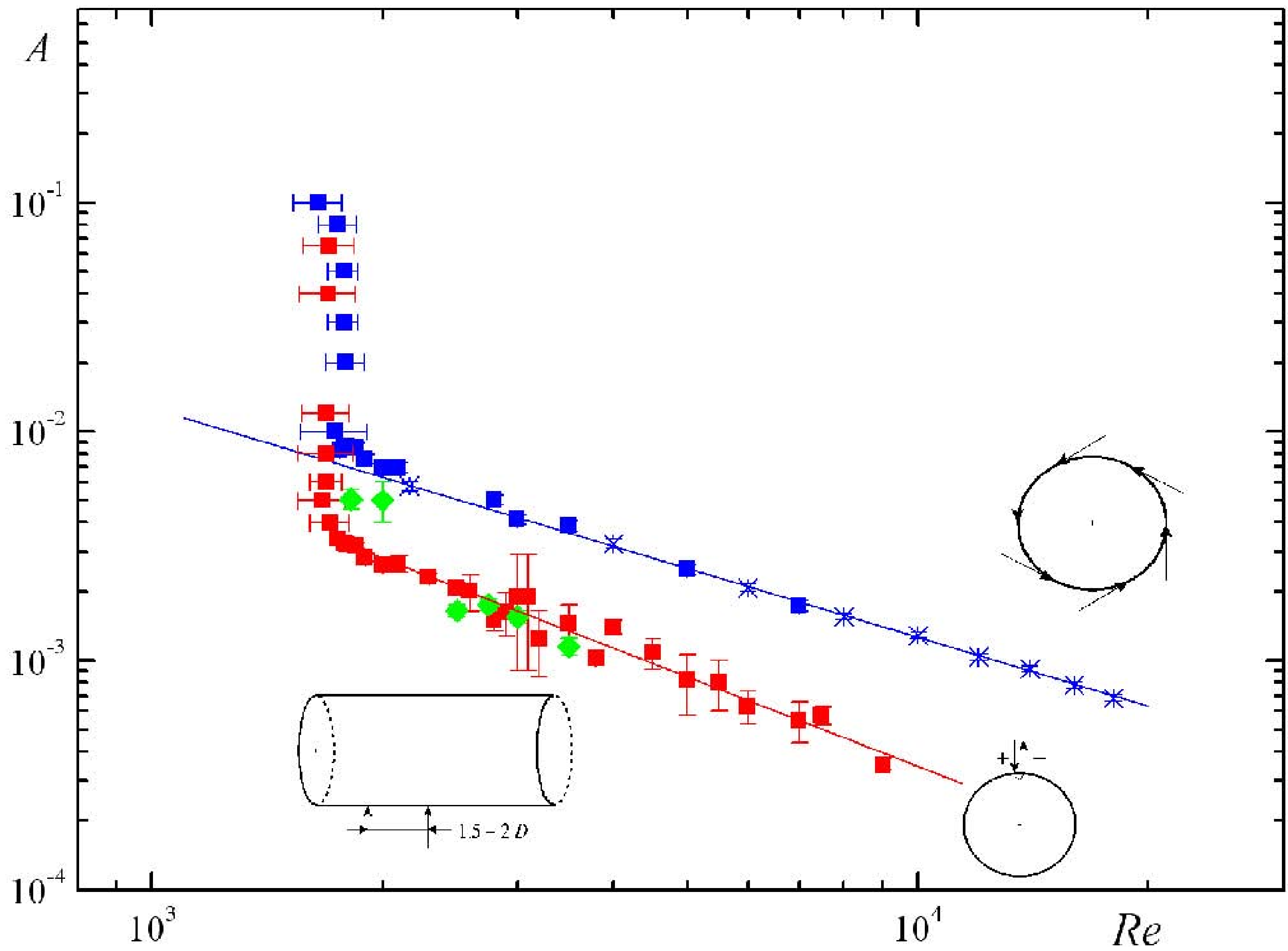
Extracted Exponents (0.01,0.1)



Threshold Curves (a) Log/Log (b) Lin/Lin



Threshold Curves with Different Perturbations.



Push-pull slope -1.4

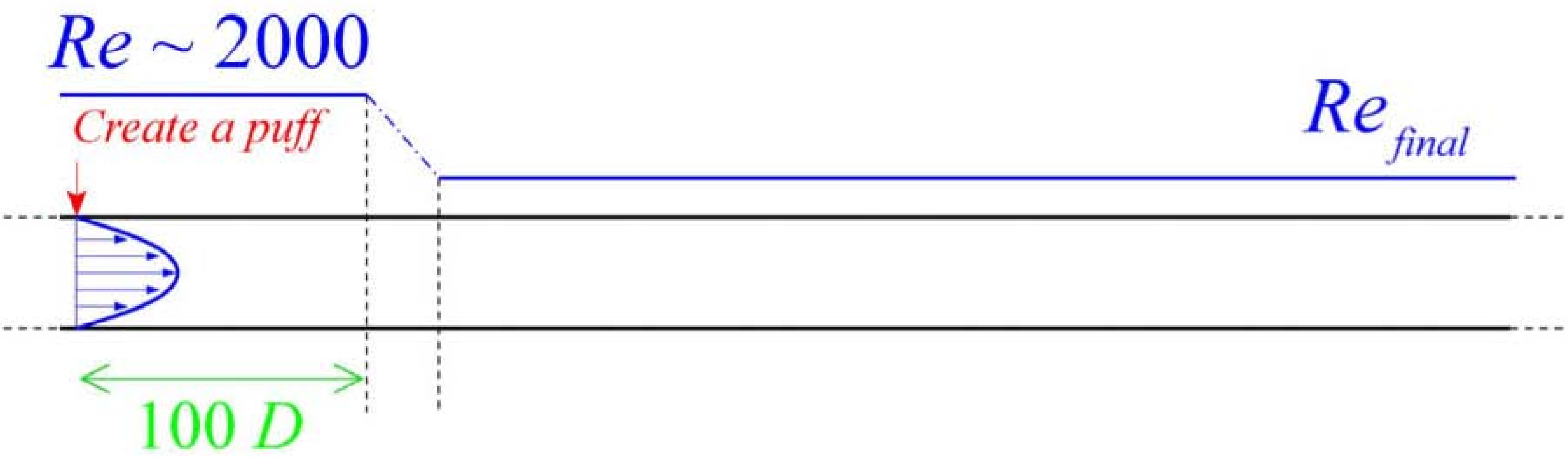
Azimuthal slope -1.0

Direct Transition is Catastrophic

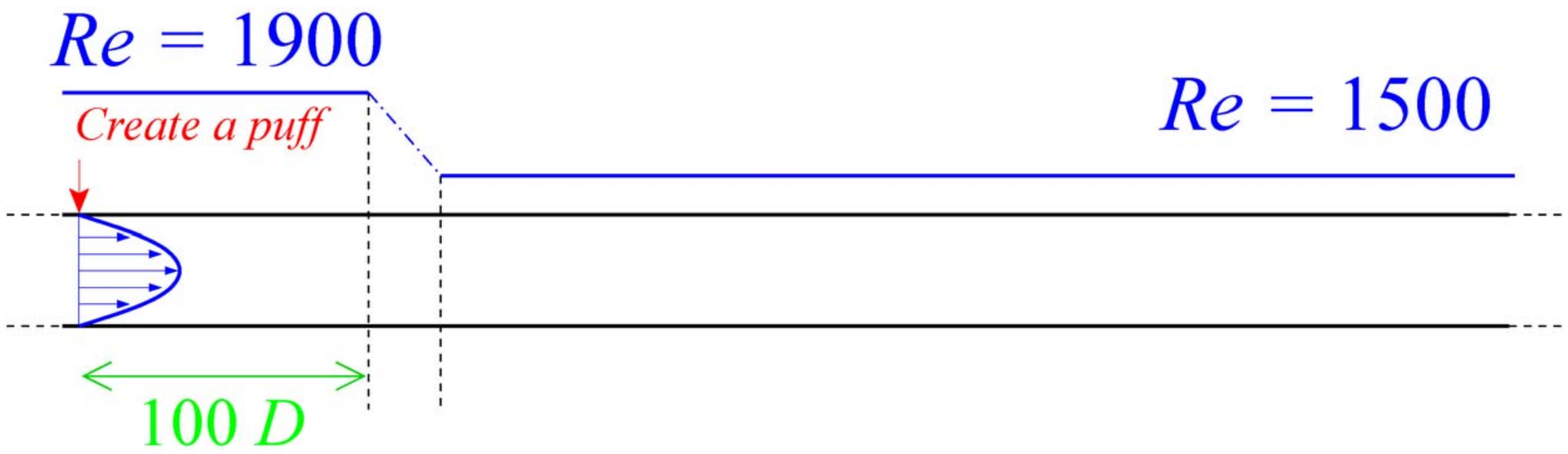
- Direction transition is very abrupt and highly dependent on the perturbation (type, amplitude, frequency …). Previous studies give rise to wide scatter in the data at low Re threshold.
- Transition from Turbulence: K. R. Sreenivasan, *Acta Mechanica* **44**, 1-48 (1982)

Fully turbulent-> Laminar Local reduction in Re by expanding pipe (Laufer 1962, Sibulkin, 1962)

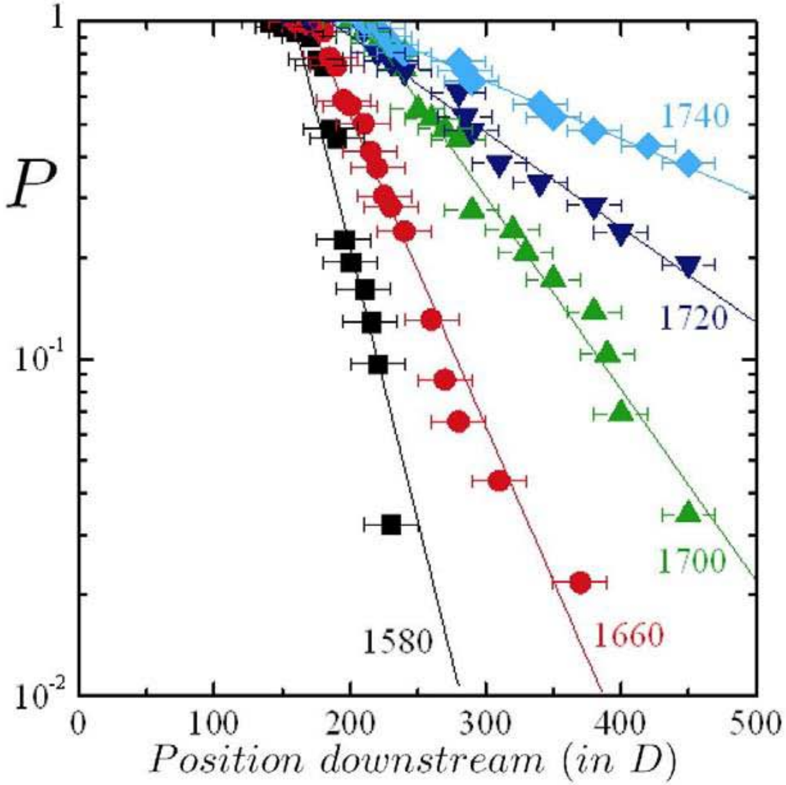
- At low Re, transition proceed via the "equilibrium puff". Our idea is to study the stability of the equilibrium puff by reducing Re



Transition *from* Turbulence

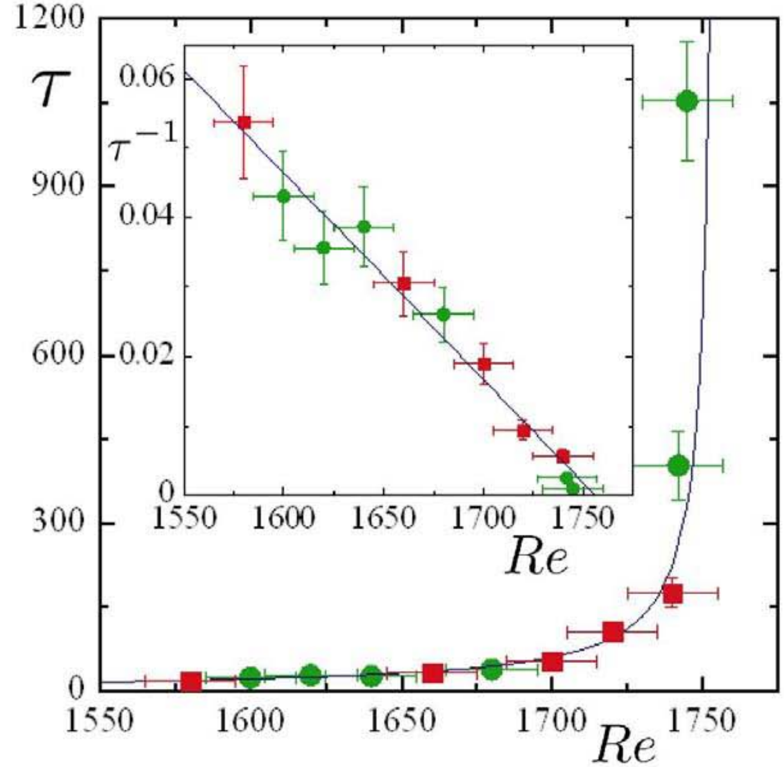


Transition *from* Turbulence



$$P(D) \propto \exp(-CD)$$

$$\tau = (\ln 2)/C$$



Additional points obtained with range of perturbations

$$Re_c = 1750 \pm 10$$

Fewer runs required to obtain good statistics than injection case.

Exponential decay in probability of observing puff downstream ----> Poisson process.

Divergence of timescales ---------> deterministic behaviour.

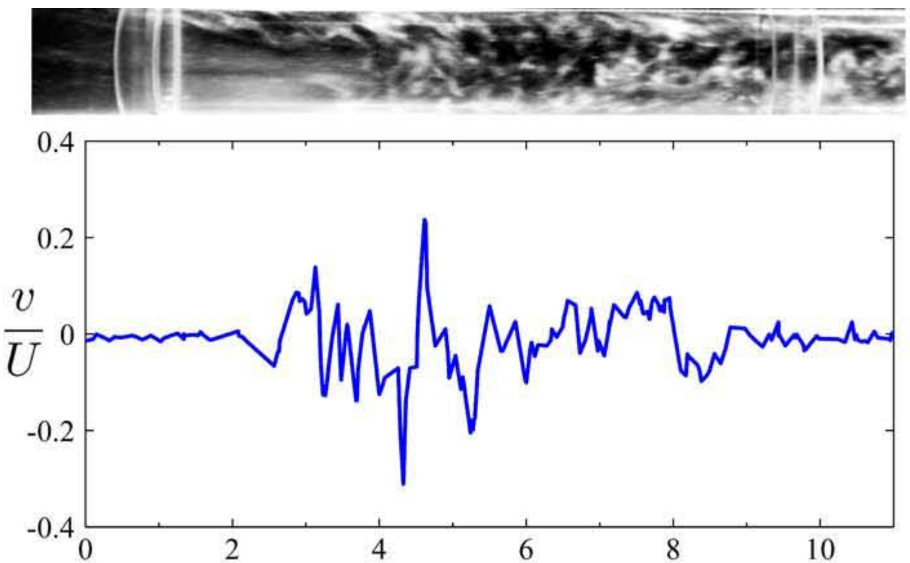
Qualitatively similar to boundary crisis of attractor Grebogi, Ott & Yorke (1986)

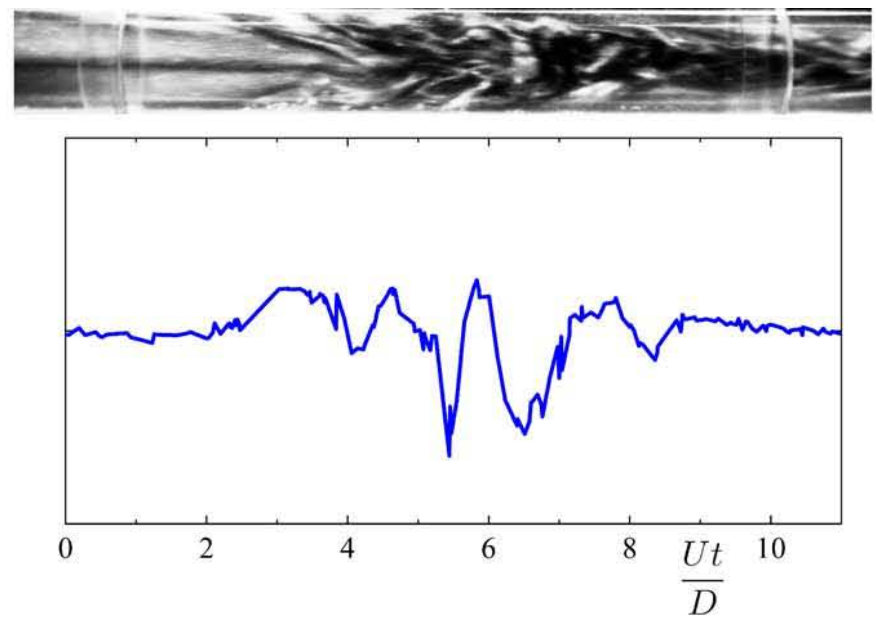
But low-d systems: exponents < 1

Wavy Patterns

Puff at $Re = 1900$

After reduction of Re down to 1750





Disordered signal

Contain wavelength of $1.5\ D$

Faisst&Eckhardt(2003)
Wedin &Kerswell(2004)

Conclusions

- A clear scaling law for the transition to turbulence has been established.

- Simplest implication given by a balance of the inertial and viscous terms in the Navier Stokes equation.

- Slowing down indicates critical behaviour; presence of waves suggest exciting prospects lie ahead.

Reference

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