Warwick turbulence symposium, Mathematics Institute, 5-9 December 2005
« **Universal features in turbulence : from quantum to cosmological scales** »

Waves and turbulence in the solar wind

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Waves and turbulence in the solar wind

- Some general properties
- Large scale fluctuations (below 1Hz - MHD scales)
- Small scale fluctuations (beyond 1Hz - Hall MHD)
- Dispersive waves and turbulence
- Conclusion

It is a short review !

For more information read, for example :

Bruno & Carbone,
« The solar wind as a turbulence laboratory »,
http://www.livingreviews.org/lprsp-2005-4, 2005
Living reviews in solar physics

The Solar Wind (E. Parker, 1958)

- Continual and variable outflow from the Sun (heliosphere ~ 100AU)

- Magnetized and collisionless plasma

- Variable from **~ 10^{-7} Hz to 10^{2} Hz**

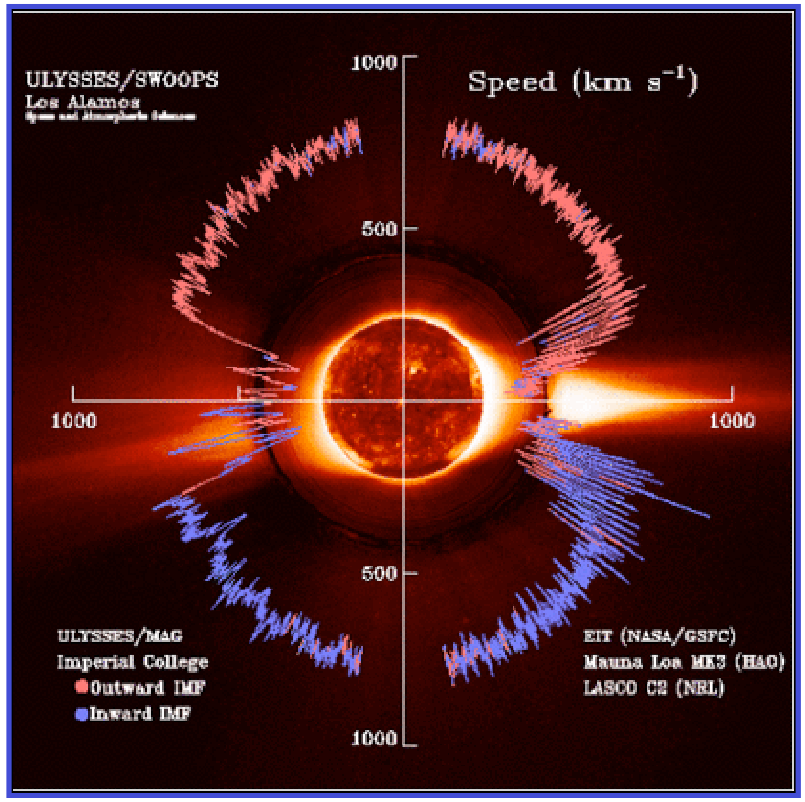
- Hot plasma > 10^5 K

- Rarefied plasma : n ~ 10^7 m^{-3} at Earth

- **Fast** and **slow** winds (> $20R_{SUN}$)

- Different polarity in each hemisphere

- Weak density variation (few %)



Scales in the Solar Wind

Spatial scales

• Heliocentric distance :	**L**	~ 10^8 km
• Ion inertial length (1AU) :	$d_i = V_A/\omega_{ci}$	~ 100 km
• Coulomb free path :	ℓ_c	~ 10^7 km

Temporal scales

• Solar rotation :	Ω_{SUN}	~ 5 10^{-7} Hz	MHD
• Alfvén waves :	$1/\tau_A$	< 0.1 Hz	
• Ion-cyclotron frequency (1AU) :	ω_{ci}	~ 0.5 Hz	HALL
• Whistler waves :	$1/\tau_W$	~ 1-10^3 Hz	

Turbulence in the heliosphere

Questions and problems:

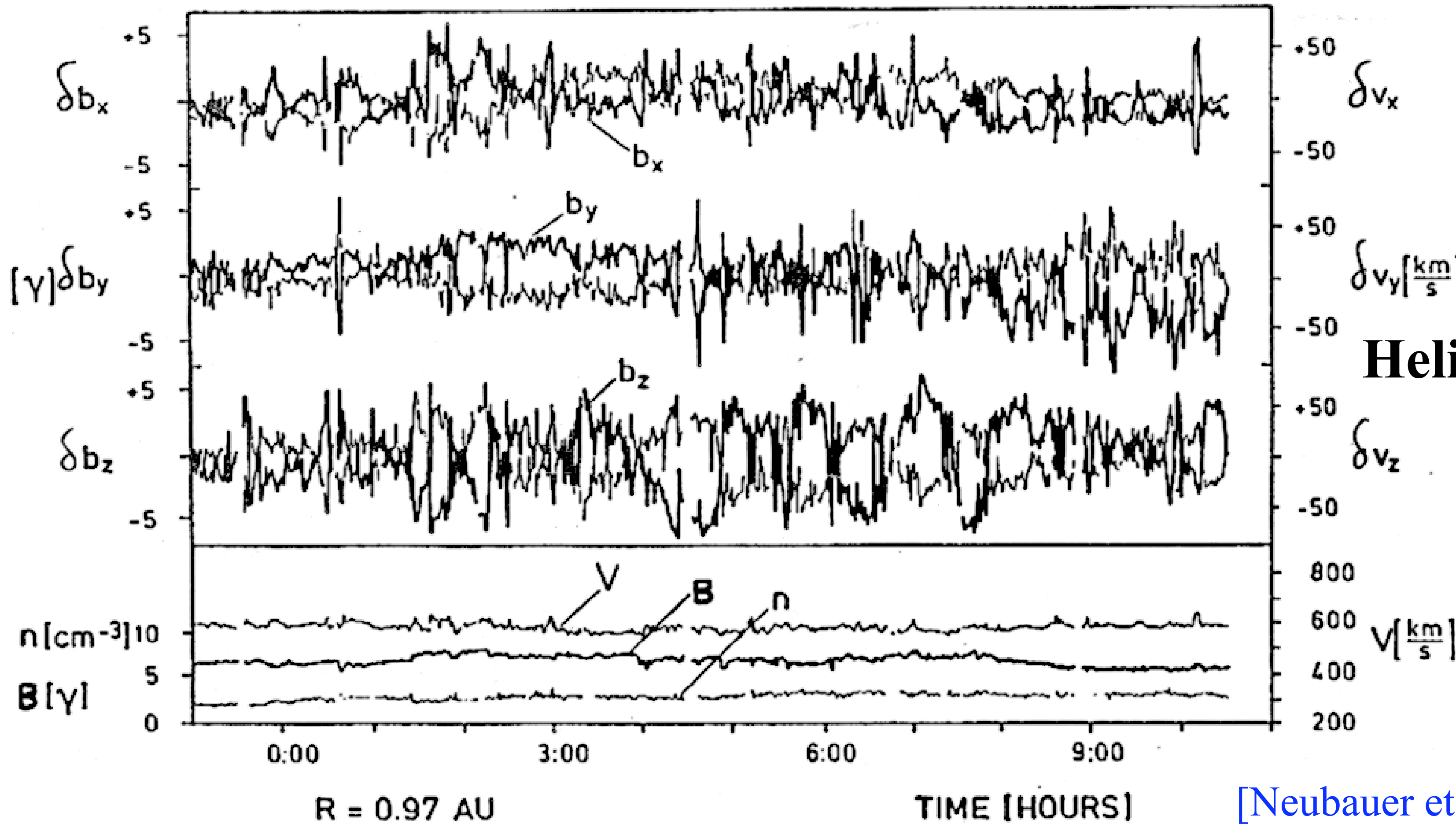
- *Nature and origin of the fluctuations*
- *Spectral transfer of turbulent energy*
- *Spatial evolution with heliocentric distance*
- *Microphysics of the « dissipative » range*

Large scale turbulence

in

the solar wind

Alfvénic fluctuations



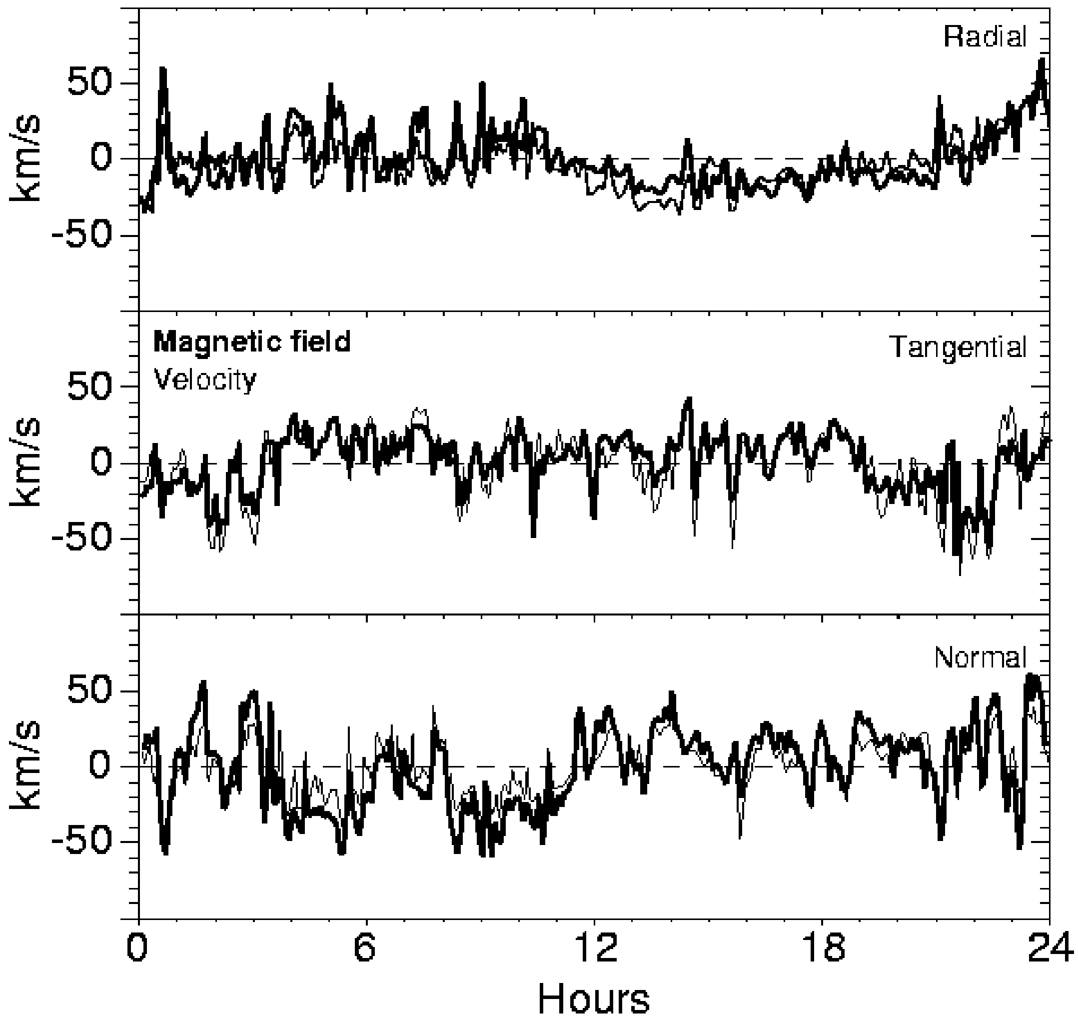
Helios data

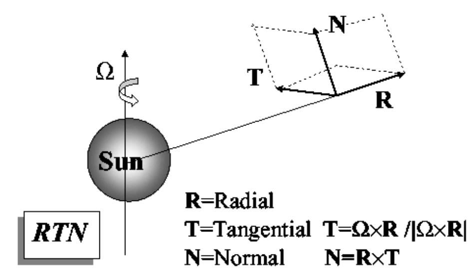
[Neubauer et al., 1977]

[Coleman, 1968]

δv and δb are anti-correlated : **δv ≈ -δb**

Alfvénic fluctuations





Ulysses data

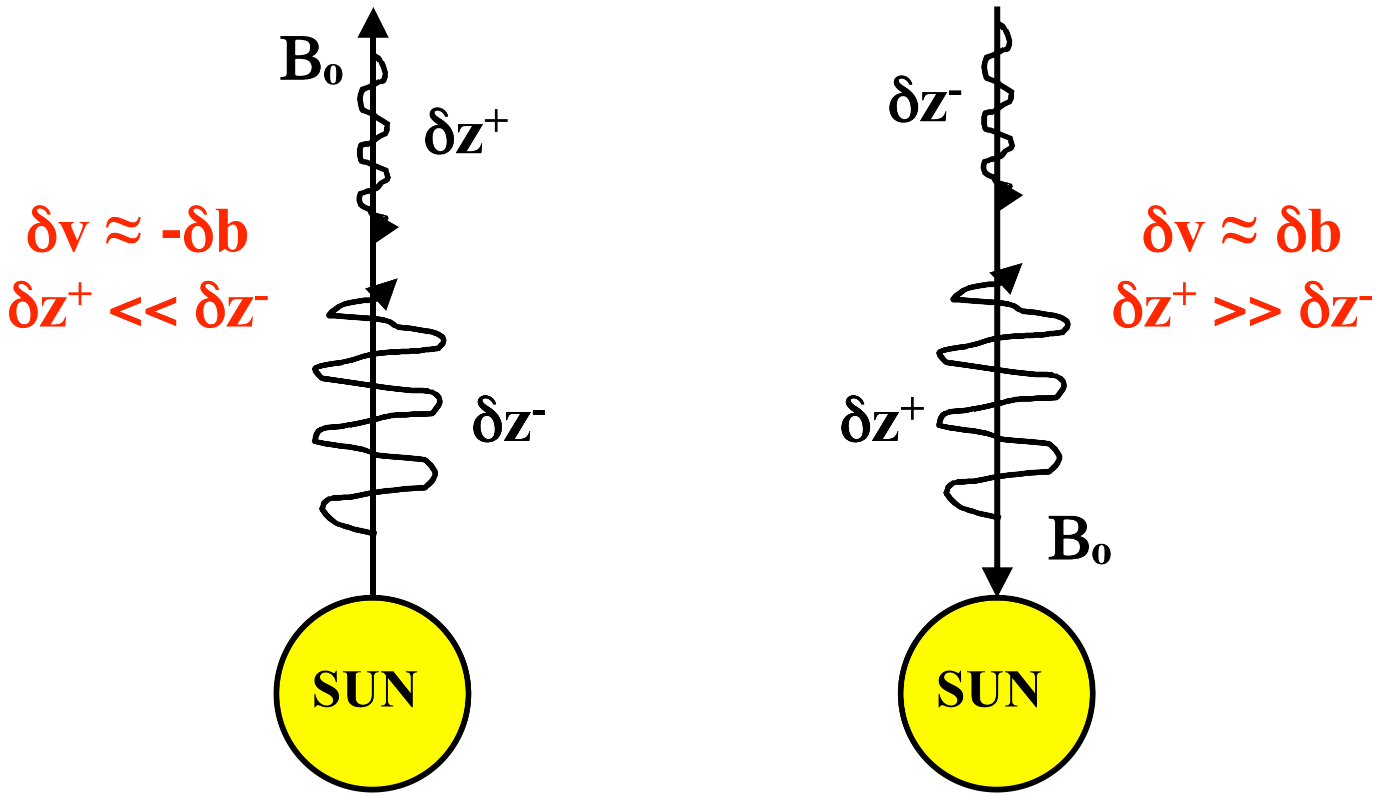
[Horbury et al., 2001]

δv and δb are correlated : **δv ≈ δb**

Outward Alfvén waves - MHD scales

[Belcher and Davis, 1971]

In the fast solar wind : $\delta\mathbf{v} \approx \pm\delta\mathbf{b}$, where $\mathbf{b} = \mathbf{B}/(\mu_0\rho)^{1/2}$



Elsässer variables : $\delta z^{\pm} = \delta\mathbf{v} \pm \delta\mathbf{b}$

Incompressible MHD approximation

Inviscid equations :

$$\nabla . \mathbf{v} = 0 \;, \quad \nabla . \mathbf{B} = 0$$

$$\partial_t \mathbf{v} + \mathbf{v} . \nabla \mathbf{v} = -\nabla \mathbf{P}_* + \mathbf{B} . \nabla \mathbf{B}$$

$$\partial_t \mathbf{B} + \mathbf{v} . \nabla \mathbf{B} = \mathbf{B} . \nabla \mathbf{v}$$

For Elsässer variables ($\mathbf{Z}^{\pm} = \mathbf{v} \pm \mathbf{B}$) :

$$\partial_t \mathbf{Z}^{\pm} + \mathbf{Z}^{\mp} . \nabla \mathbf{Z}^{\pm} = -\nabla \mathbf{P}_*$$

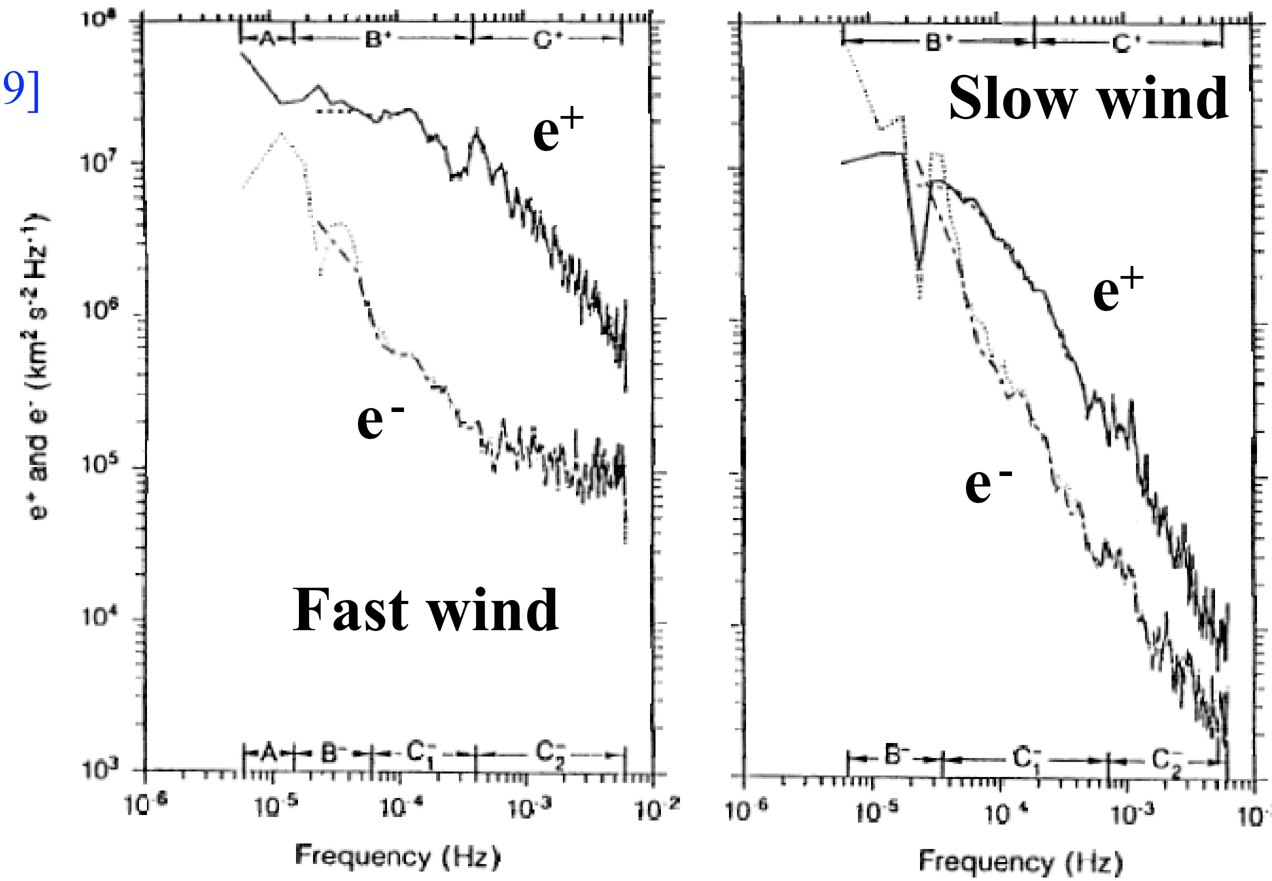
$$\nabla . \mathbf{Z}^{\pm} = 0$$

If : $\mathbf{B} = \mathbf{B_o} + \mathbf{b} = B_o \, \mathbf{e}_{//} + \mathbf{b}$ → Alfvén waves

Inward/outward power energy spectra

[Tu et al., 1989]

0.3 AU

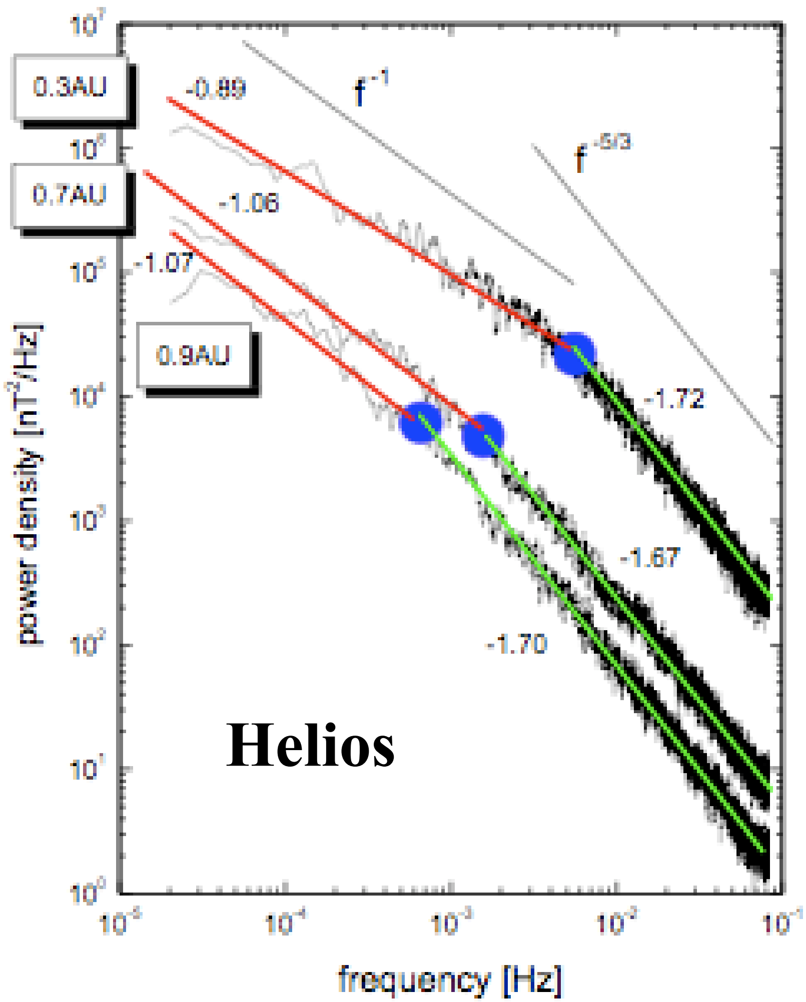


Energy spectrum : $1/2(\delta z^{\pm})^2 \rightarrow e^{\pm}(f) \sim f^{-\alpha_{\pm}}$ → $e^{\pm}(k) \sim k^{-\alpha_{\pm}}$

Taylor hypothesis : $k \approx 2\pi f / V_{SW}$

Dynamical evolution over distance

Magnetic fluctuation power law spectra in the fast solar wind

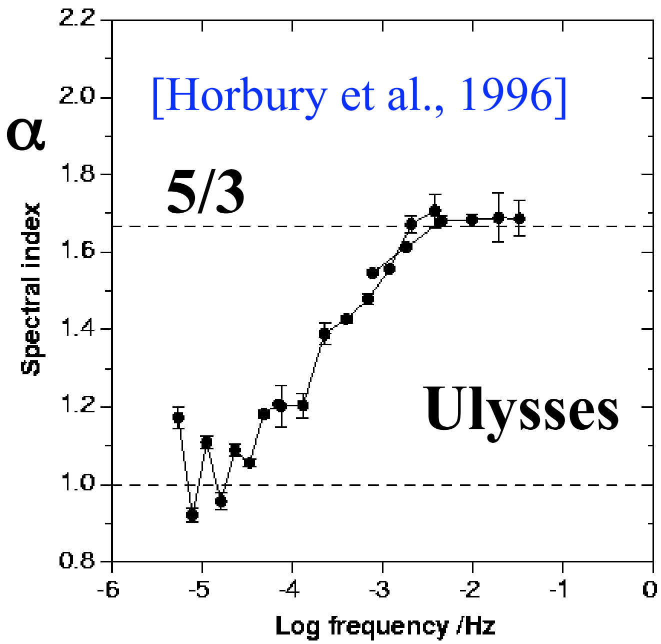


The (blue) knee **moves** to lower frequency as the heliocentric distance increases

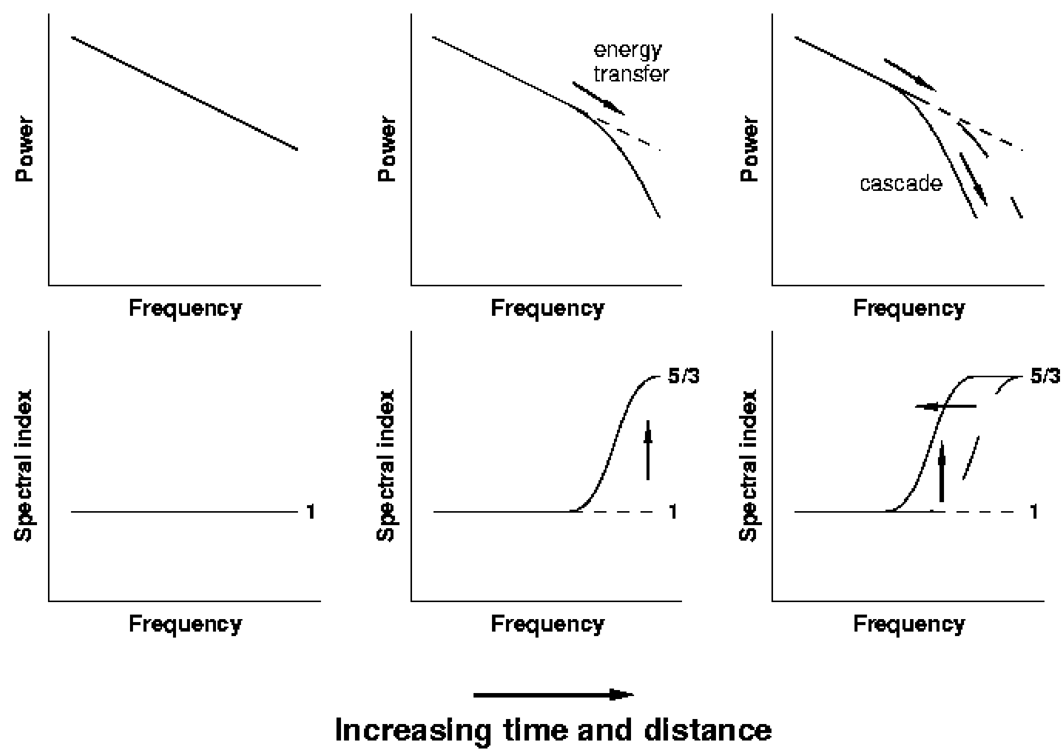
Turbulence intensity **declines** with solar distance

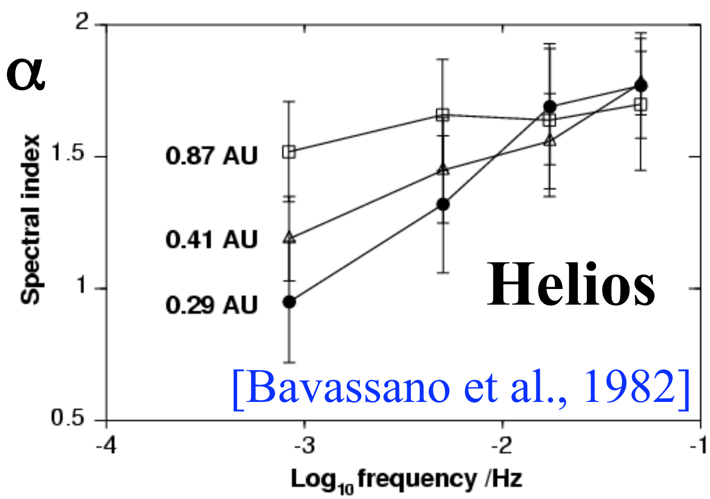
[Bruno et al., 2005]

A signature of a turbulent MHD cascade

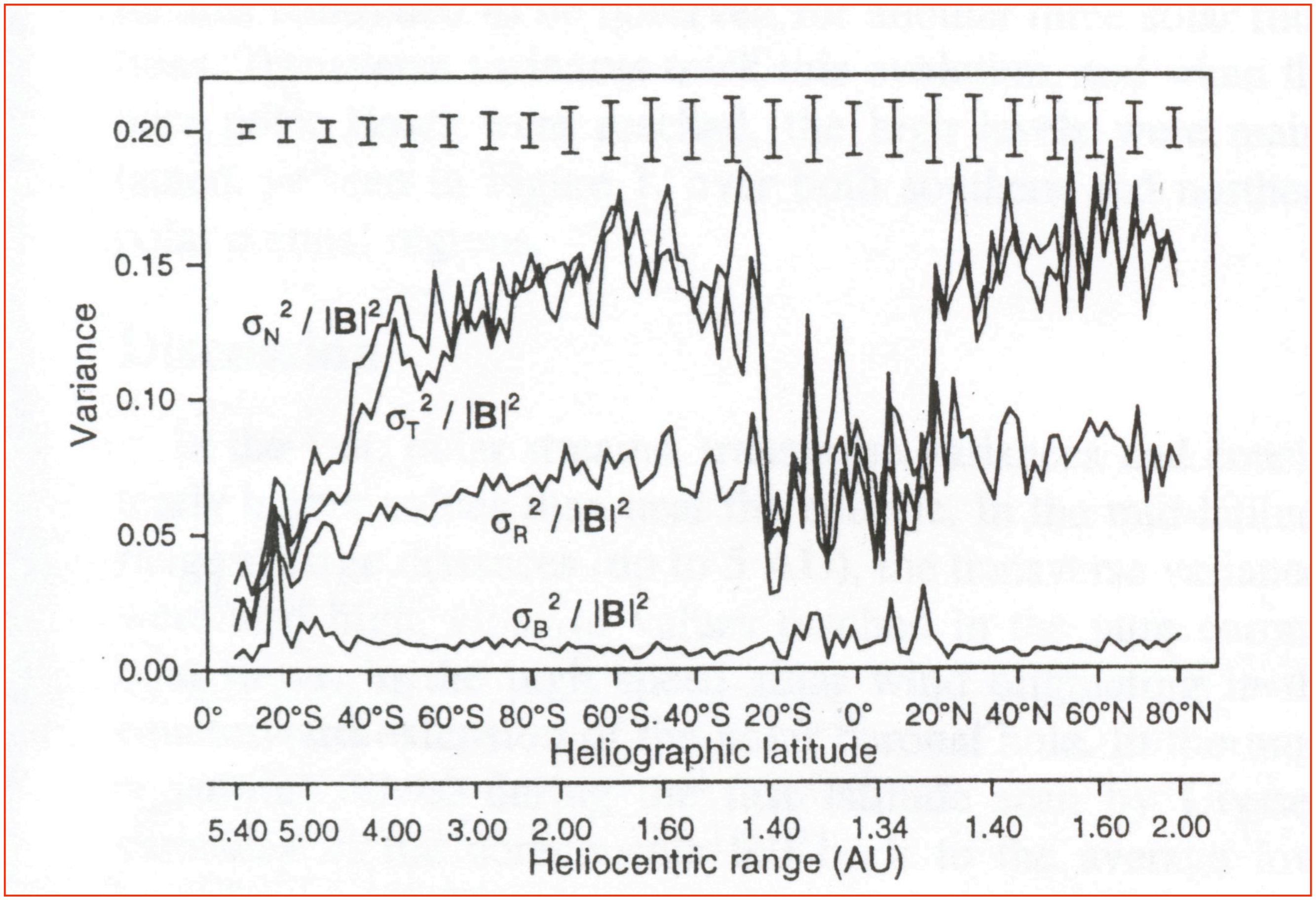


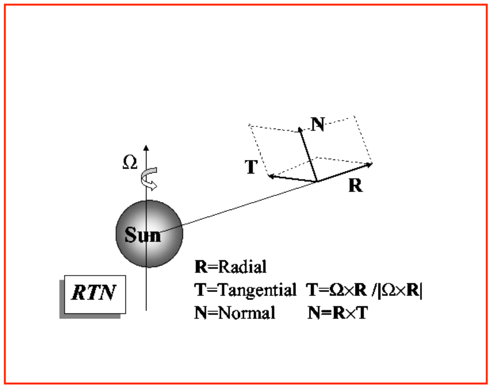
The energy spectrum steepens further from the Sun ⇔ **non linear transfer**





Magnetic field vector properties





Ulysses data

[Forsyth et al., 1996]

$\delta b_{rms} / |B_{total}| \approx 0.1$ → B_{total} « randomly » walks with **small** variations in magnitude

Presence of anisotropy

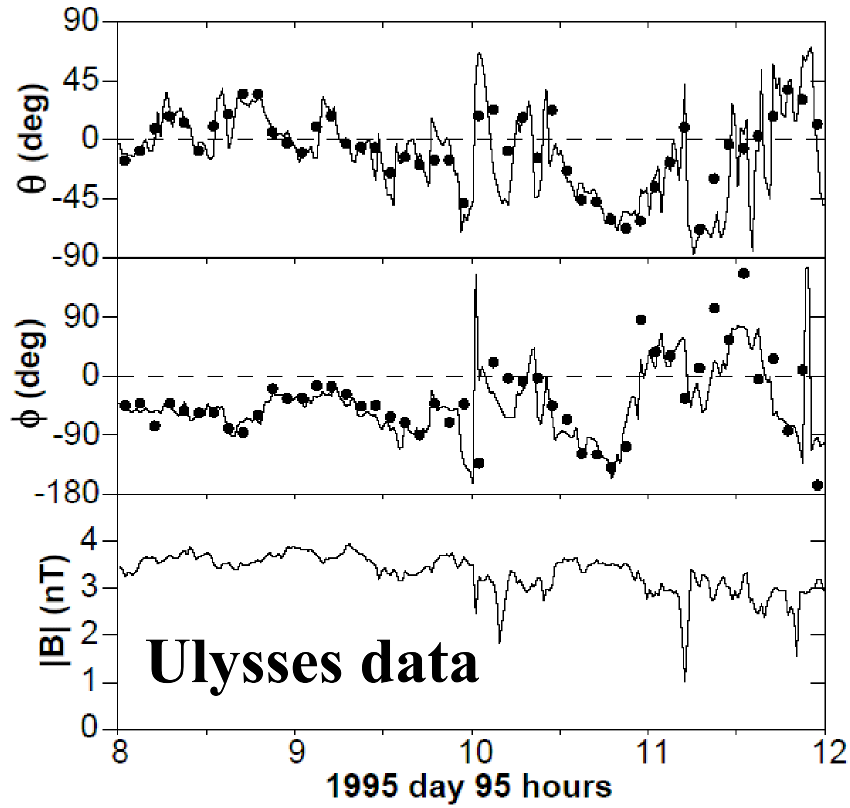
- $\delta b_{//} / \delta b_{\perp} \approx 1/30$ $(\mathbf{B}_{\mathbf{tot}} = B_{tot}\, \mathbf{e}_{//})$ → **anisotropy** [Belcher & Davis, 1971]

- Study of the local minimum variance direction :

→ It tracks large scale changes in field direction

→ Small scale fluctuations are mainly **perpendicular** to $\mathbf{B}_{\mathbf{tot}}$

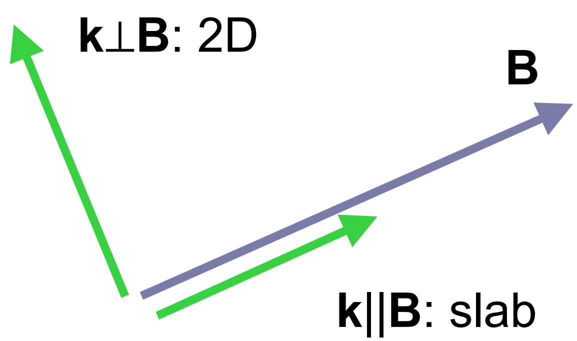
[Horbury, 1999]



Presence of spectral anisotropy

Single spacecraft measurements **are not adequate** to determine the 3D wavevector spectrum

- **Indirect** lines of evidence for magnetic **spectral anisotropy** :
  - ➔ 85% of energy is 2D and 15% is slab [Bieber et al., 1996]
  - ➔ Anisotropic 2D correlations [Matthaeus et al., 1996]



Small scale turbulence

in

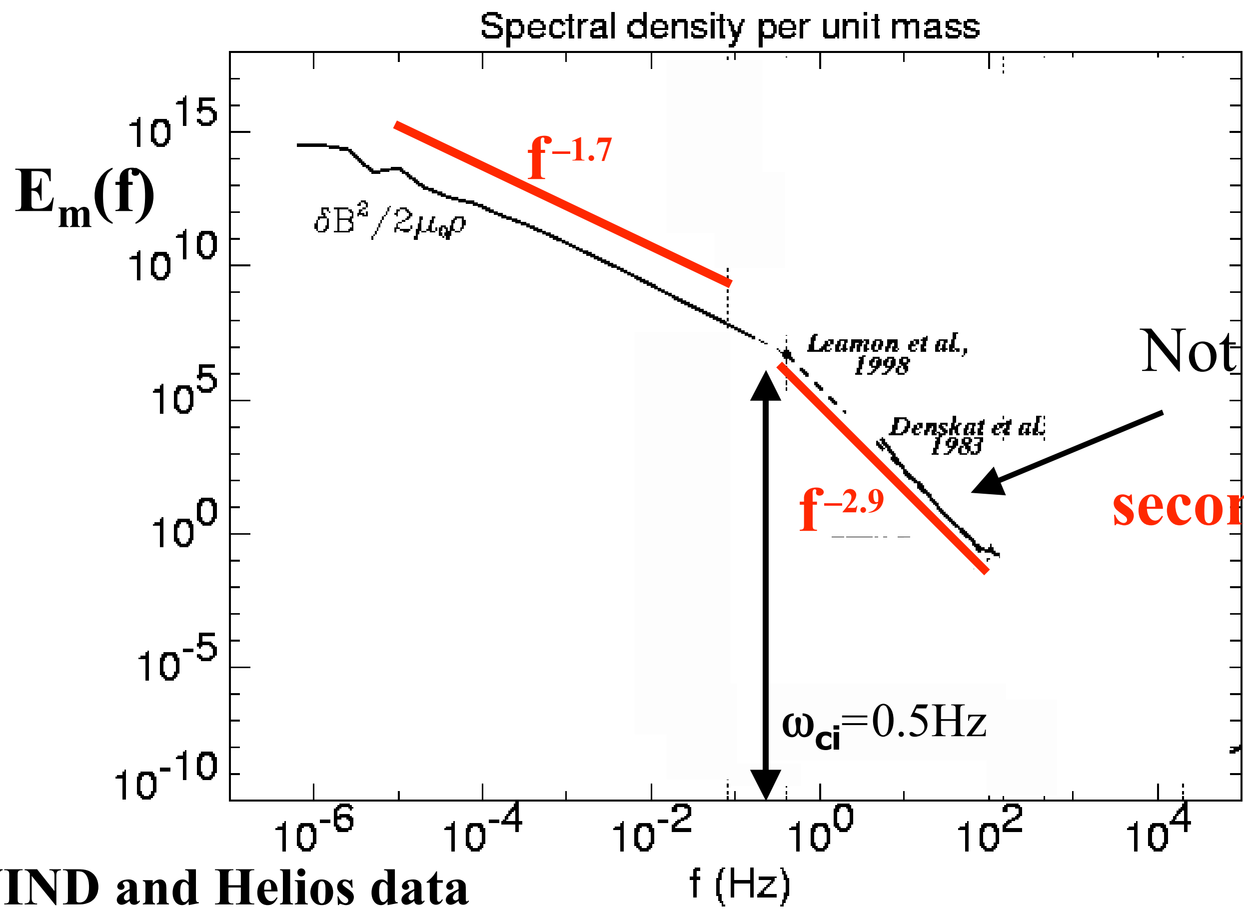
the solar wind

Small-scale turbulence

– Standard Magnetohydrodynamics (MHD) is not valid –

Steepening of the magnetic fluctuation power law spectra: $f^{-1.7} \rightarrow f^{-2.9}$

[Coroniti et al., 1982; Denskat et al., 1983; Leamon et al., 1999; Bale et al., 2005]



Not pure dissipation but **second inertial range !**

[Salem, PhD, 2000]

WIND and Helios data

Magnetic field power spectrum

- Spectral steepening found in fast and slow winds, from 0.3 to 5AU

- Steeper power law may be attributed to **nonlinear dispersive** processes rather than dissipation [Ghosh et al., 1996; Stawicki et al., 2001]

  → It is not an exponential decay

  → It is mainly due to **whistler waves**

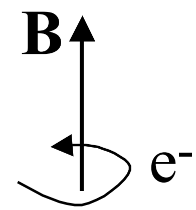
  Cyclotron absorption of left circularly polarized waves

[Coroniti et al., 1982; Denskat et al., 1983; Goldstein et al., 1994; Leamon et al., 1999]

Reduced magnetic helicity spectrum

$\sigma_m = k\, H_m / E_m = k < A\bullet B > / < B^2 >$

For pure circularly polarized waves $\sigma_m = \pm 1$

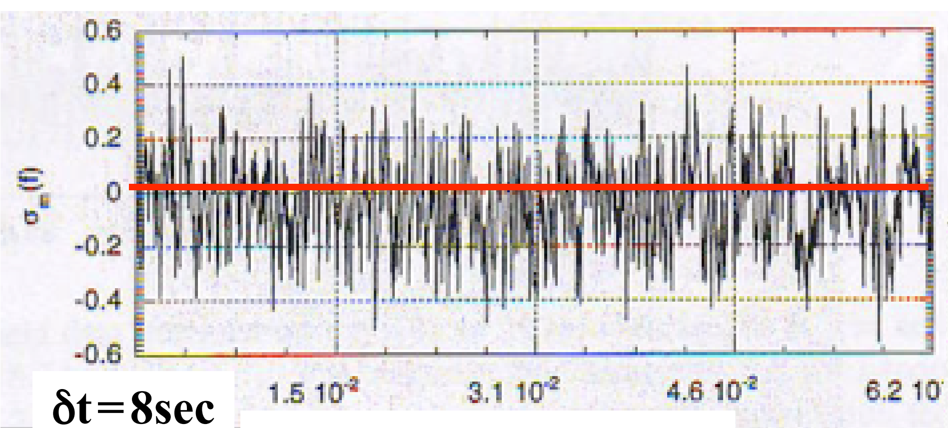


Assuming outwards propagating waves => **whistler waves**

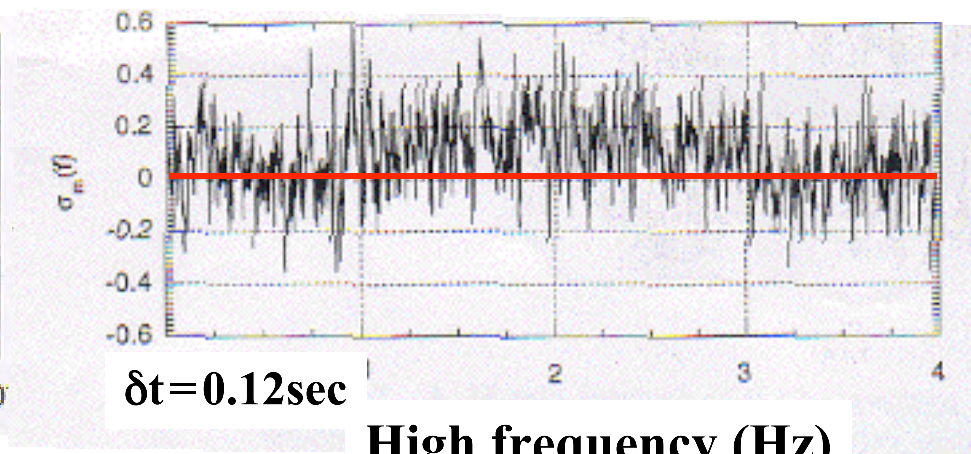
(result in agreement with cyclotron absorption)

$<\sigma_m(f)> = 0$

$<\sigma_m(f)>$ is positive



Low frequency (Hz)



High frequency (Hz)

[Goldstein et al., 1994]

Small-scale turbulence

- The same signature of whistler waves is found in Hall MHD direct numerical simulations [Ghosh et al., 1996]

- Signatures of **(spectral) anisotropy** is also observed

Waves and turbulence

in

Hall MHD

Incompressible Hall MHD turbulence

Inviscid equations :

$$\nabla . \mathbf{v} = 0$$

$$\partial_t \mathbf{v} + \mathbf{v} . \nabla \mathbf{v} = -\nabla P_* + \mathbf{B} . \nabla \mathbf{B}$$

$$\partial_t \mathbf{B} + \mathbf{v} . \nabla \mathbf{B} = \mathbf{B} . \nabla \mathbf{v} - d_i \nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}]$$

$$\nabla . \mathbf{B} = 0$$

- Ion inertial length : $d_i = B_o / \omega_{ci}$; $\mathbf{B} = \mathbf{B_o} + \mathbf{b} = B_o \mathbf{e}_{//} + \mathbf{b}$
- If $d_i k \ll 1$ $(\omega \ll \omega_{ci})$ => standard MHD
- If $d_i k \gg 1$ $(\omega < \omega_{ce})$ => electron MHD and « ion MHD »

Incompressible Hall MHD turbulence

- Inviscid invariants (B is frozen in the electron flow) :

$$E = (1/2) \int (v^2 + B^2)\, d^3x$$ **Total energy**

$$H_m = (1/2) \int A.B\, d^3x$$ **Magnetic helicity**

$$H_G = (1/2) \int (A + d_i v).(B + d_i \nabla \times v)\, d^3x$$ **Hybrid helicity**

- There are linear incompressible waves ($sk_{//} > 0$) :

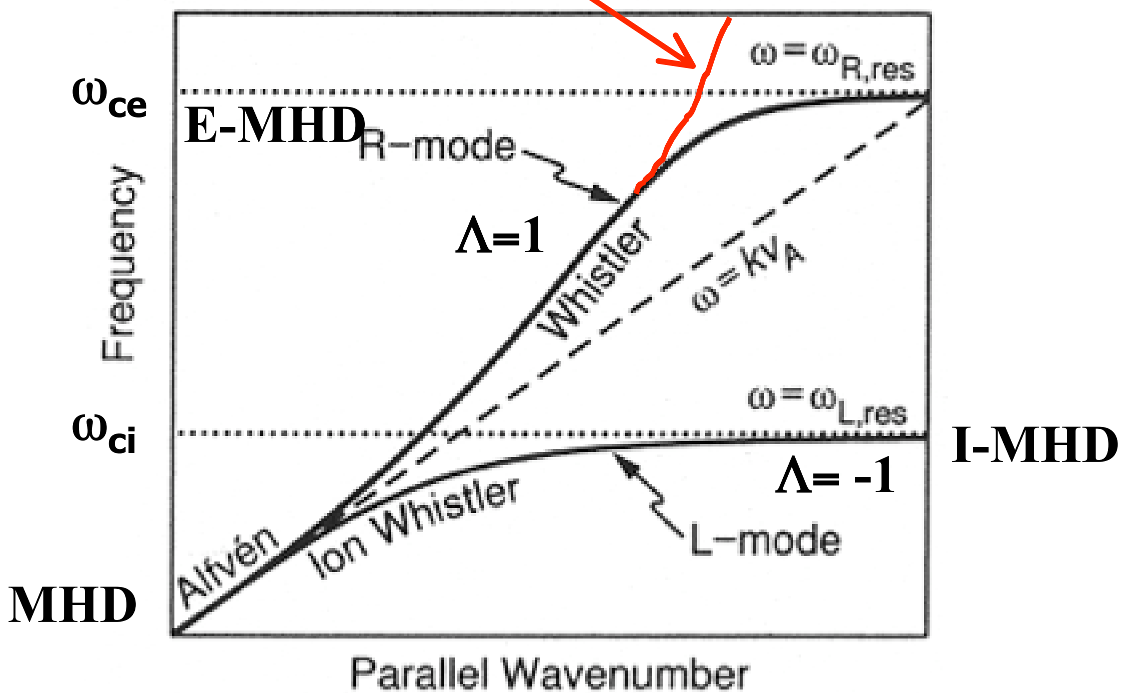
$$\omega_\Lambda^s(k) = B_0\, sk_{//}\, d_i k \left(s\Lambda + \sqrt{1 + 4/(d_i k)^2}\right) / 2$$

$$\begin{cases} \text{left circularly polarized wave } (\Lambda = -1) \\ \text{right circularly polarized wave } (\Lambda = 1) \text{ - whistler} \end{cases}$$

Incompressible Hall MHD waves

In Hall MHD electrons are massless

$k = k_{//}$



$$\omega_{\Lambda}^{S}(k) = B_o\, s\, d_i\, k^2\, (s\Lambda + \sqrt{1 + 4/(d_i k)^2}\,) / 2$$

Wave turbulence in Hall MHD

We shall describe the small scale solar wind conditions

- We introduce : $\mathbf{B}(\mathbf{x},t) = B_o \mathbf{e}_{//} + \boldsymbol{\varepsilon}\mathbf{b}(\mathbf{x},t)$ with $0 < \varepsilon << 1$

  but $\mathbf{B_o}$ is in a **fixed** direction

- We develop perturbatively (in Fourier) the Hall MHD equations

- We derive the asymptotically exact wave kinetic equations

  [Zakharov et al, 1992; Newell et al, 2001]

→ **Dynamical description for energy and helicity spectra (3-wave interactions)**

Wave turbulence in Hall MHD

- Use a complexe **helicity decomposition** (HMHD waves are helical) :

$$\mathbf{h}^\Lambda(\mathbf{k}) \equiv \mathbf{h}_\mathbf{k}^\Lambda = \hat{\mathbf{e}}_\theta + i\Lambda\hat{\mathbf{e}}_\Phi \qquad \begin{cases} \hat{\mathbf{e}}_\theta = \hat{\mathbf{e}}_\Phi \times \hat{\mathbf{e}}_k \\ \hat{\mathbf{e}}_\Phi = \dfrac{\hat{\mathbf{e}}_\parallel \times \hat{\mathbf{e}}_k}{|\hat{\mathbf{e}}_\parallel \times \hat{\mathbf{e}}_k|} \end{cases}$$

Λ is the wave polarization ($\Lambda = \pm 1$) ; $\mathbf{k} \cdot \mathbf{h}_\mathbf{k}^\Lambda = 0 \,, \; \hat{\mathbf{e}}_k \times \mathbf{h}_\mathbf{k}^\Lambda = -i\Lambda\, \mathbf{h}_\mathbf{k}^\Lambda$

[Craya, 1958; Kraichnan, 1973; Cambon et al., 1989; Turner, 2000; Galtier 2003]

$$\rightarrow \quad \begin{cases} \mathbf{v}_\mathbf{k} = \sum_\Lambda \mathcal{U}_\Lambda(\mathbf{k})\, \mathbf{h}_\mathbf{k}^\Lambda = \sum_\Lambda \mathcal{U}_\Lambda\, \mathbf{h}_\mathbf{k}^\Lambda \,, \\ \mathbf{b}_\mathbf{k} = \sum_\Lambda \mathcal{B}_\Lambda(\mathbf{k})\, \mathbf{h}_\mathbf{k}^\Lambda = \sum_\Lambda \mathcal{B}_\Lambda\, \mathbf{h}_\mathbf{k}^\Lambda \,. \end{cases}$$

Wave turbulence in Hall MHD

- We introduce the **generalized Elsässer variables** :

$$\mathcal{Z}_\Lambda^s \equiv \mathcal{U}_\Lambda + \xi_\Lambda^s \mathcal{B}_\Lambda \, ,$$

$$\xi_\Lambda^s(k) = \xi_\Lambda^s = -\frac{s d_i k}{2}\left(s\Lambda + \sqrt{1 + \frac{4}{d_i^2 k^2}}\right) \, .$$

Such that :

$$\begin{cases} \partial_t \mathcal{Z}_\Lambda^s = -i\,\omega_\Lambda^s \mathcal{Z}_\Lambda^s \\ \omega_\Lambda^s(k) = B_o\, s k_{//}\, d_i k\, \left(s\Lambda + \sqrt{1 + 4/(d_i k)^2}\right) / 2 \end{cases}$$

Wave turbulence in Hall MHD

- Introduction of the orthonormal basis vector to get a **polar form** :

$$\vec{O}^{(1)}(\vec{p}) = \vec{n} \times \vec{e}_{\mathbf{p}} \,, \quad \vec{O}^{(2)}(\vec{p}) = \vec{n} \,, \quad \vec{O}^{(3)}(\vec{p}) = - \vec{e}_{\mathbf{p}}$$

$\vec{n} \perp$ to the triangle $\vec{k} = \vec{p} + \vec{q}$ and $\vec{n} = (\vec{k} \times \vec{p}) / |\vec{k} \times \vec{p}| = \ldots$

[Turner, 2000]

- We obtain the wave amplitude equation :

$$\partial_t a_\Lambda^s = \frac{\epsilon}{4\, d_i} \int \sum_{\substack{\Lambda_p : \Lambda_q \\ s_p : s_q}} \xi_\Lambda^{s\,2} \, \frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{\xi_\Lambda^s - \xi_\Lambda^{-s}} \, M_{\ -k\, p\, q}^{\ \Lambda \Lambda_p \Lambda_q}{}_{\, s\, s_p\, s_q} \, a_{\Lambda_p}^{s_p} \, a_{\Lambda_q}^{s_q} \, e^{-i\Omega_{pq,k} t} \, \delta_{pq,k} \, d\mathbf{p} \, d\mathbf{q}$$

The matrix **M** has **all symmetries** you need to do wave turbulence

Wave kinetic equations of Hall MHD

- We derive the **3D** wave kinetic equations for **energies** and **helicities**

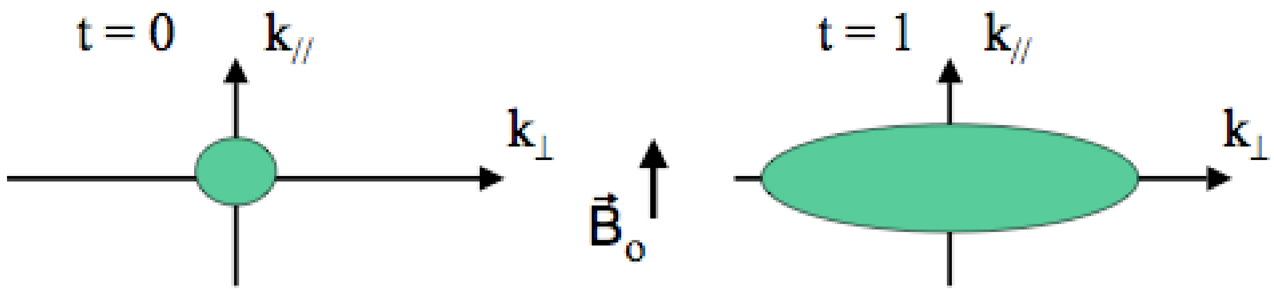
- We **recover** standard ($d_i k \ll 1$) and electron ($d_i k \gg 1$) MHD as two limits [Galtier, Nazarenko, Newell & Pouquet, 2000; Galtier & Bhattacharjee, 2003]

- Standard MHD limit is **singular** → Principal value terms appear

- Detailed conservation of invariants for each triad **k**, **p** and **q**

Wave HMHD turbulence properties

- Global tendency (at any scales) towards spectral **anisotropy** :



- The master equations are :

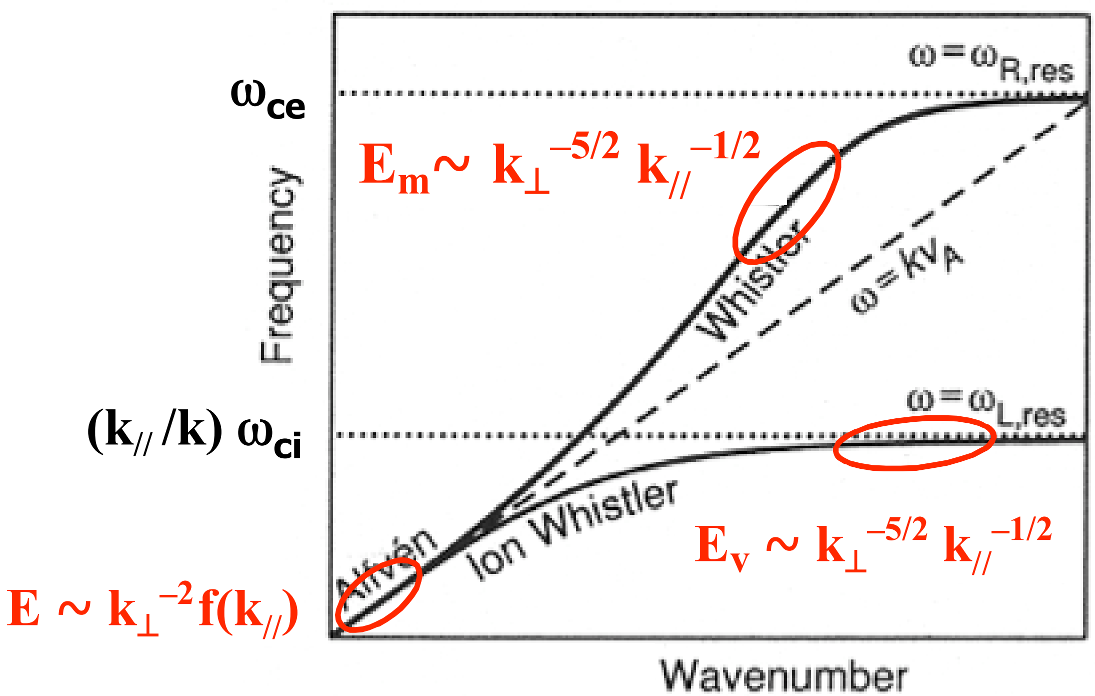
$$\partial_t \begin{Bmatrix} E^V(\mathbf{k}) \\ E^B(\mathbf{k}) \end{Bmatrix} = \frac{\pi \epsilon^2}{8\, d_i^2 B_0^2} \int \sum_{\substack{\Lambda, \Lambda_p, \Lambda_q \\ s, s_p, s_q}} \left(\frac{\sin \psi_k}{k}\right)^2 \frac{(\Lambda k + \Lambda_p p + \Lambda_q q)^2 \left(1 - \xi_\Lambda^{-s^2} \xi_{\Lambda_p}^{-s_p{}^2} \xi_{\Lambda_q}^{-s_q{}^2}\right)^2}{(1 + \xi_\Lambda^{-s^2})(1 + \xi_{\Lambda_p}^{-s_p{}^2})(1 + \xi_{\Lambda_q}^{-s_q{}^2})}$$

$$\left(\frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{k_\parallel}\right)^2 \begin{Bmatrix} \xi_\Lambda^{-s^2} \\ 1 \end{Bmatrix} \frac{\omega_\Lambda^s \omega_{\Lambda_p}^{s_p}}{\xi_\Lambda^{-s^2} + 1} \left(\frac{\xi_{\Lambda_q}^{-s_q{}^2} E^V(\mathbf{q}) - E^B(\mathbf{q})}{\xi_{\Lambda_q}^{-s_q{}^2} - 1}\right)$$

$$\left[\left(\frac{\xi_{\Lambda_p}^{-s_p{}^2} E^V(\mathbf{p}) - E^B(\mathbf{p})}{\xi_{\Lambda_p}^{-s_p{}^2} - 1}\right) - \left(\frac{\xi_\Lambda^{-s^2} E^V(\mathbf{k}) - E^B(\mathbf{k})}{\xi_\Lambda^{-s^2} - 1}\right)\right] \delta(\Omega_{k,pq})\, \delta_{k,pq}\, d\mathbf{p}\, d\mathbf{q}\,.$$

[Galtier, 2005]

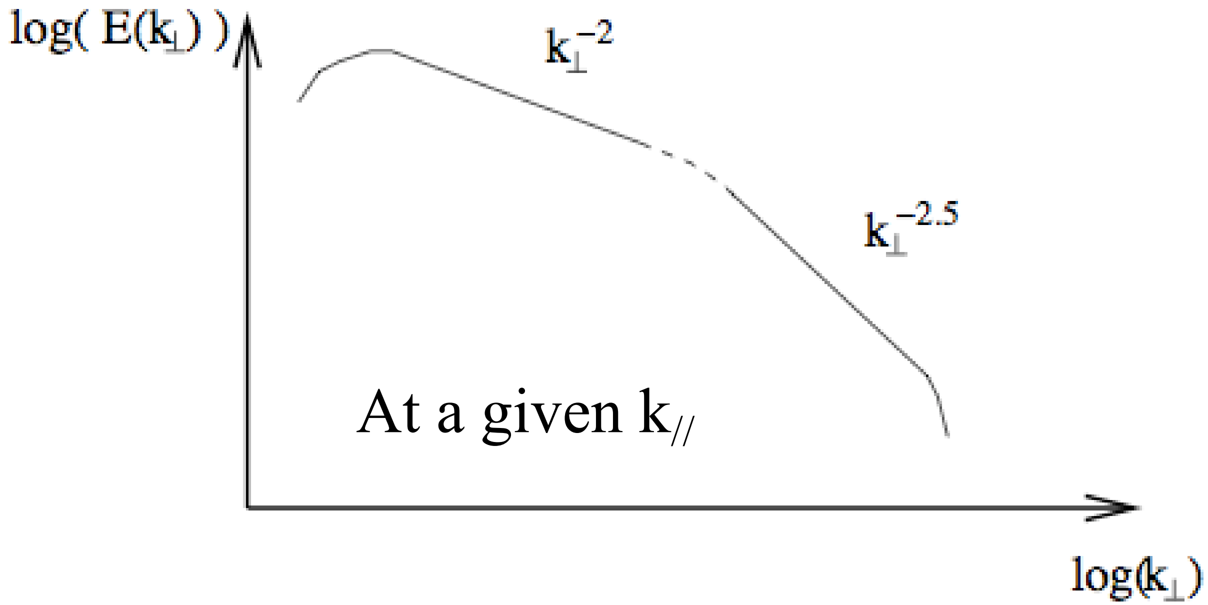
Wave HMHD turbulence properties



- The exact power law solutions show a **steepening** at small scales

Wave HMHD turbulence properties

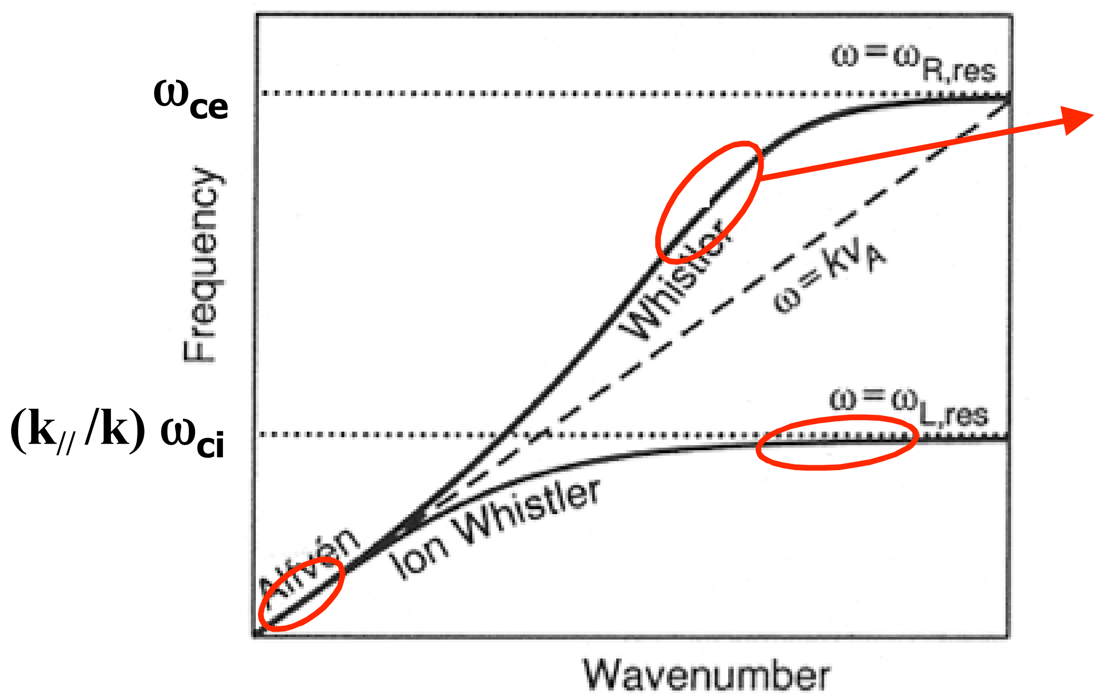
$$E(k_\perp, k_\parallel) \sim \sqrt{\Pi B_0}\, k_\perp^{-2} k_\parallel^{-1/2} (1 + k_\perp^2 d_i^2)^{-1/4}$$



→ We recover a **knee** as observed in the solar wind

→ A global anisotropic **phenomenology** is given

Wave HMHD turbulence properties



It is like strongly rotating Navier Stokes fluids !

- **Nonlocal** interactions between whistler and Alfvén waves are possible as soon as E_v and E_b are **different for Alfvén waves**

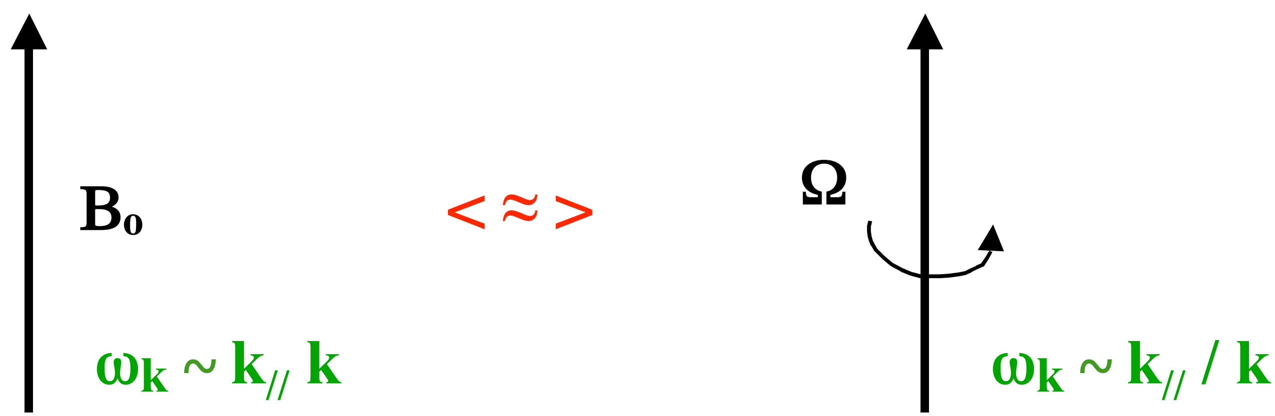
Rotating Navier-Stokes turbulence

$$\partial_t \mathbf{w} - 2(\boldsymbol{\Omega}.\nabla)\mathbf{v} = \mathbf{w}.\nabla\mathbf{v} - \mathbf{v}.\nabla\mathbf{w}$$

$$\mathbf{w} = \nabla \times \mathbf{v} \quad \text{and} \quad \nabla . \mathbf{v} = 0$$

Very strong analogy between wave turbulence in EMHD and in Navier-Stokes fluids under rapid rotation

[Galtier, 2003]

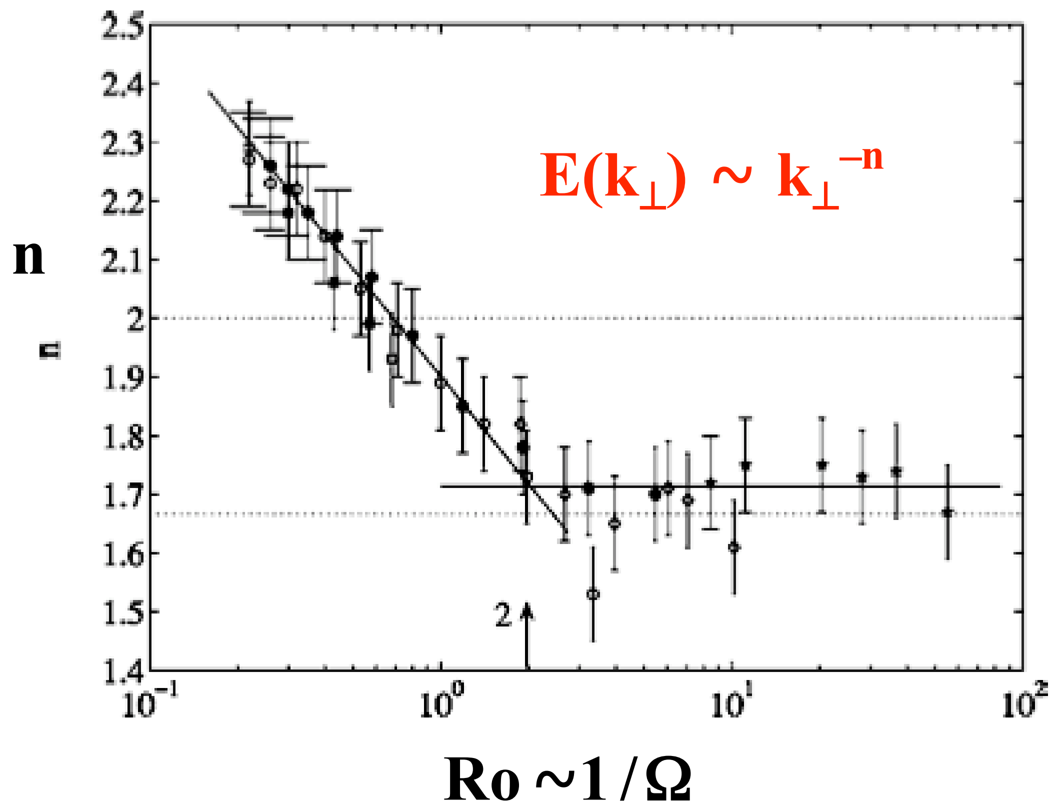


Whistler waves

Inertial waves

Helical waves

Rotating Navier-Stokes turbulence



[Morize et al., 2005]

Experimental measurements made at FAST-Paris XI

Conclusion

• The solar wind is a **vast turbulent laboratory** :

→ Alfvén waves, active cascades, anisotropy, intermittency…

• Small scale solar wind turbulence **exists** (Hall MHD scales)

→ Useful for the inner corona…

• **Questions** : initial forcing, cross-helicity, power law exponents…

• Lack of data to understand the **3D structure** of turbulence

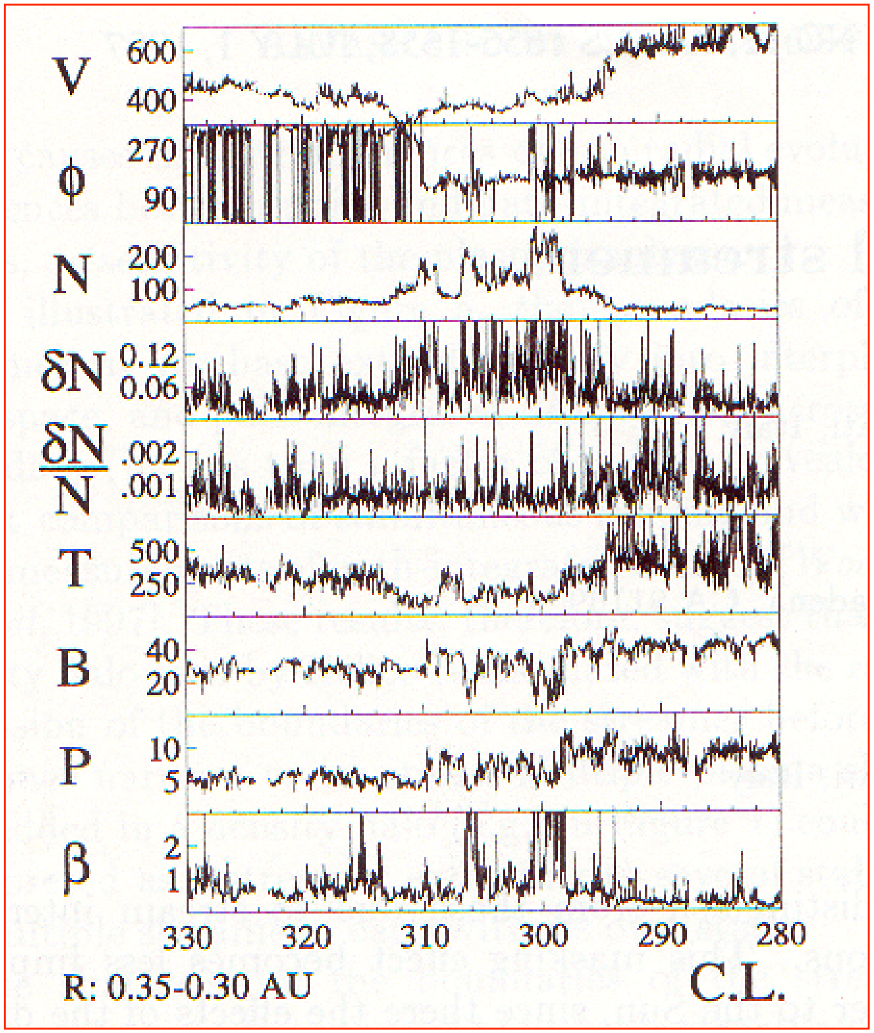
→ Current data **are not** sufficient [Bigot et al., 2005]

→ Crucial for **many** problems in astrophysics

Other slides

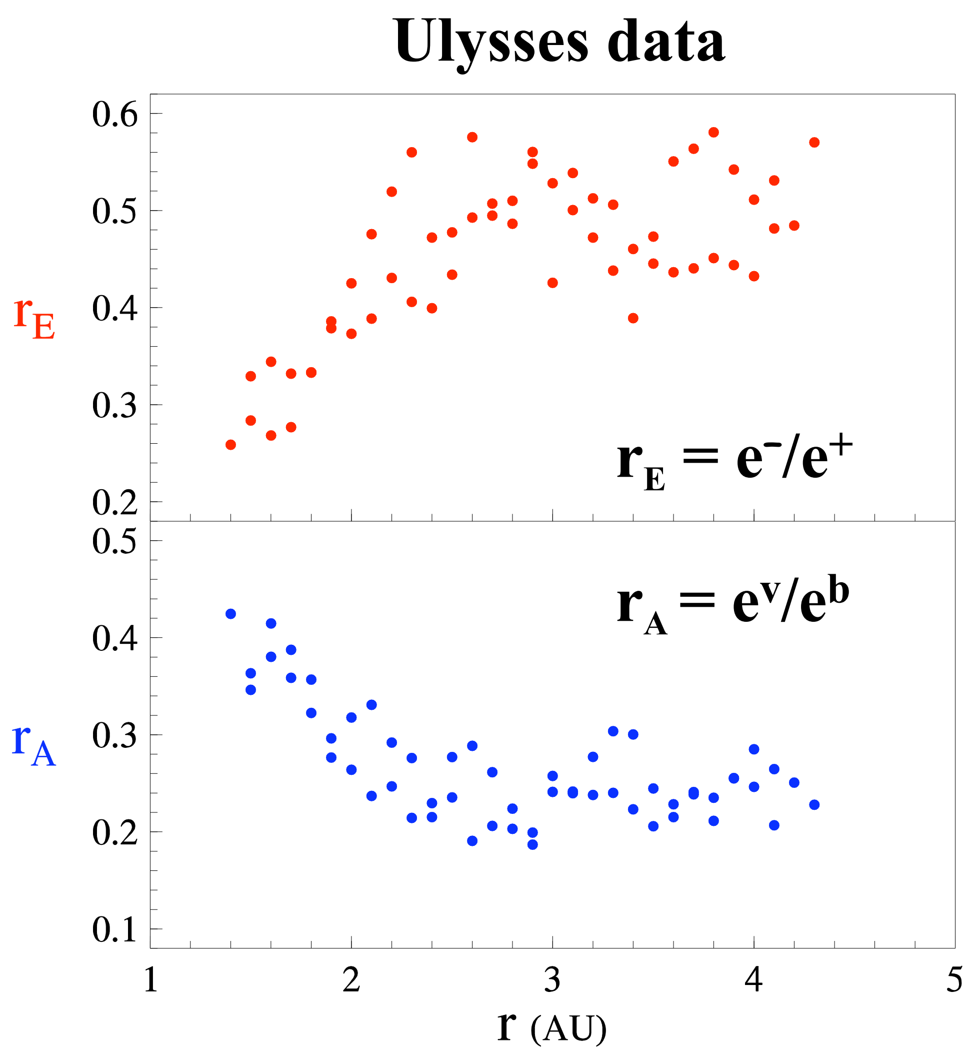
$\delta N / N < 0.05$

[Bavassano et al., 1997]



Slow solar wind

Other radial evolutions...

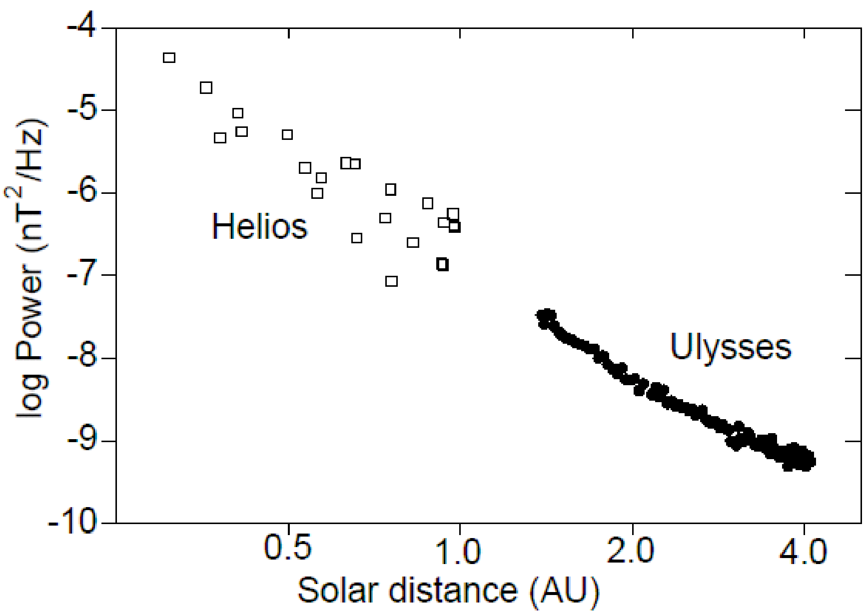


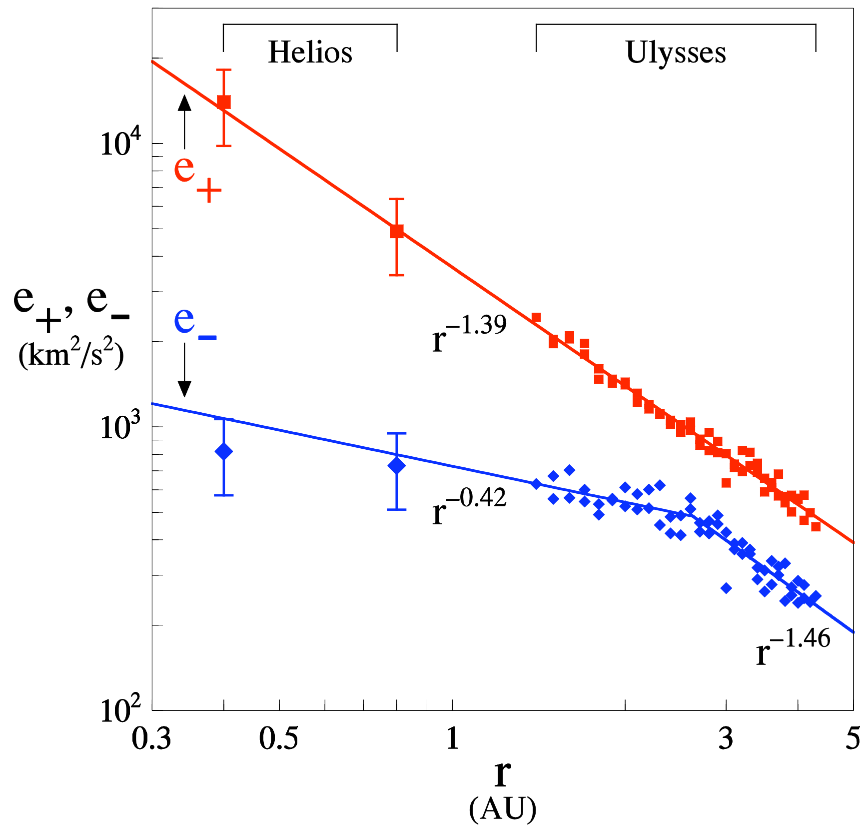
The predominance of outward modes is preserved but **saturates** at ≈ 0.5

The imbalance in favor of the magnetic energy **saturates** at ≈ 0.25

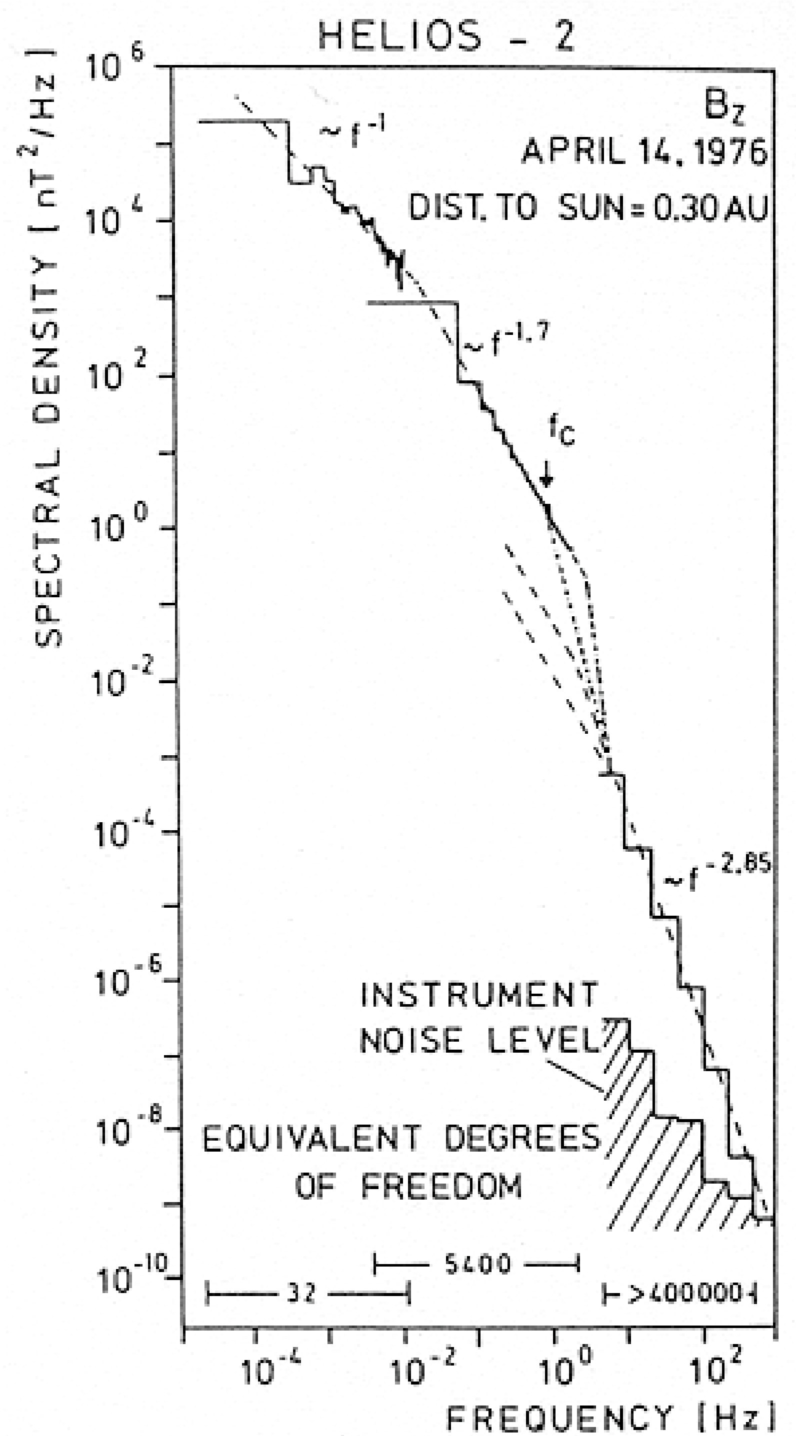
[Bavassano et al., 2000]

Radial evolutions…





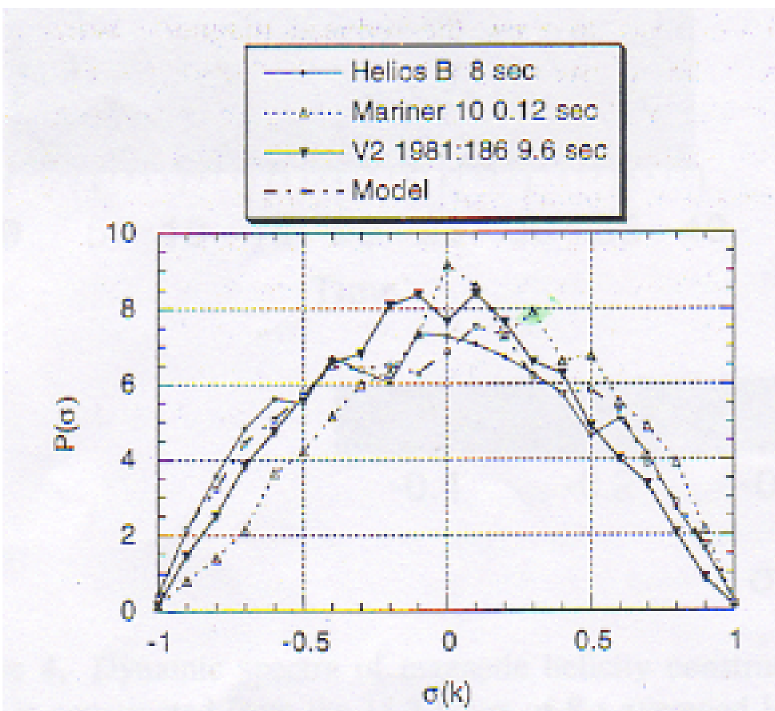
Magnetic spectrum

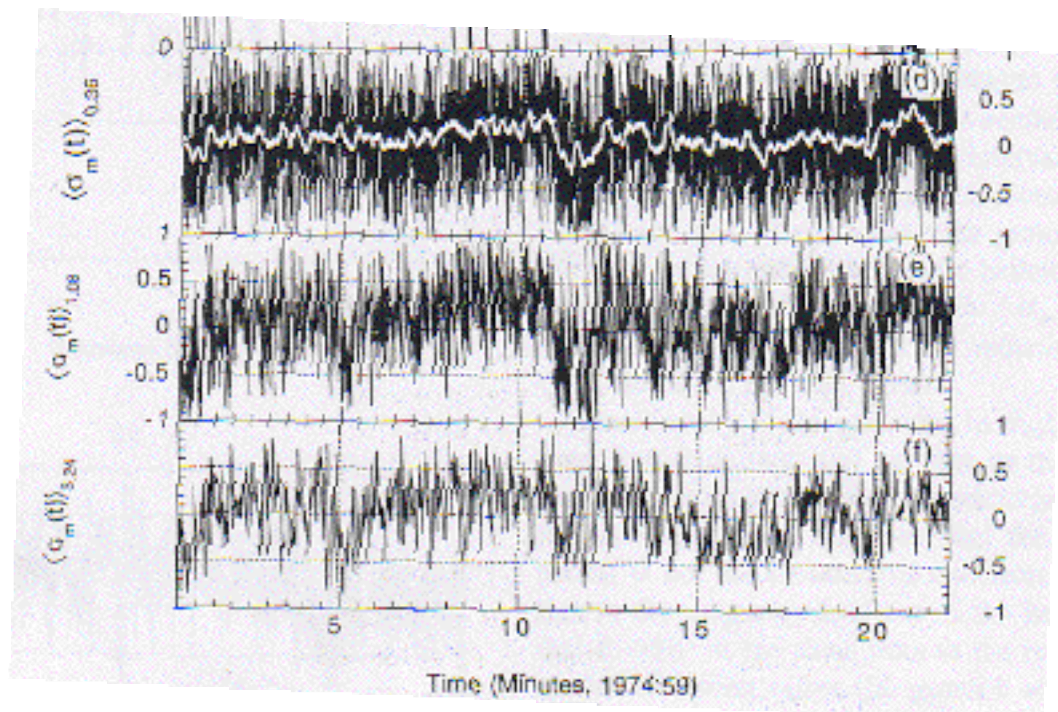


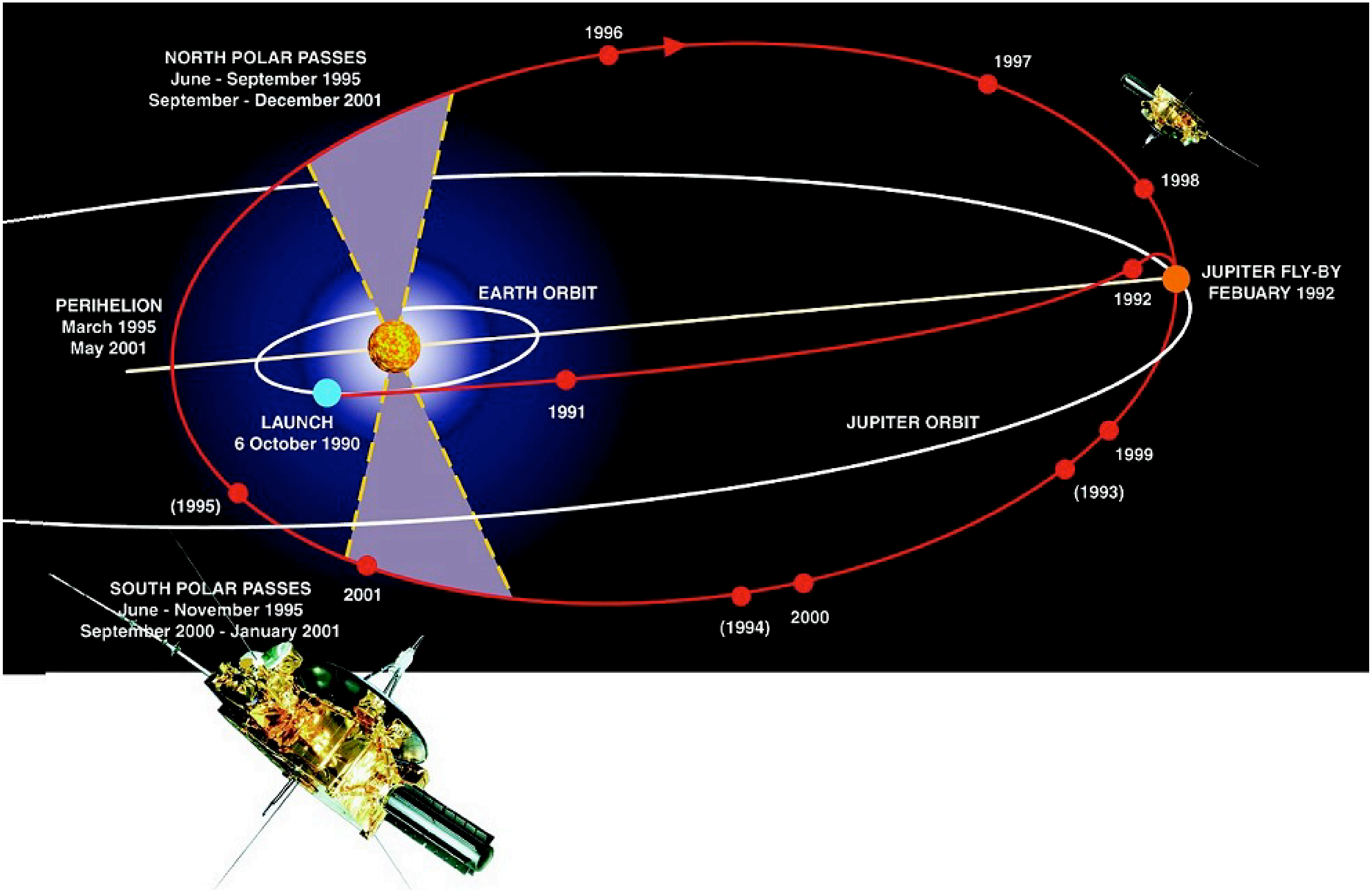
- Abrupt decline at ω_{ci} indicates cyclotron absorption
- Steep spectrum at high frequencies above ~ 1Hz is mainly due to whistler waves

[Denskat et al., 1983]

Reduced magnetic helicity spectrum







Observations out of the ecliptic by Ulysses

Wave HMHD turbulence properties

- The resonance may occur at frequency **lower** than ω_{ci} if the small scale turbulence is anisotropic : $\omega_{res} = (k_{//}/k)\,\omega_{ci}$

→ Possible source of heating at **larger** scales