Propagation of vortex front in rotating superfluid

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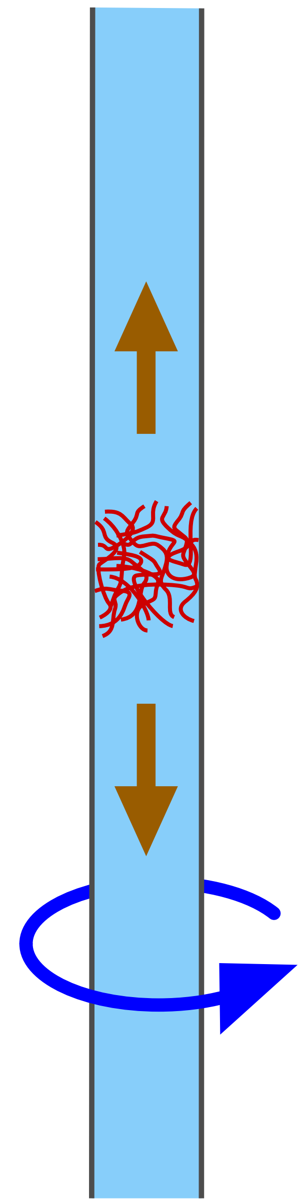
OVERVIEW

1. Experiment: injection of vortex lines into vortex-free flow and NMR study of their evolution.

2. Transition in vortex dynamics from regular at higher temperatures to turbulent at lower temperatures.

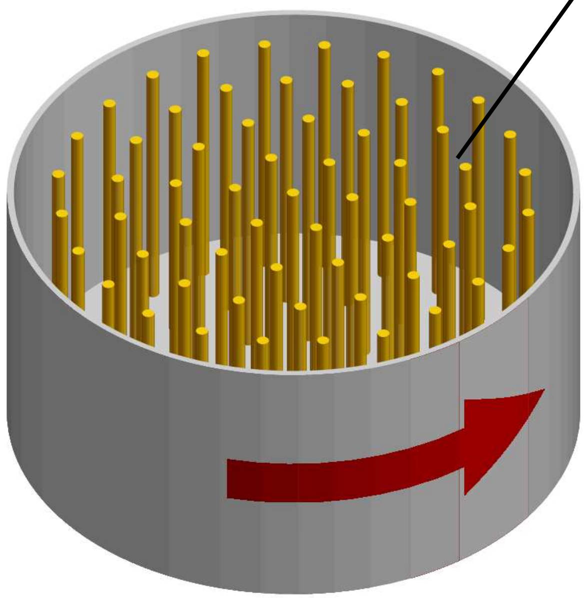
3. Expansion of vorticity in rotating column as a propagating front: Experimental results, numerical simulations and analytic studies.



ROTATING SUPERFLUID

Minimum free energy

$\mathbf{v}_\mathrm{n} \approx \mathbf{v}_\mathrm{s}$

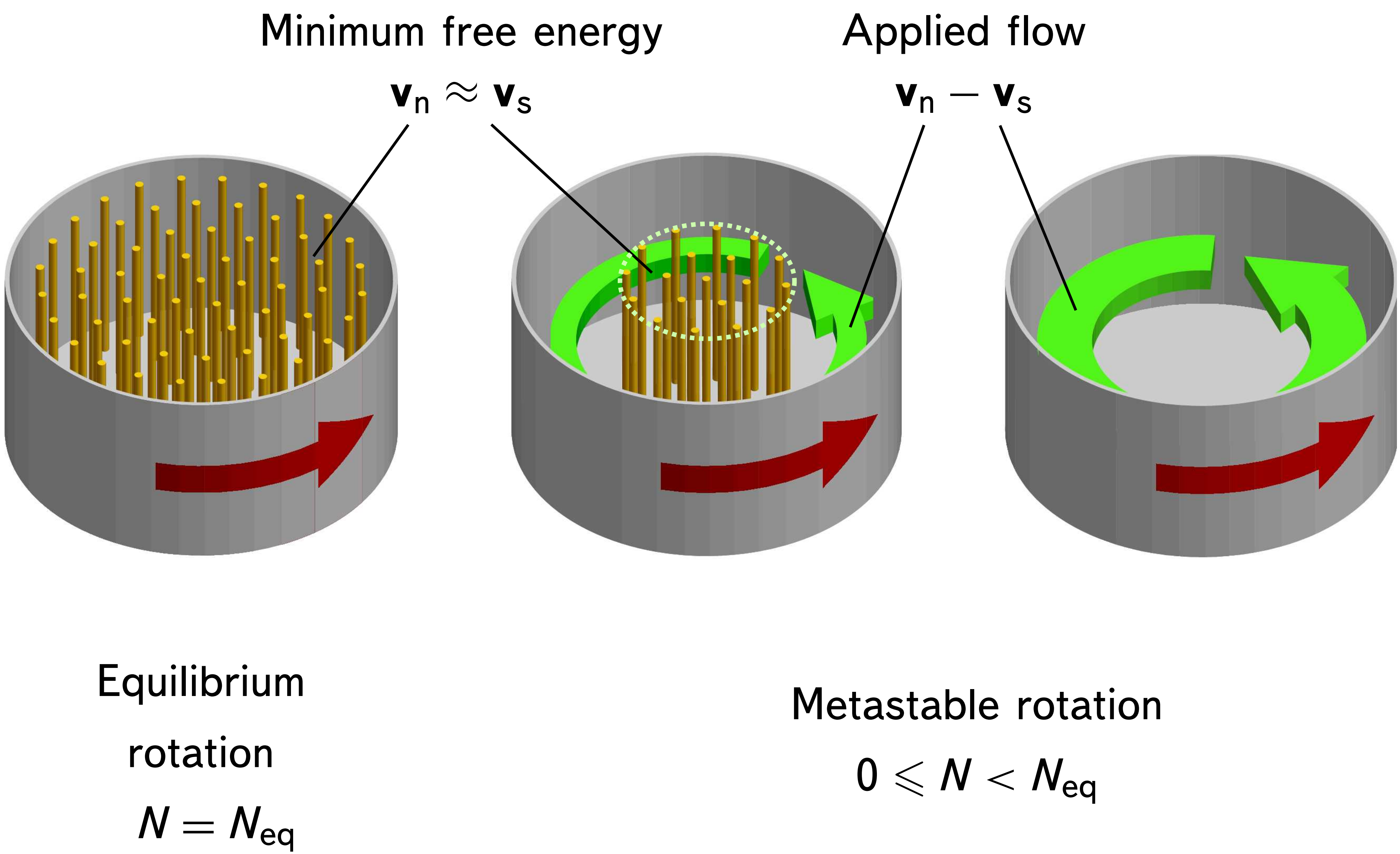


Equilibrium

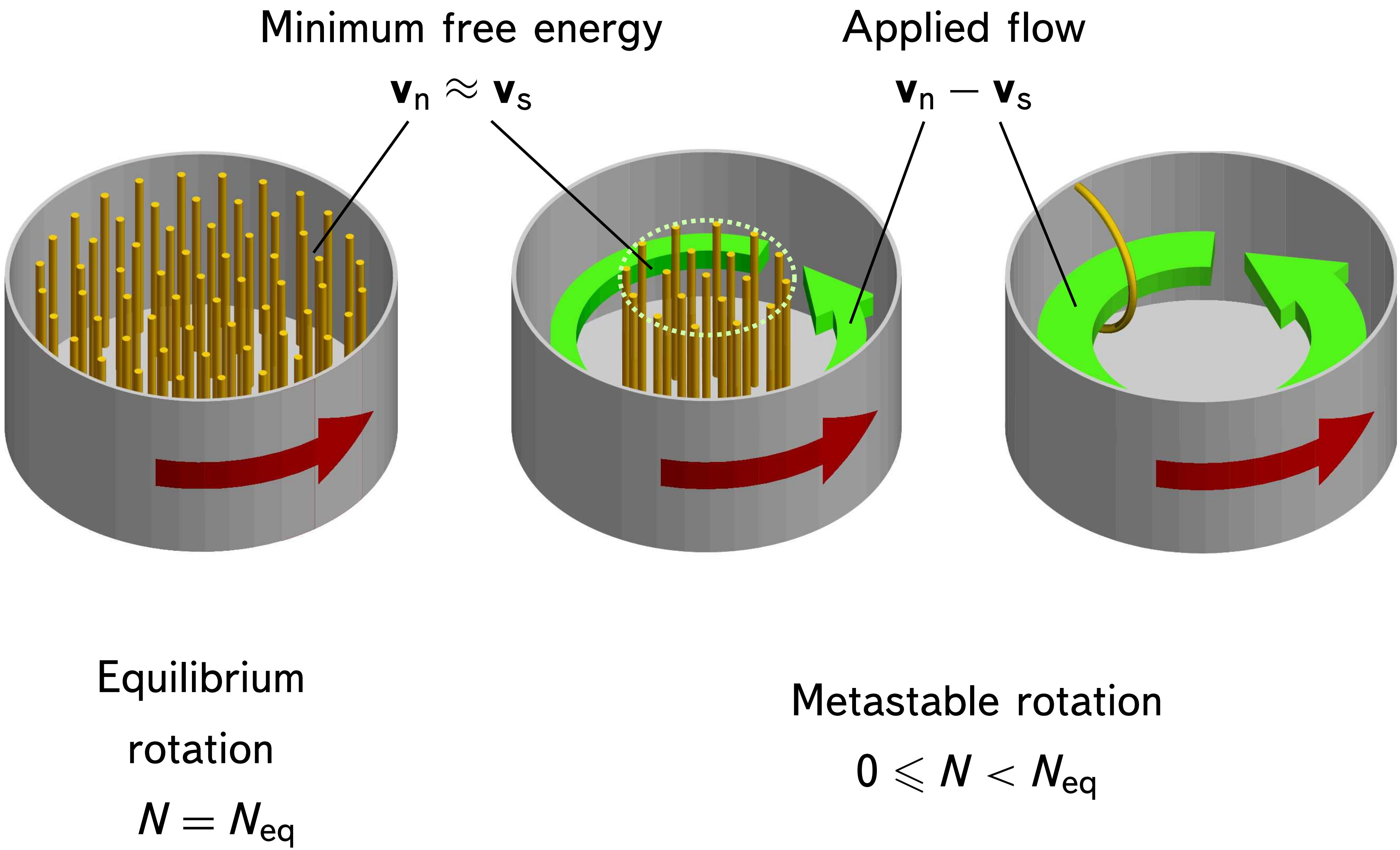
rotation

$N = N_\mathrm{eq}$

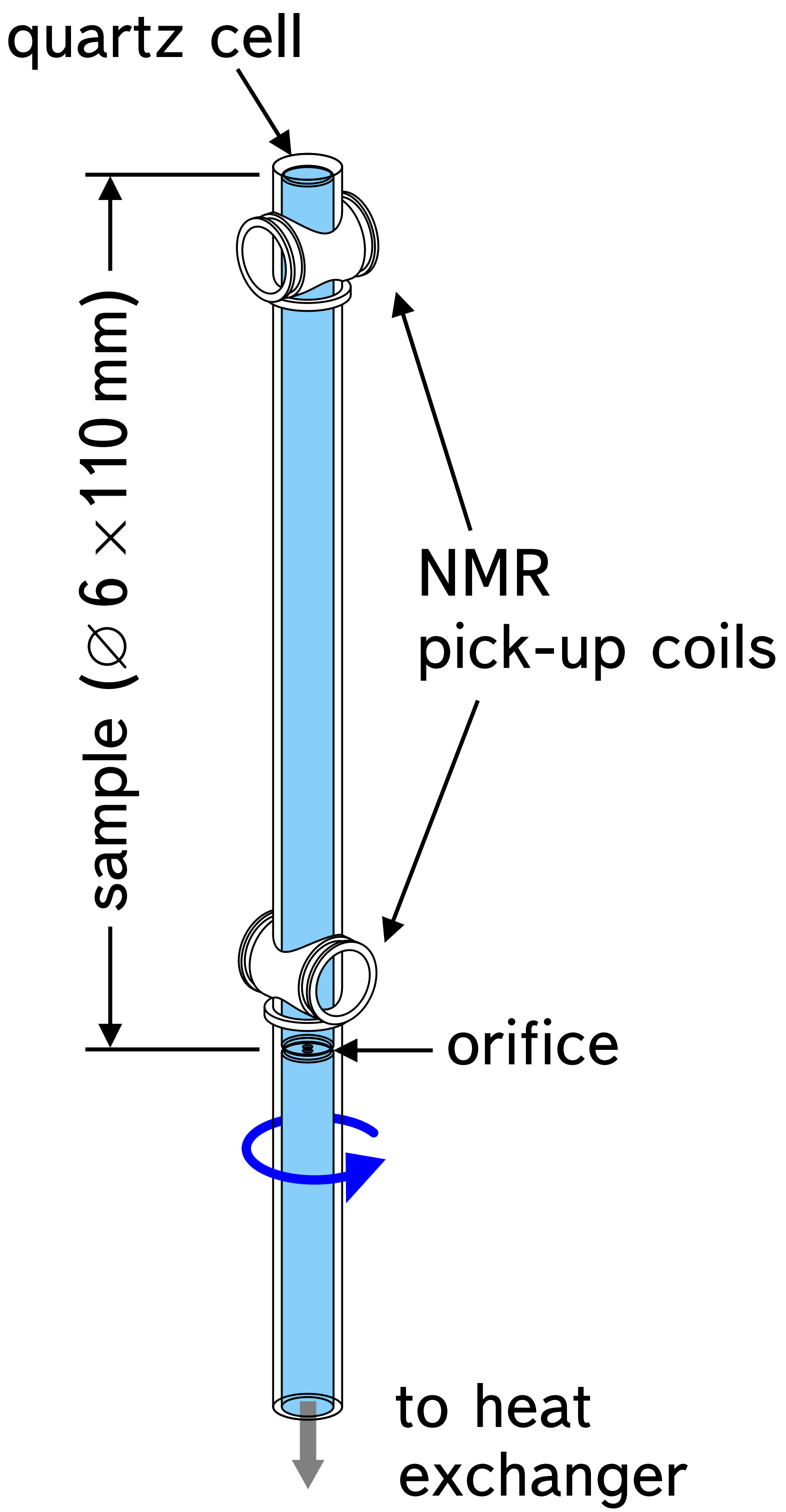
ROTATING SUPERFLUID



ROTATING SUPERFLUID



SAMPLE AND NMR



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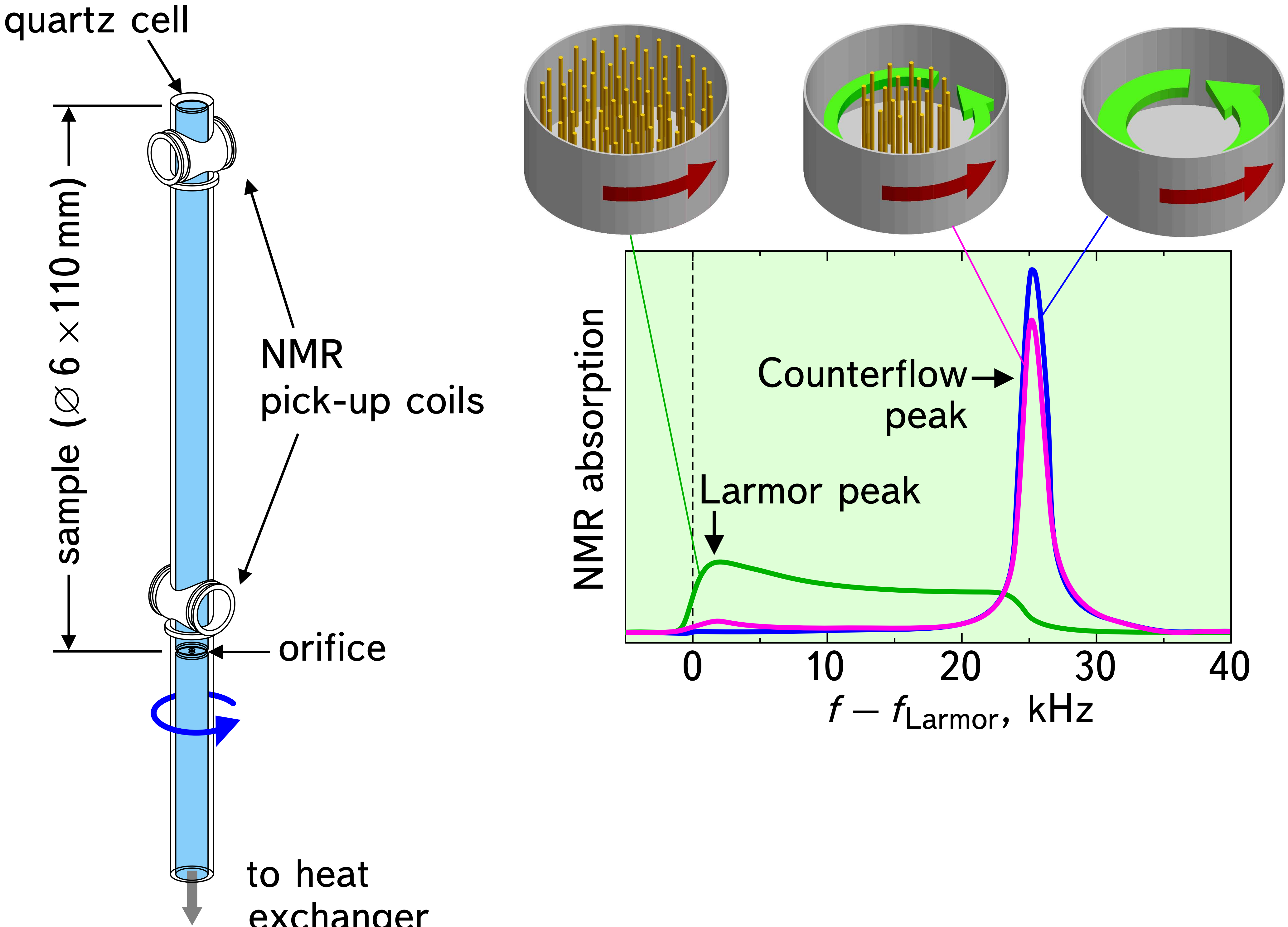
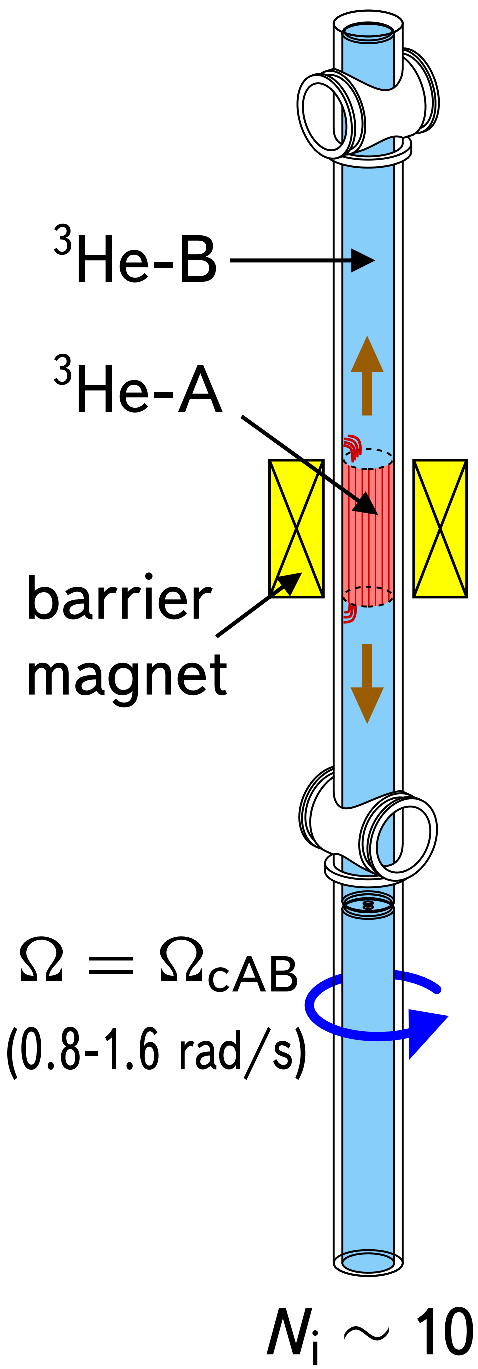
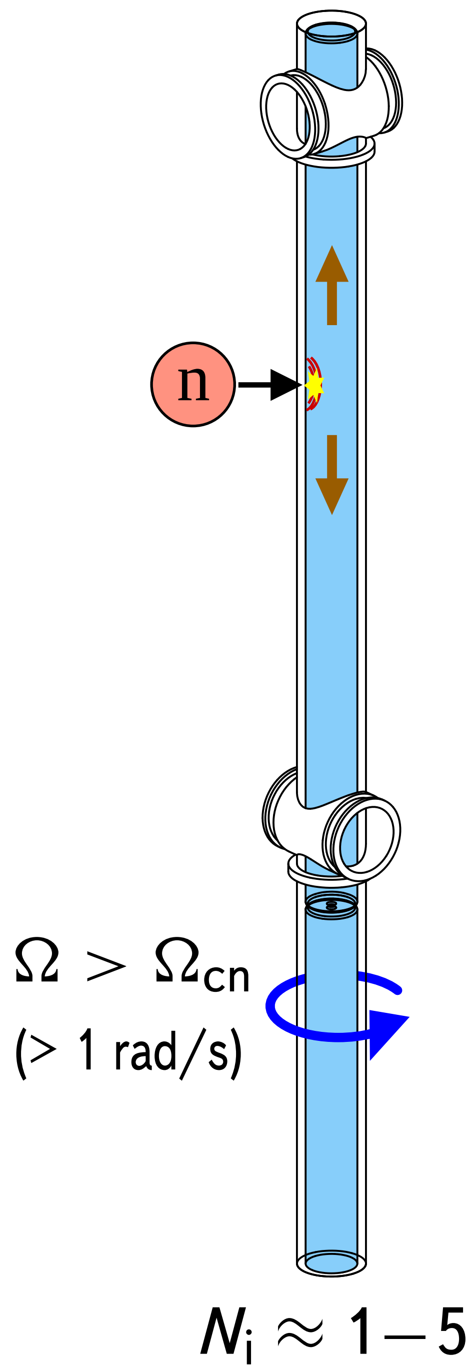


VORTEX INJECTION

AB interface instability



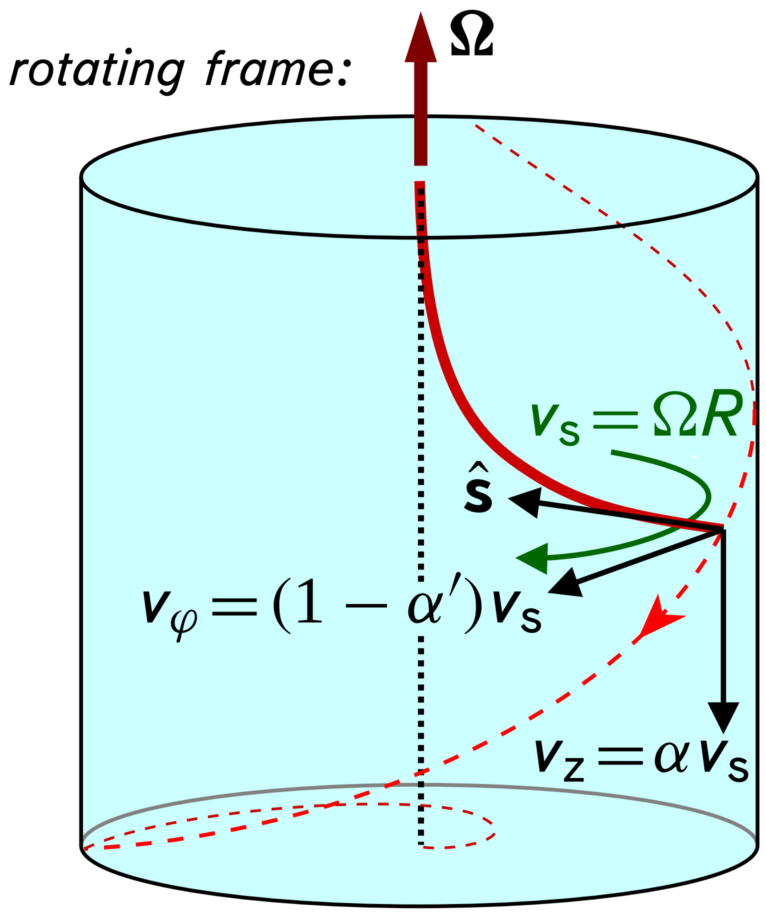
neutron absorption



MOTION OF A SINGLE VORTEX

Vortex motion under influence of the flow and of the mutual friction:

$$\mathbf{v}_{\mathrm{vort}} = \mathbf{v}_{\mathrm{s}} + \alpha(T,P)\,\hat{\mathbf{s}} \times (\mathbf{v}_{\mathrm{n}} - \mathbf{v}_{\mathrm{s}}) + \alpha'(T,P)\,(\mathbf{v}_{\mathrm{n}} - \mathbf{v}_{\mathrm{s}})_{\perp}$$



spiralling motion

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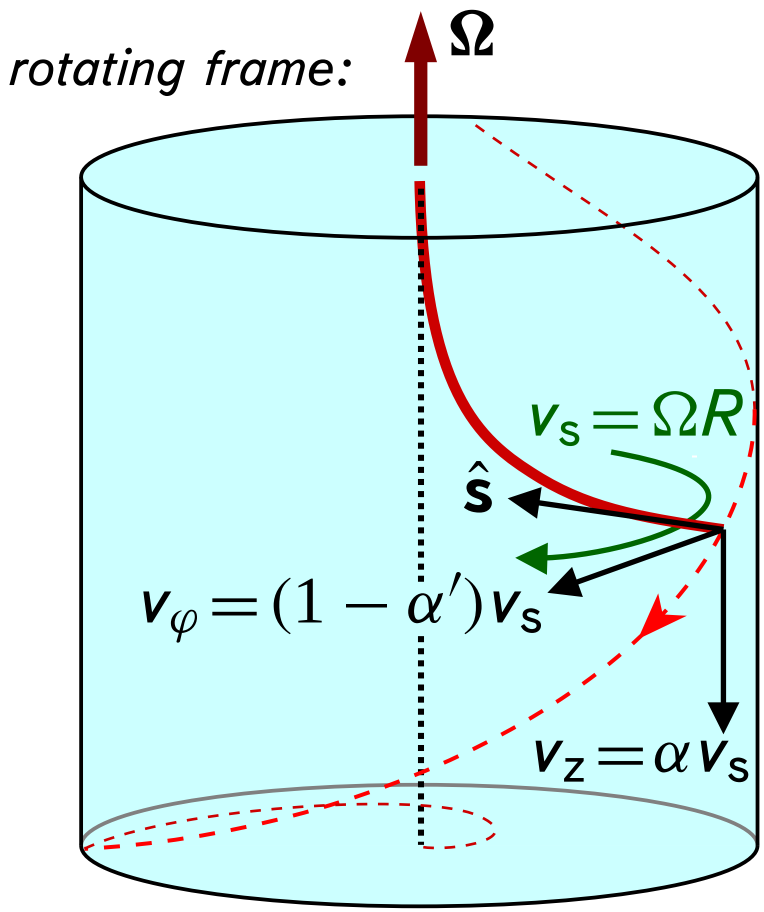
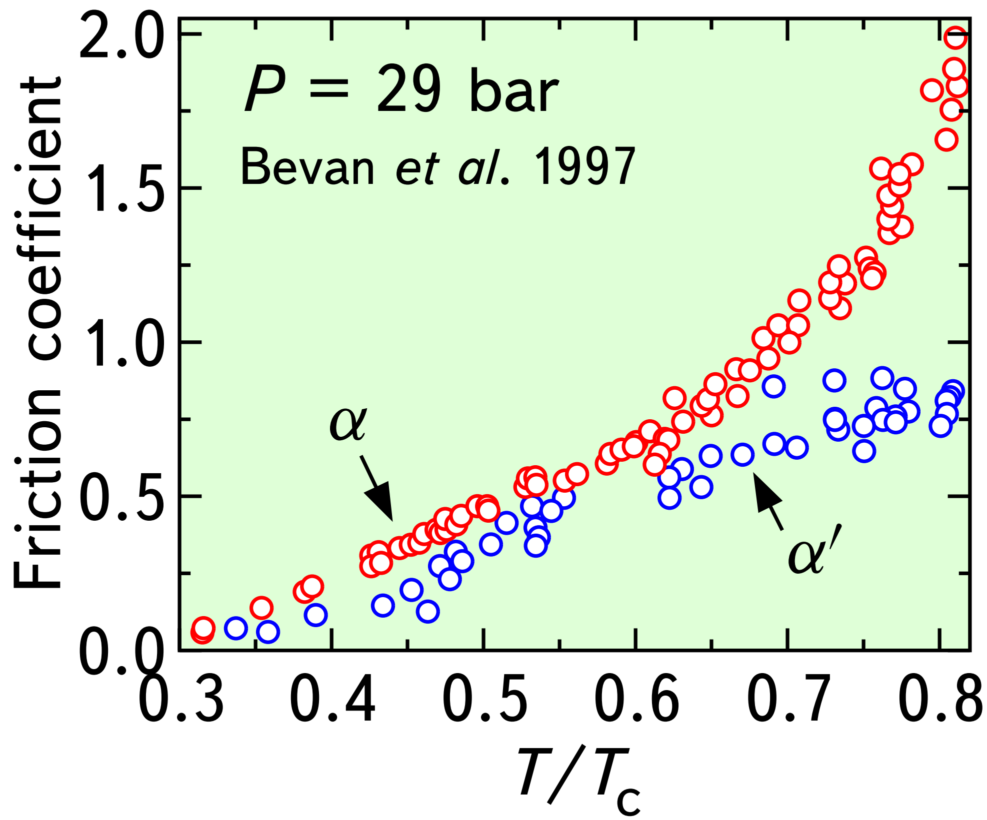


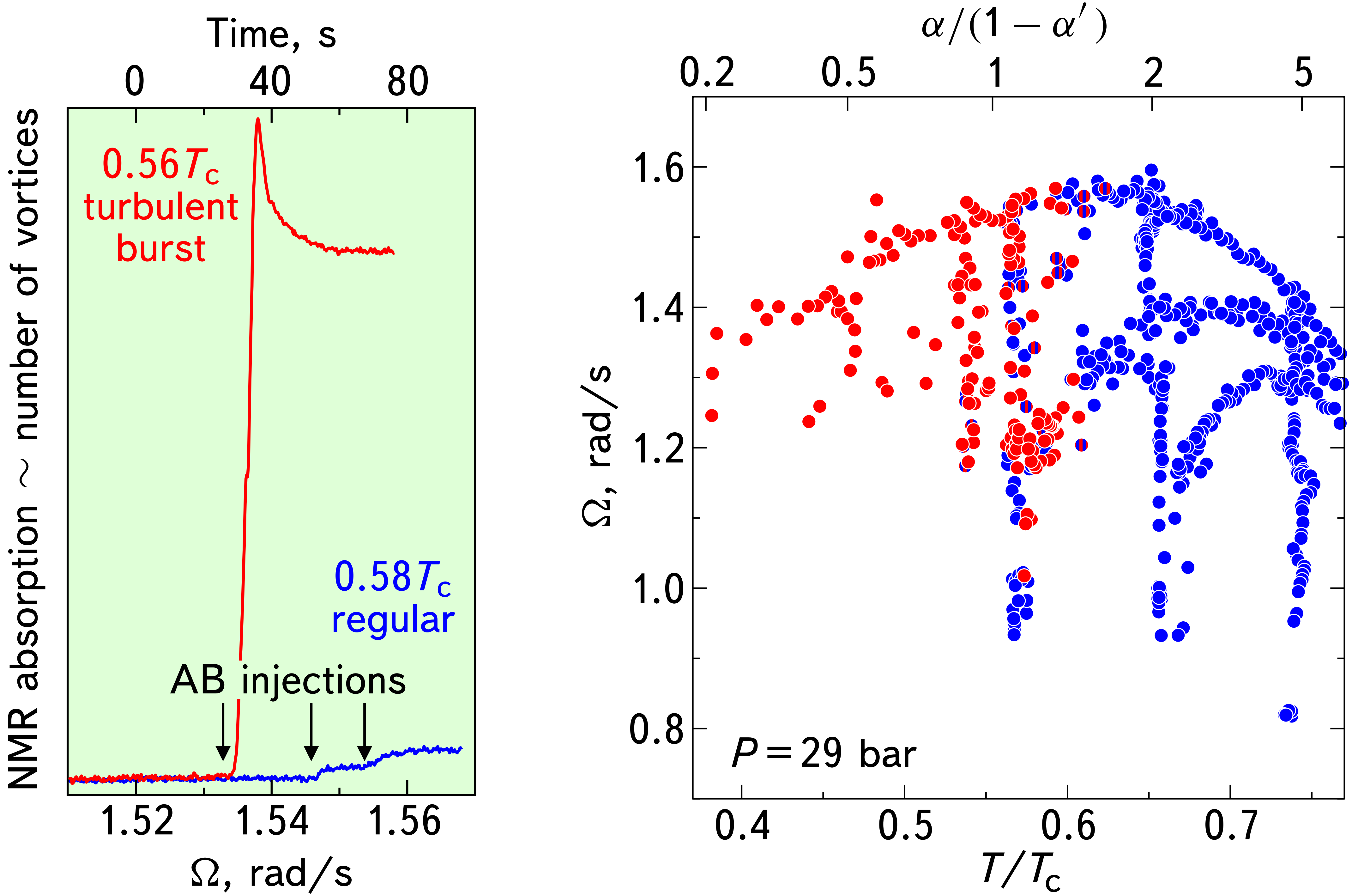
spiralling motion



For Ω = 1 rad/s, Δz = 10 cm:

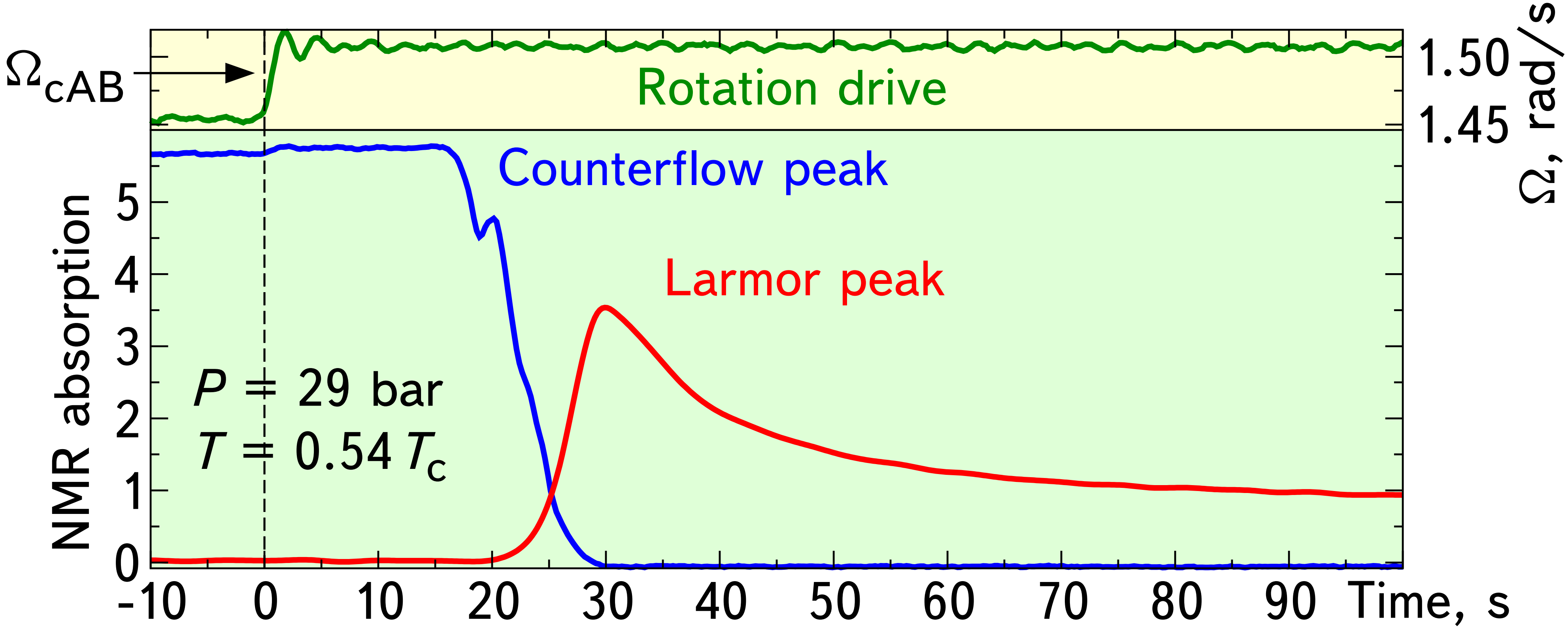
Temperature	$0.8T_{\rm c}$	$0.3T_{\rm c}$
Expansion time (s)	19	670
Number of turns	0.6	115

TRANSITION BETWEEN REGULAR AND TURBULENT DYNAMICS

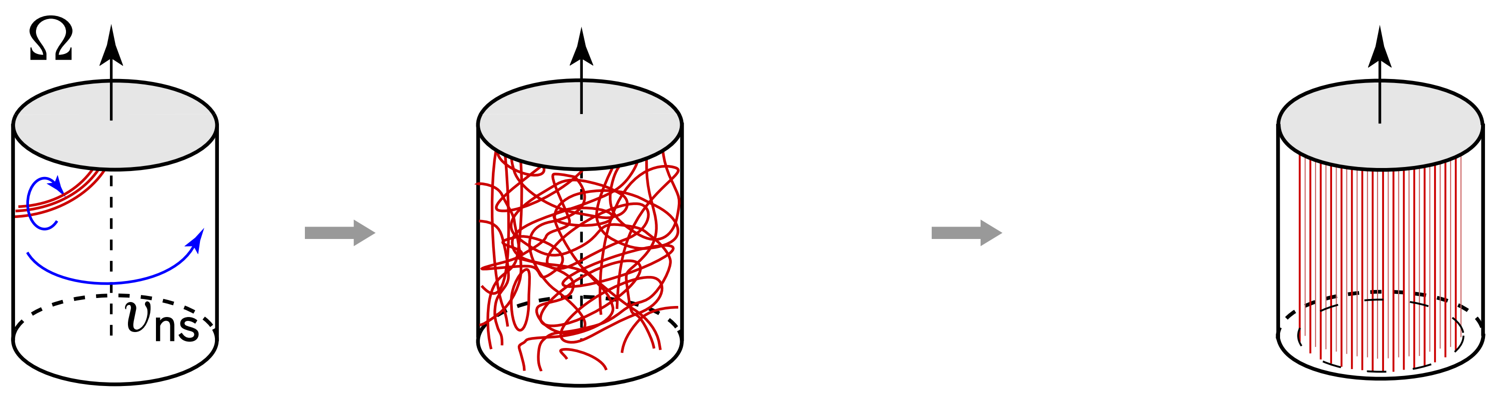


HOW DOES TURBULENCE EVOLVE IN ROTATING COLUMN?

Measurements:



Possible interpretation:



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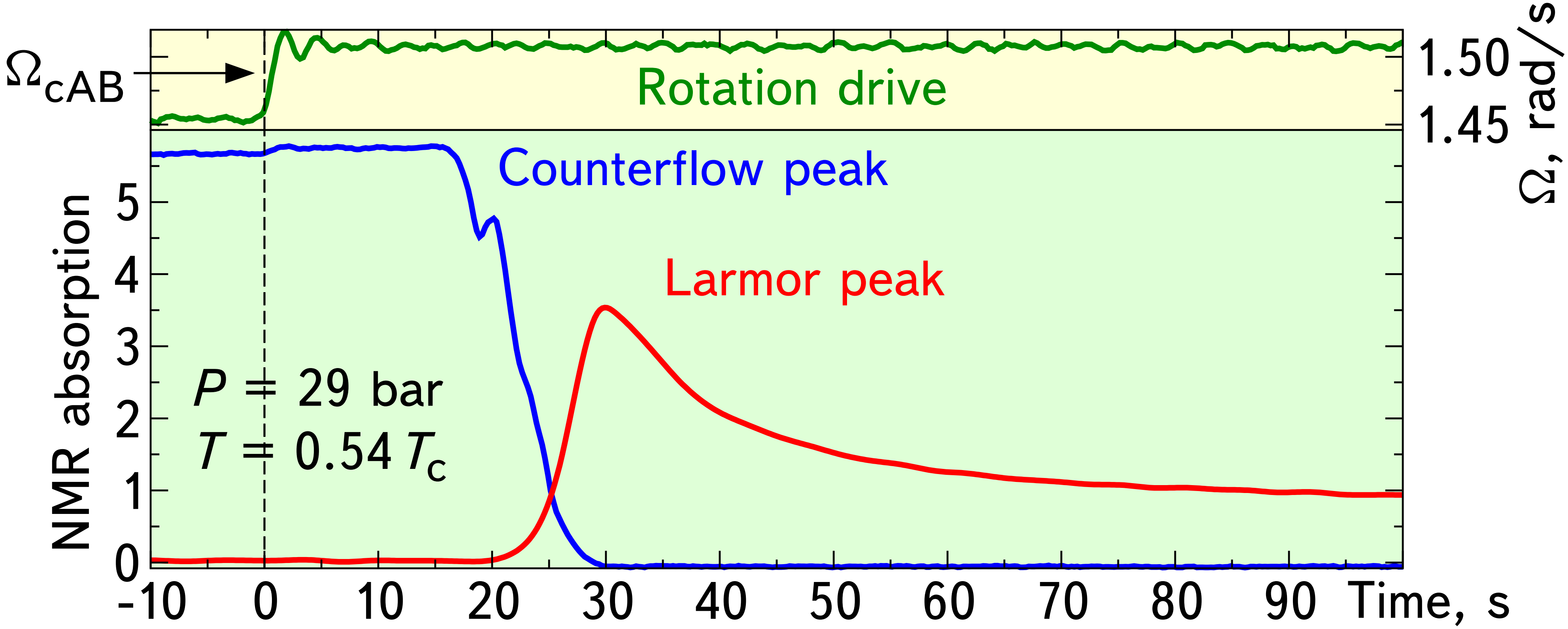

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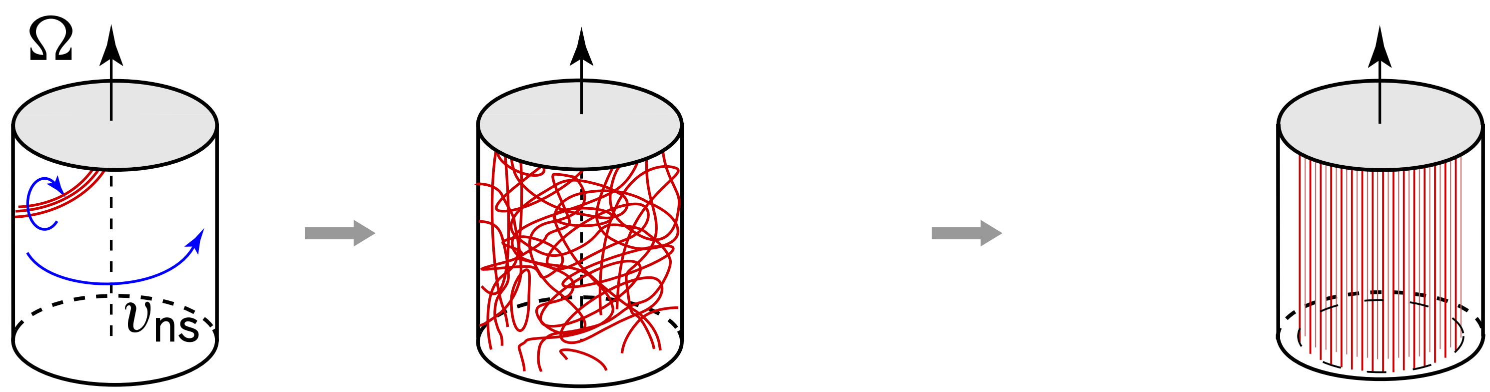
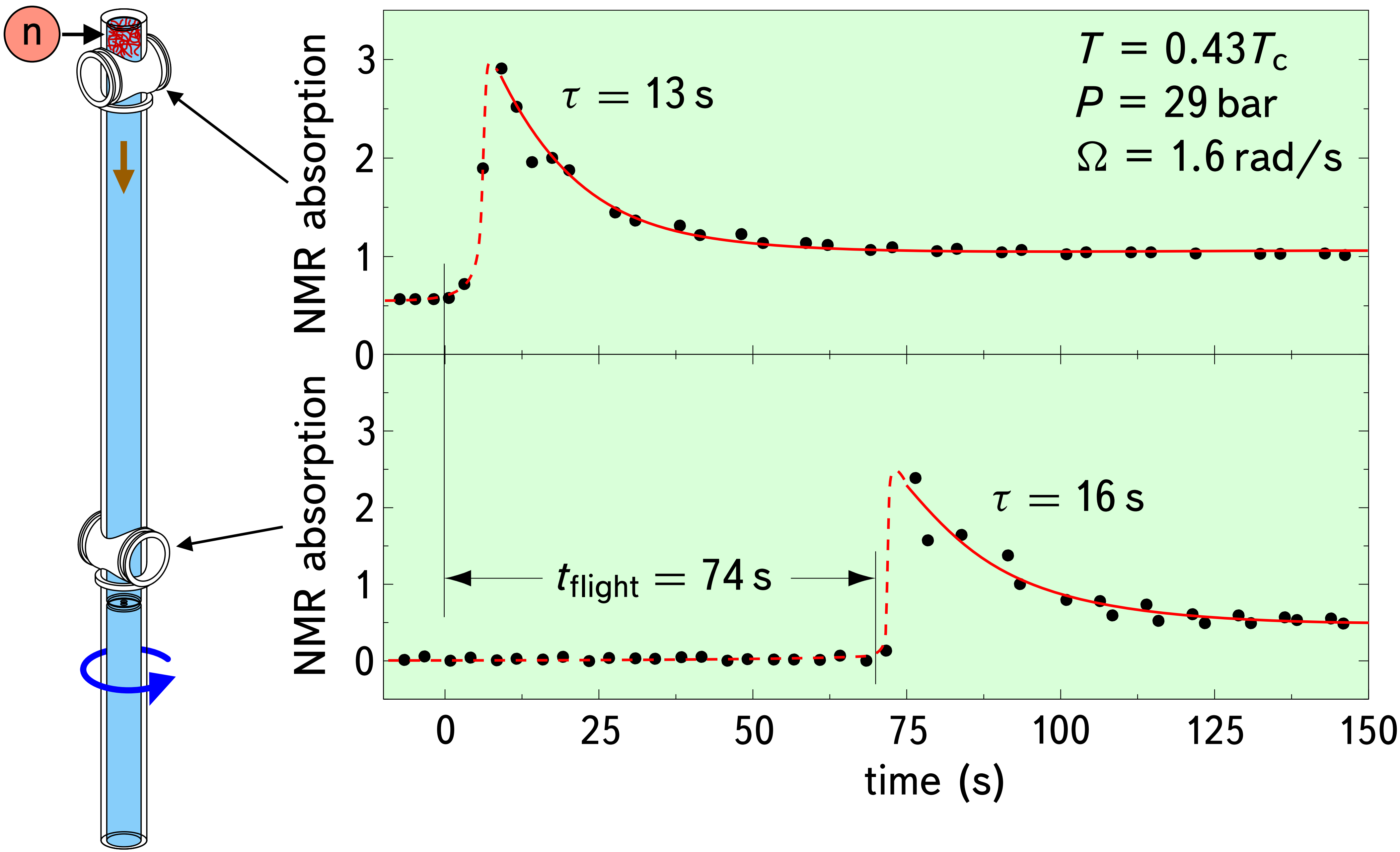
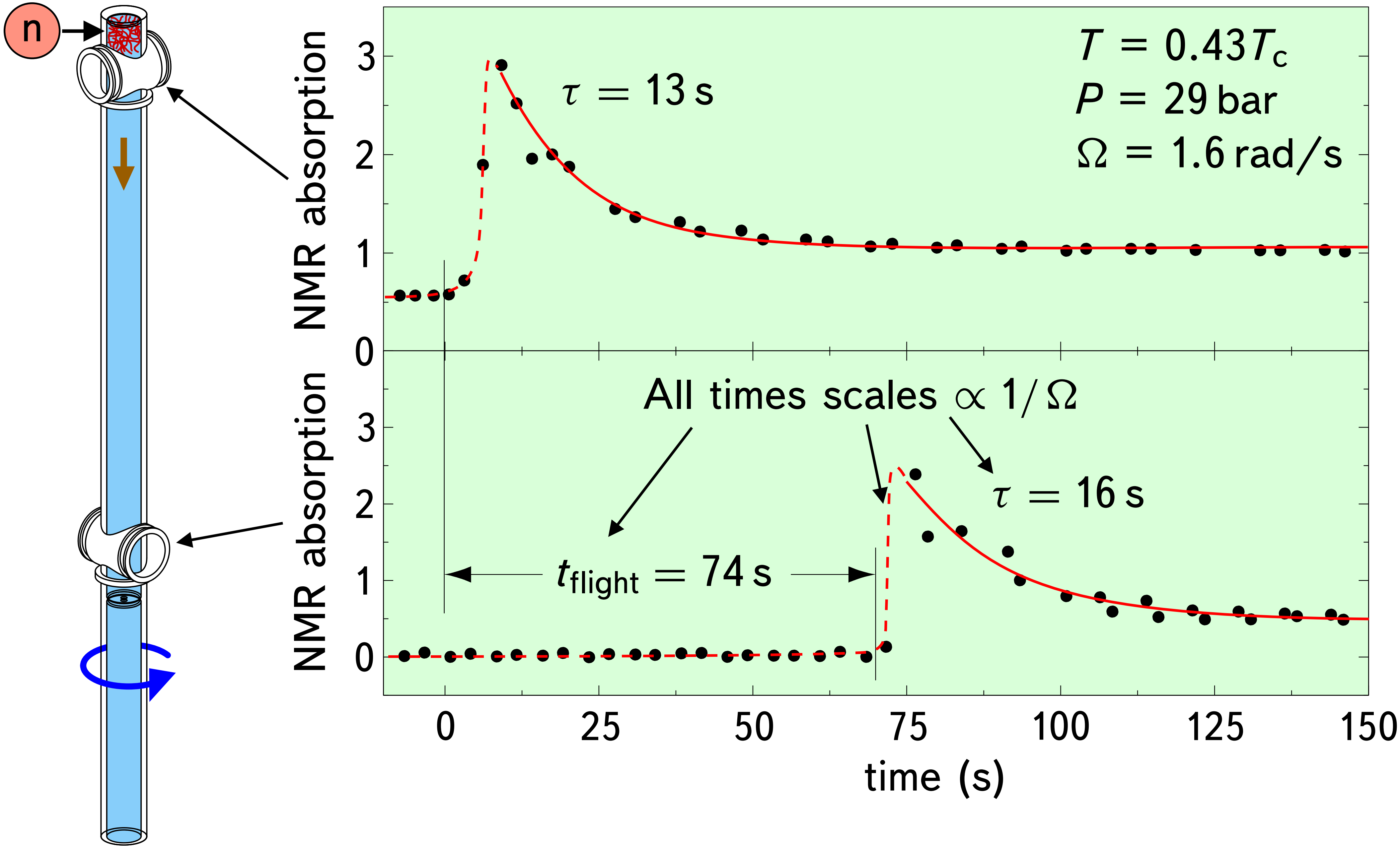


This interpretation is not correct!

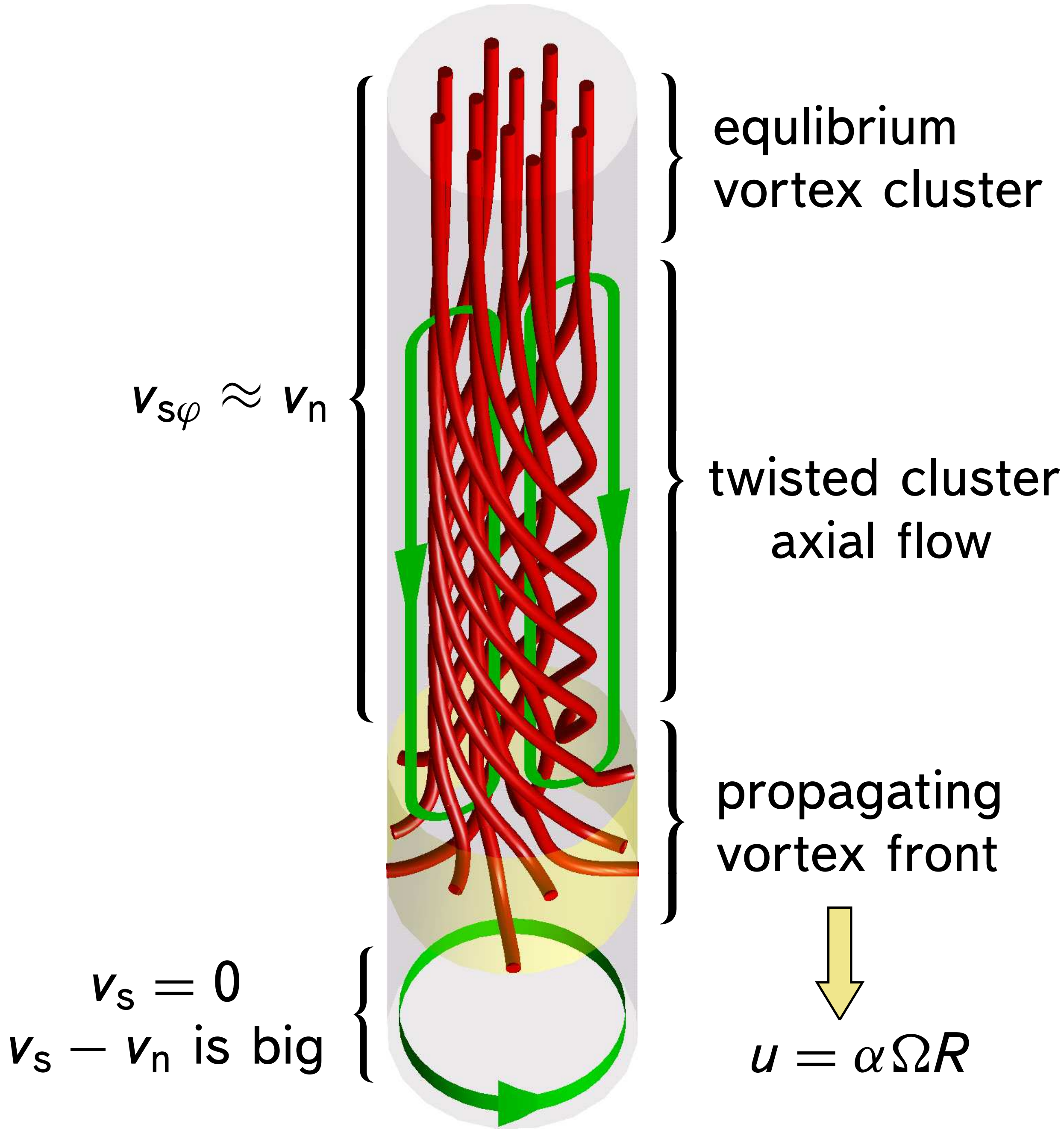
EVIDENCE FOR THE FRONT



EVIDENCE FOR THE FRONT

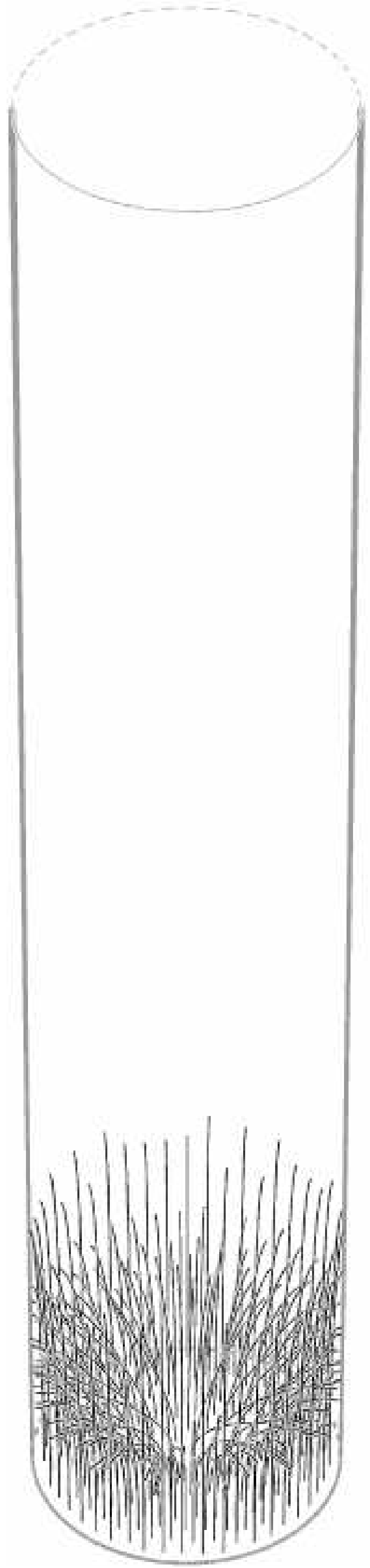


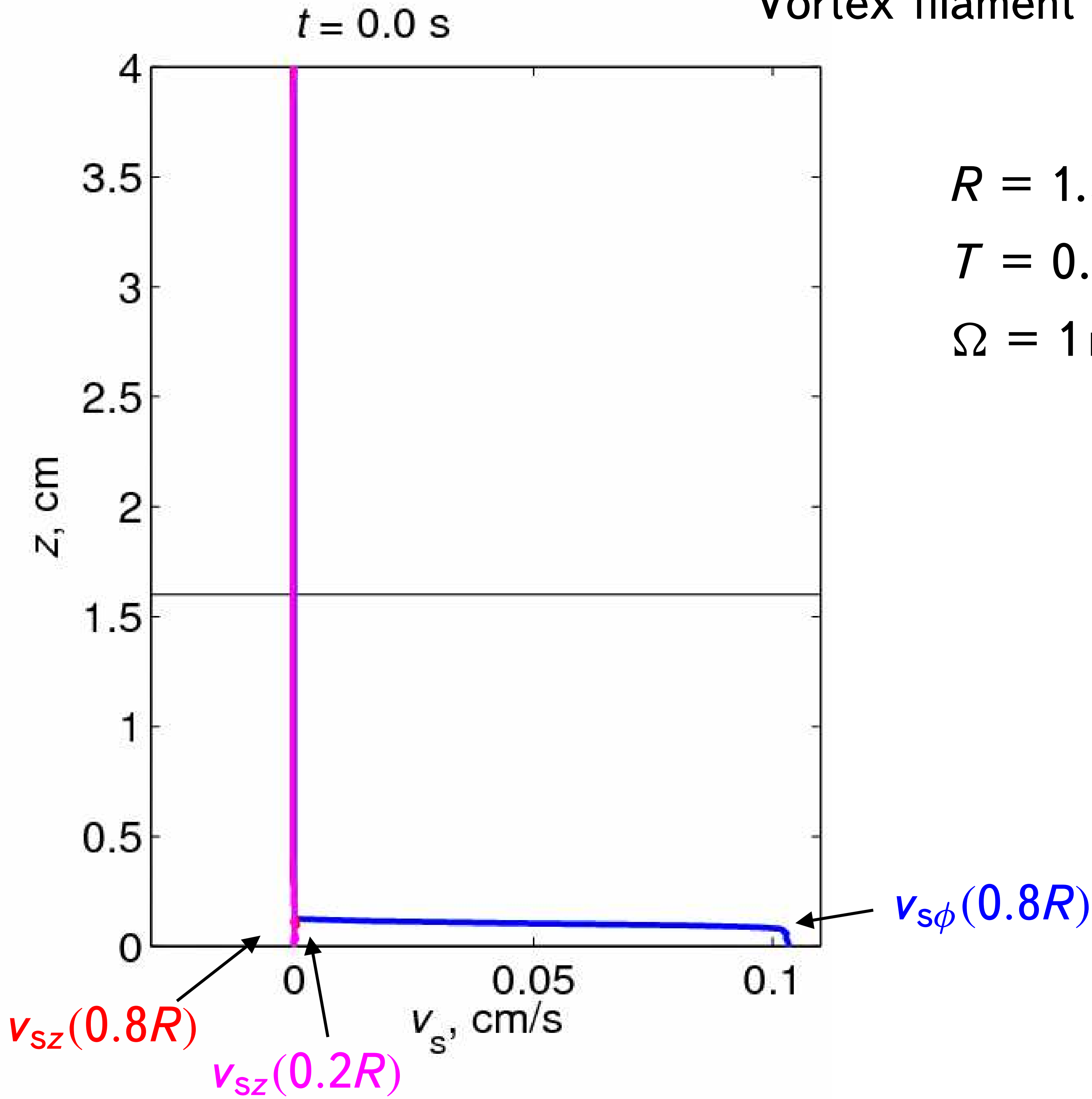
PROPAGATION OF VORTICITY IN ROTATING COLUMN



VORTEX FRONT IN NUMERICAL SIMULATIONS

Vortex filament model





$R = 1.5$ mm

$T = 0.4\, T_c$

$\Omega = 1$ rad/s

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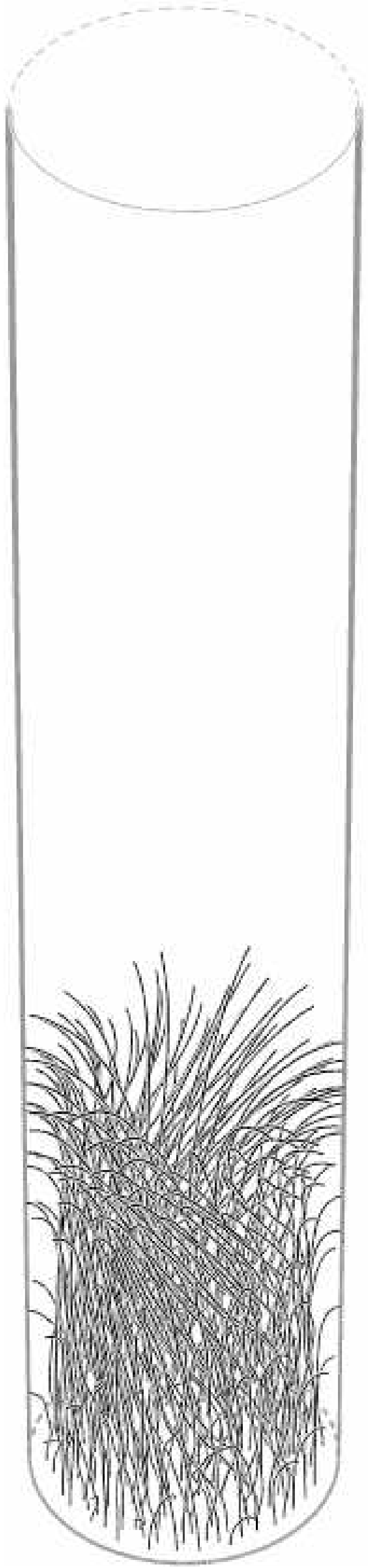

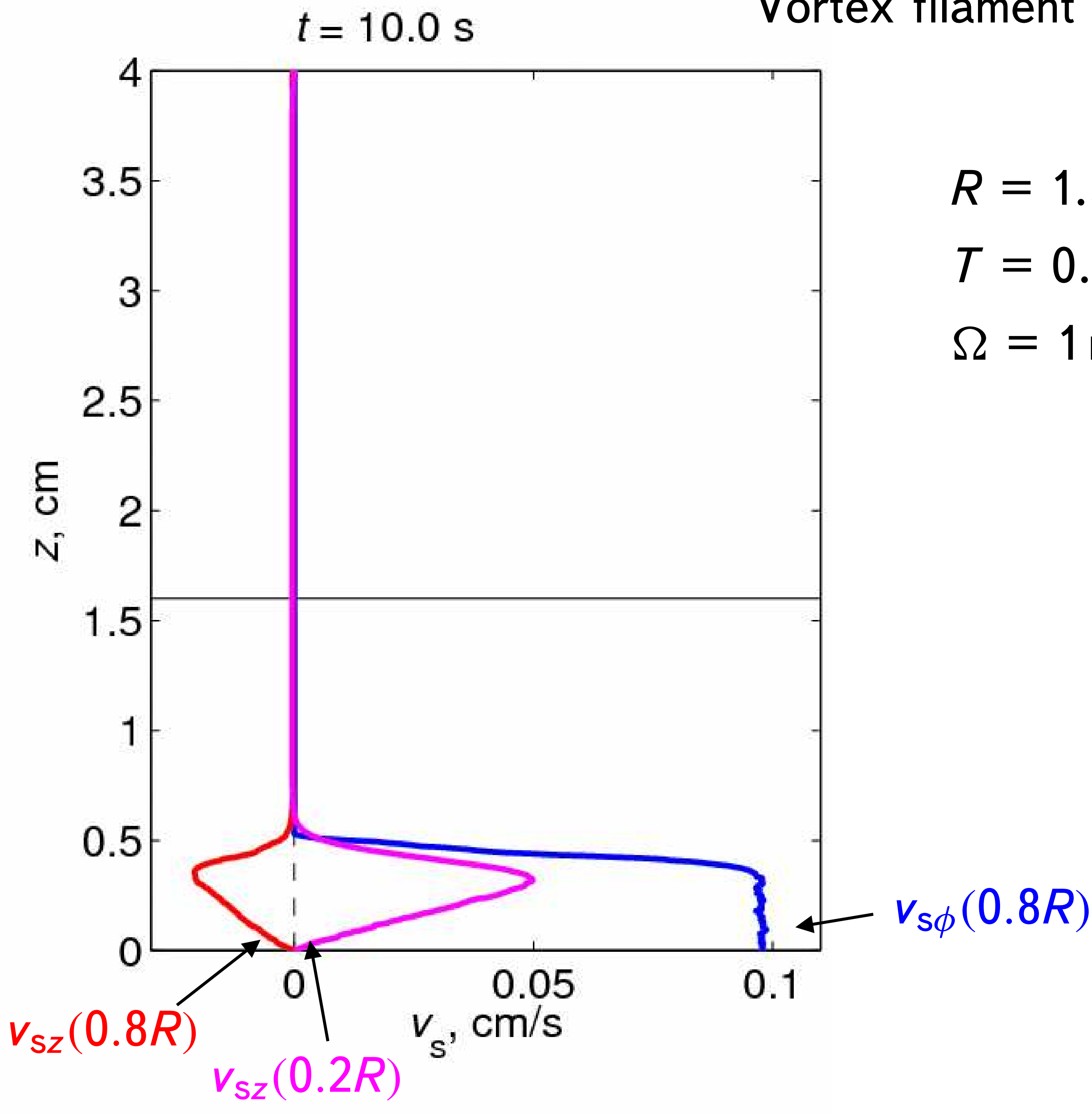

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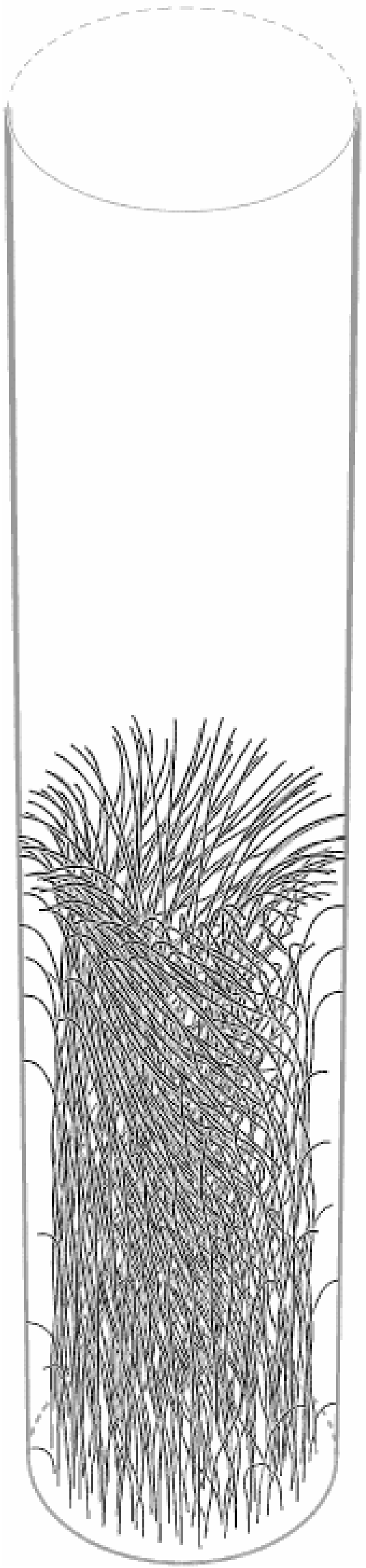

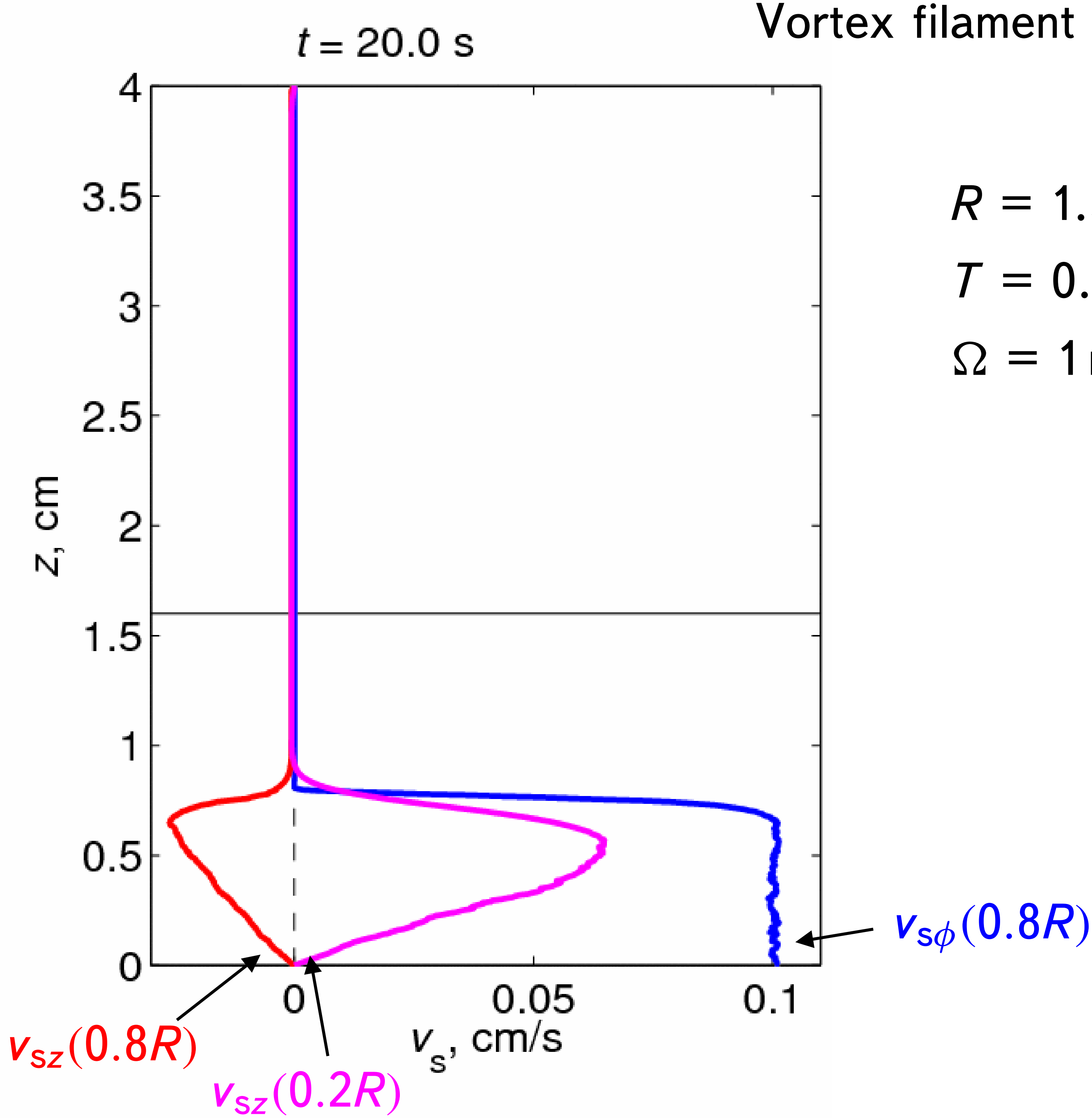

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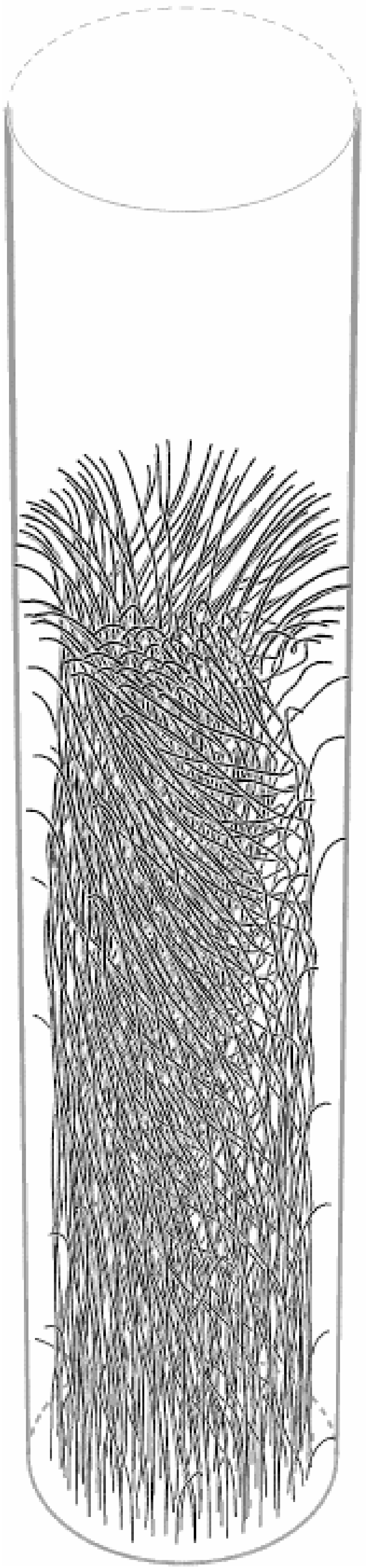

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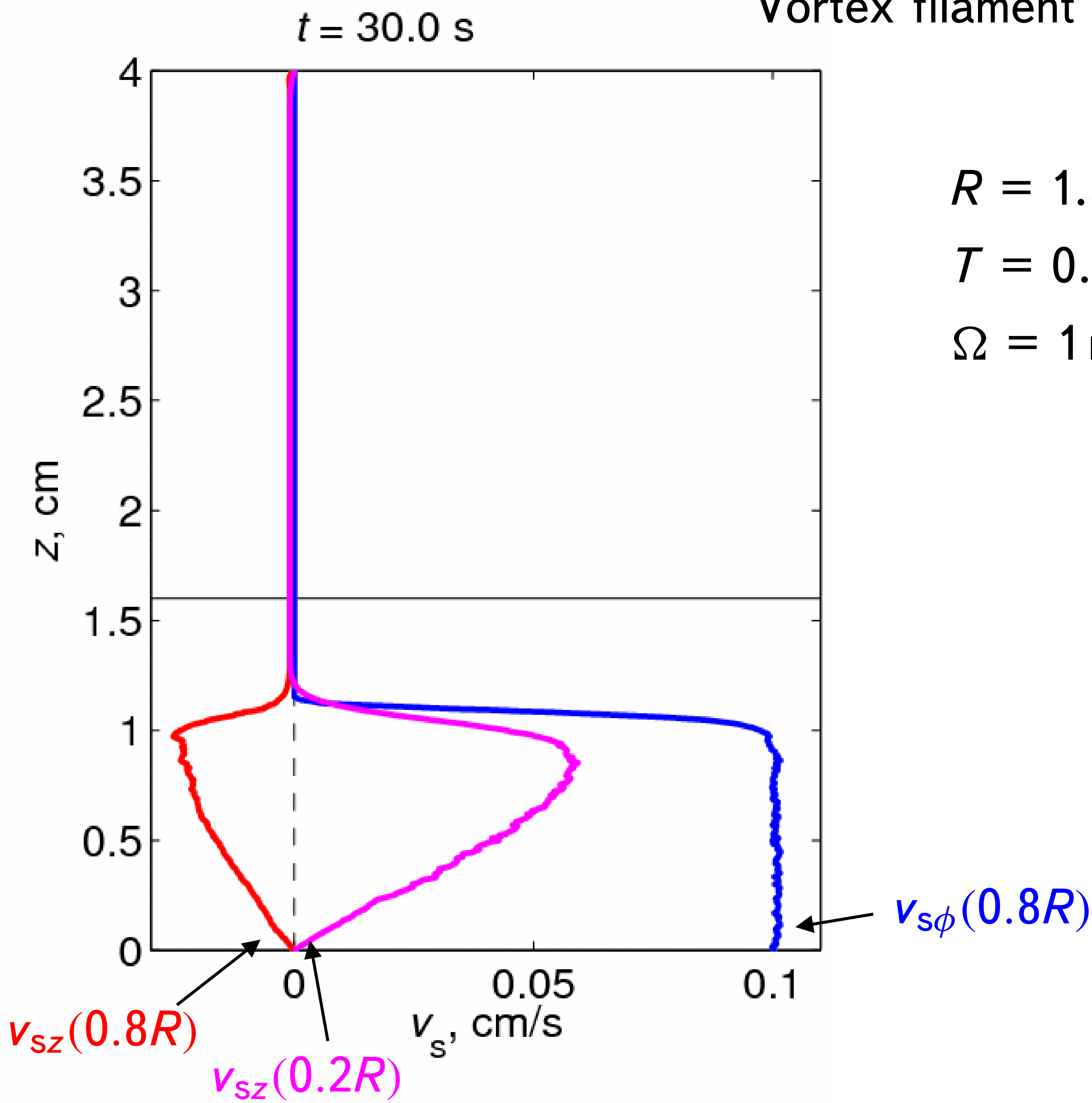

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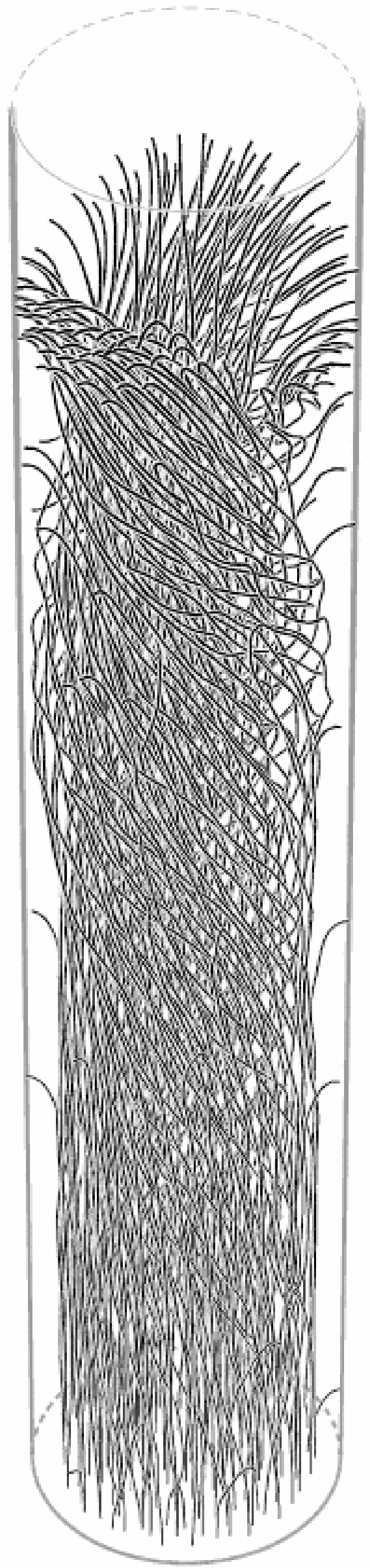

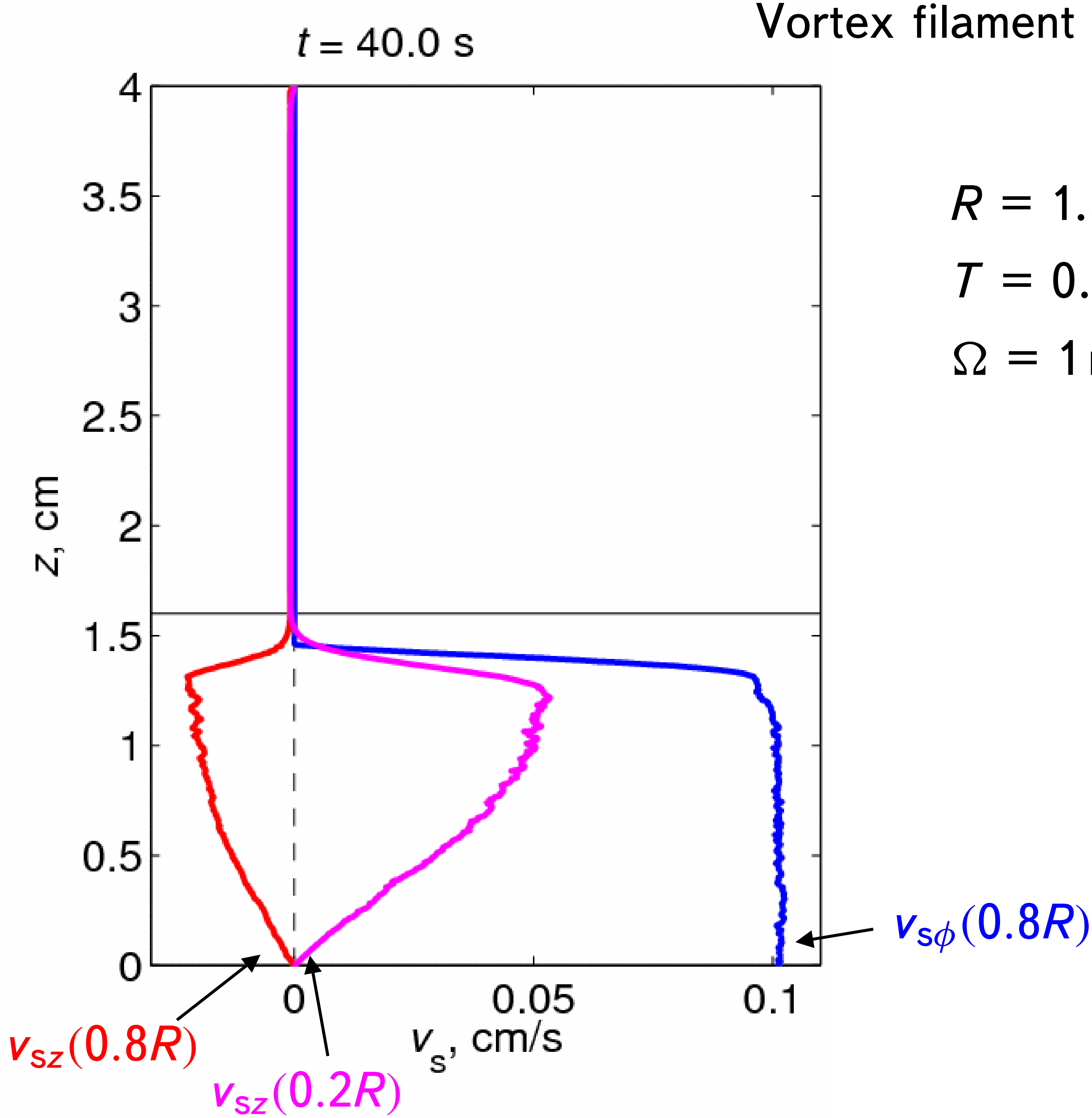

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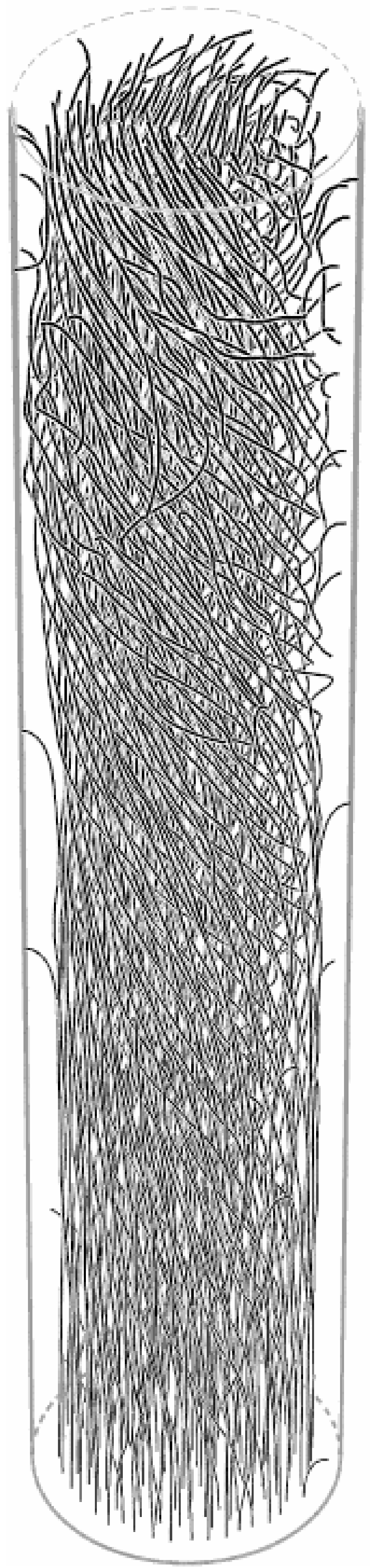

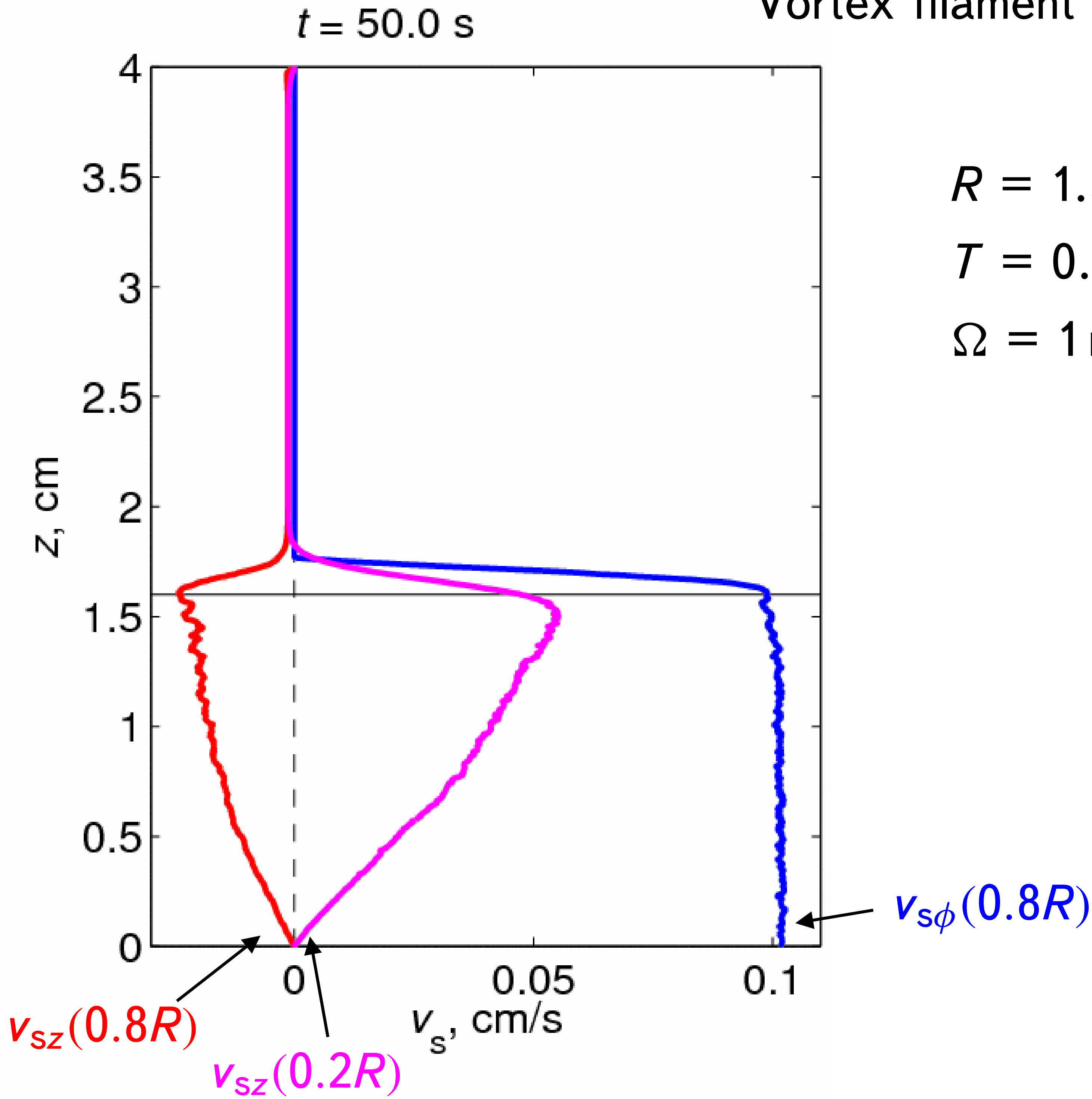

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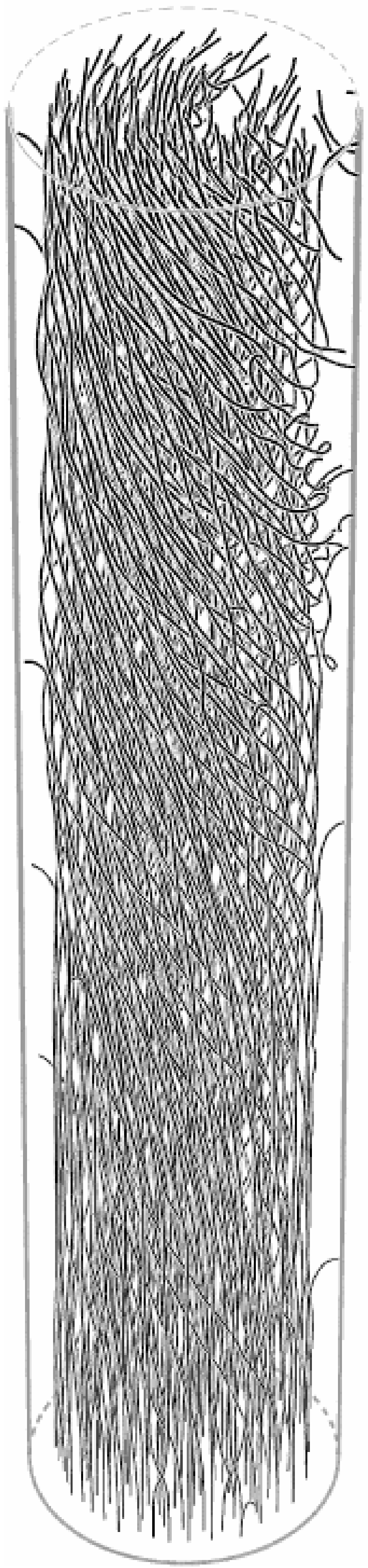

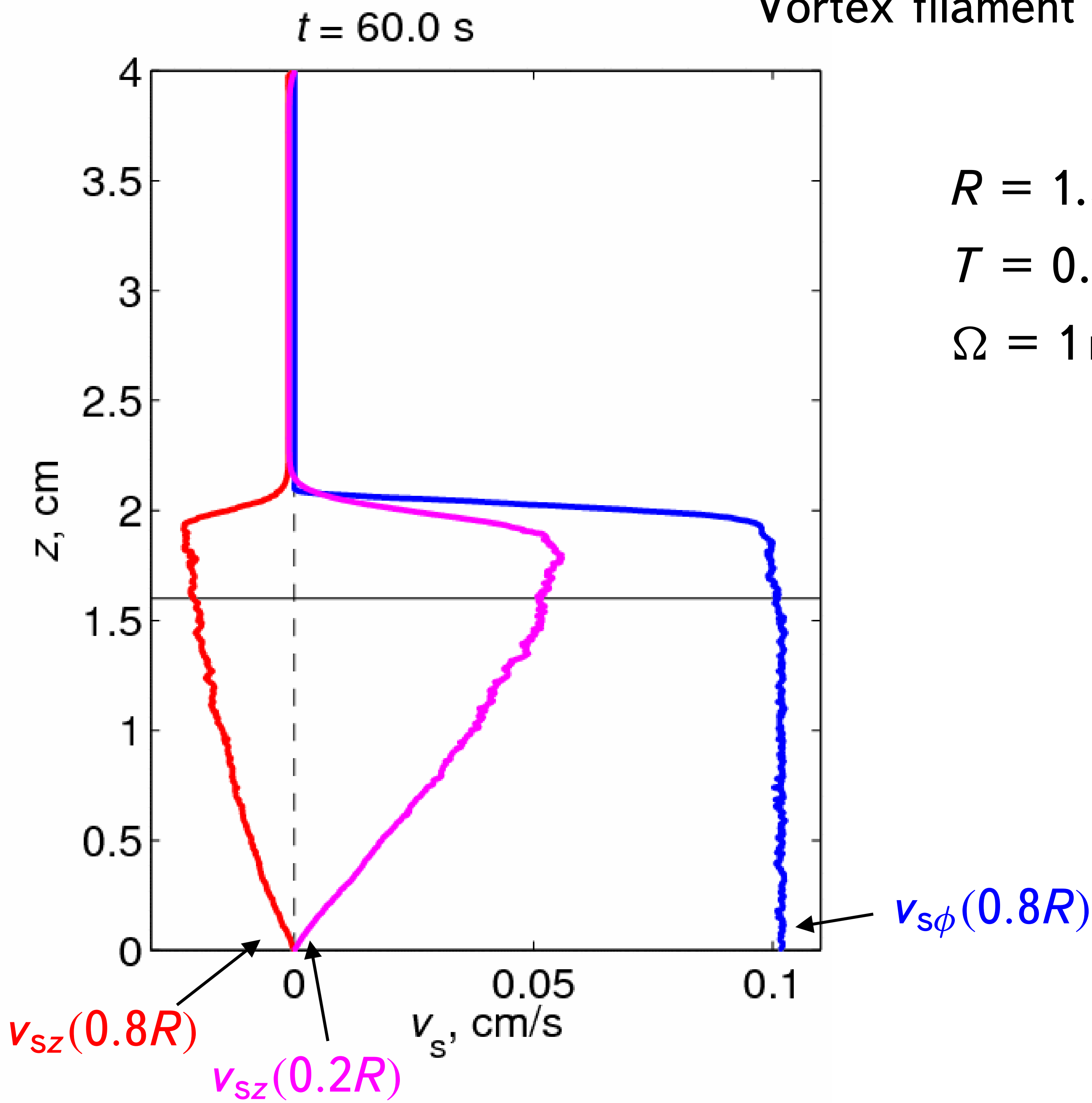

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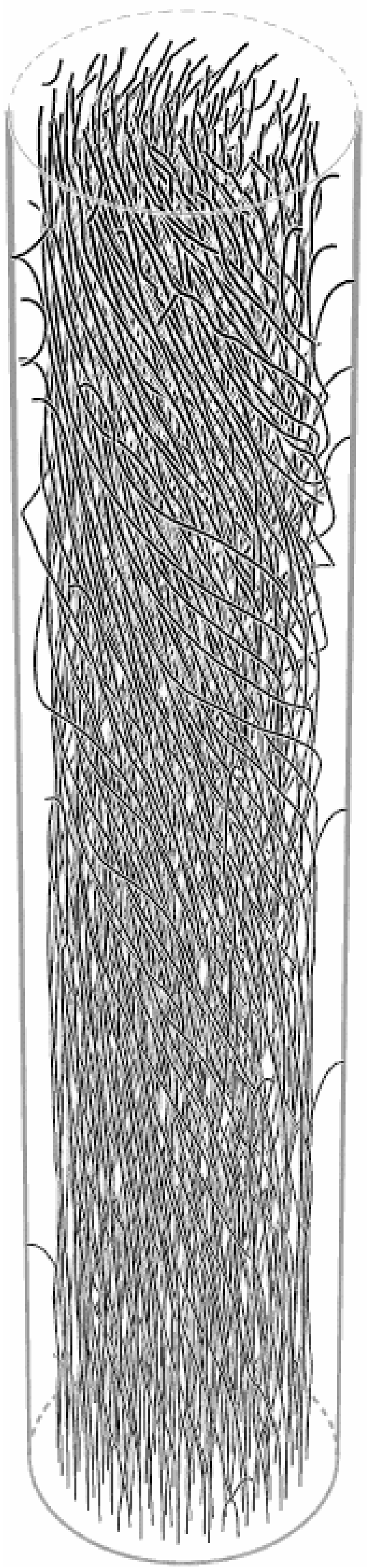

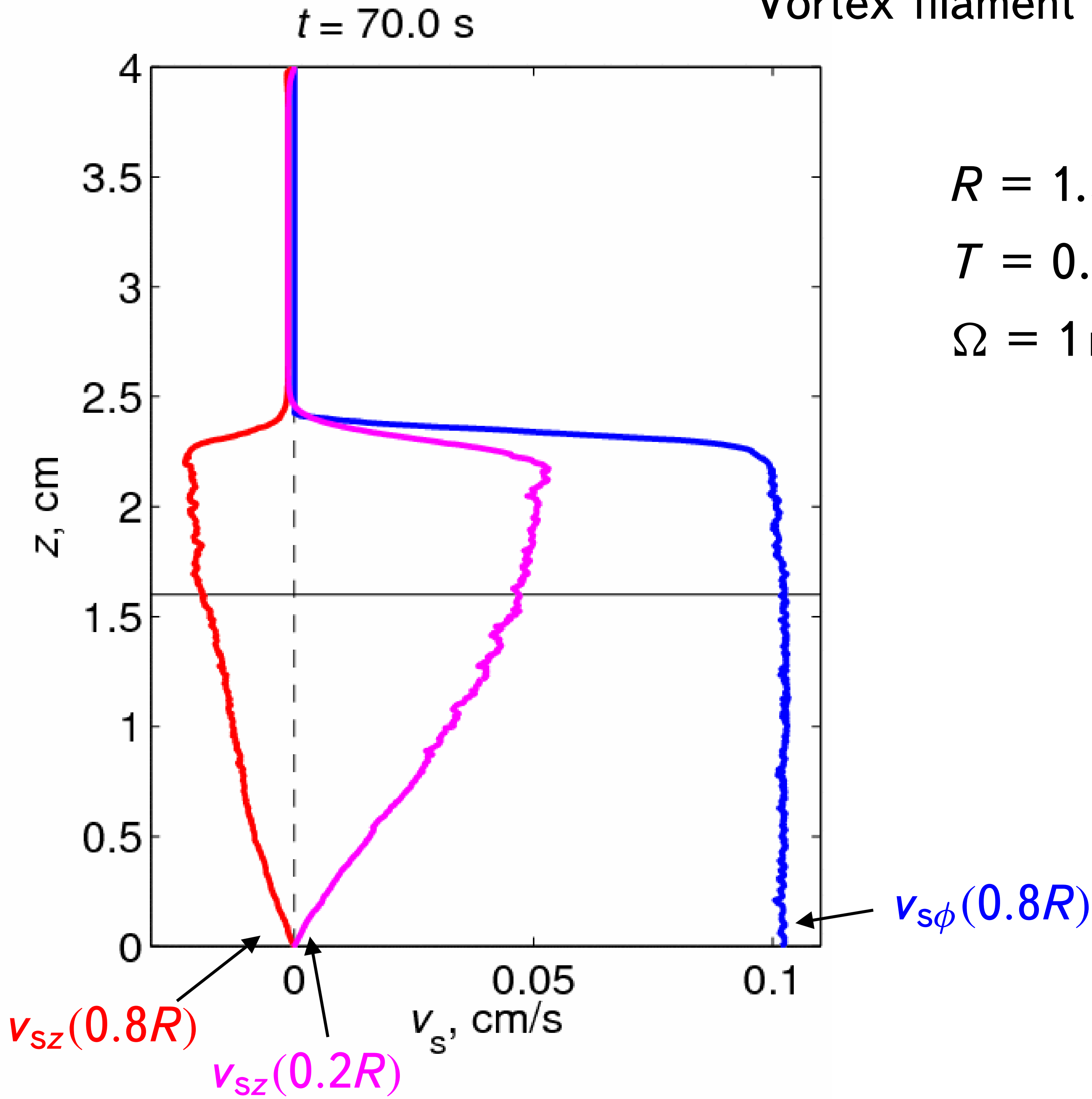

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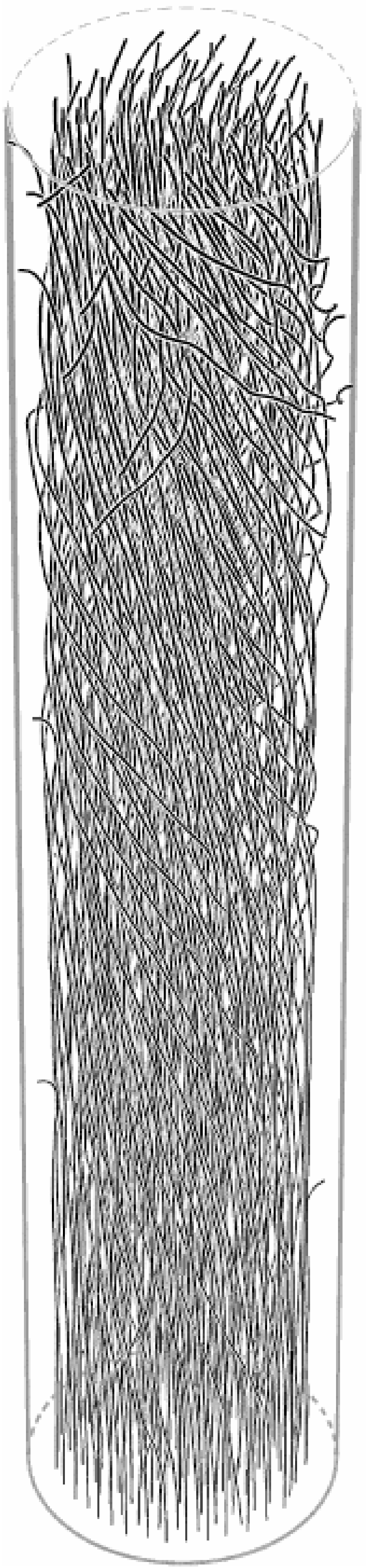

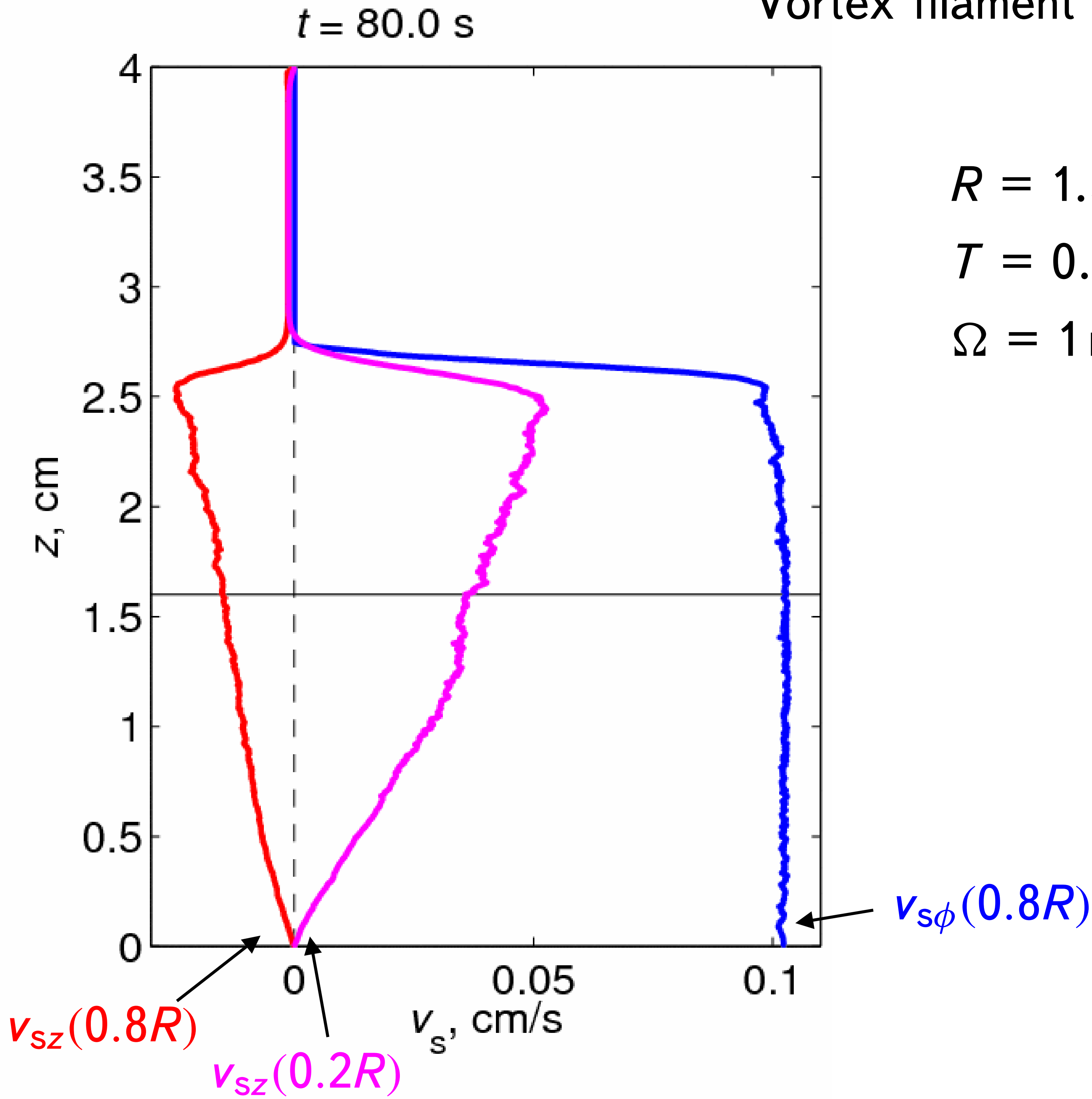

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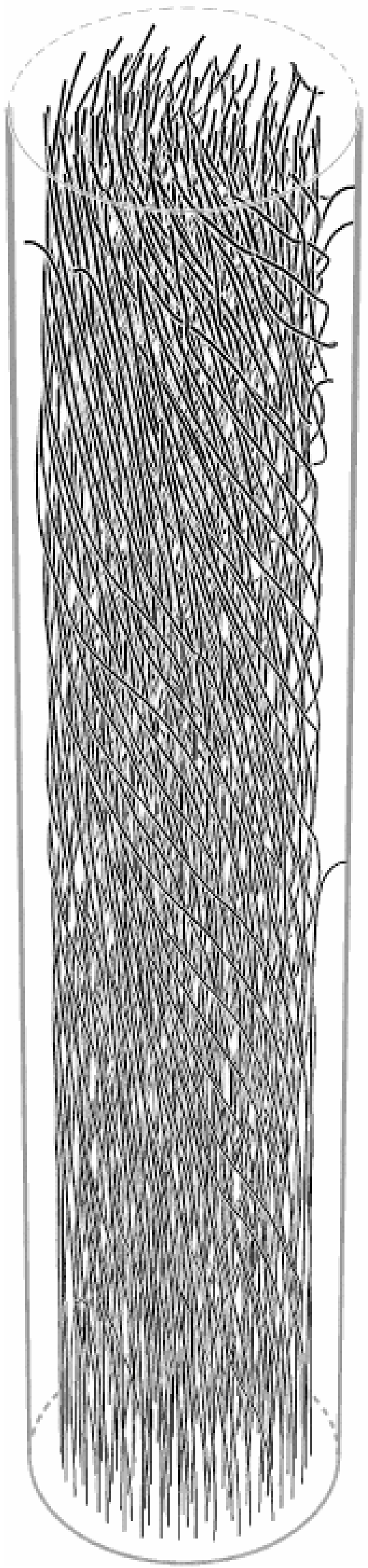

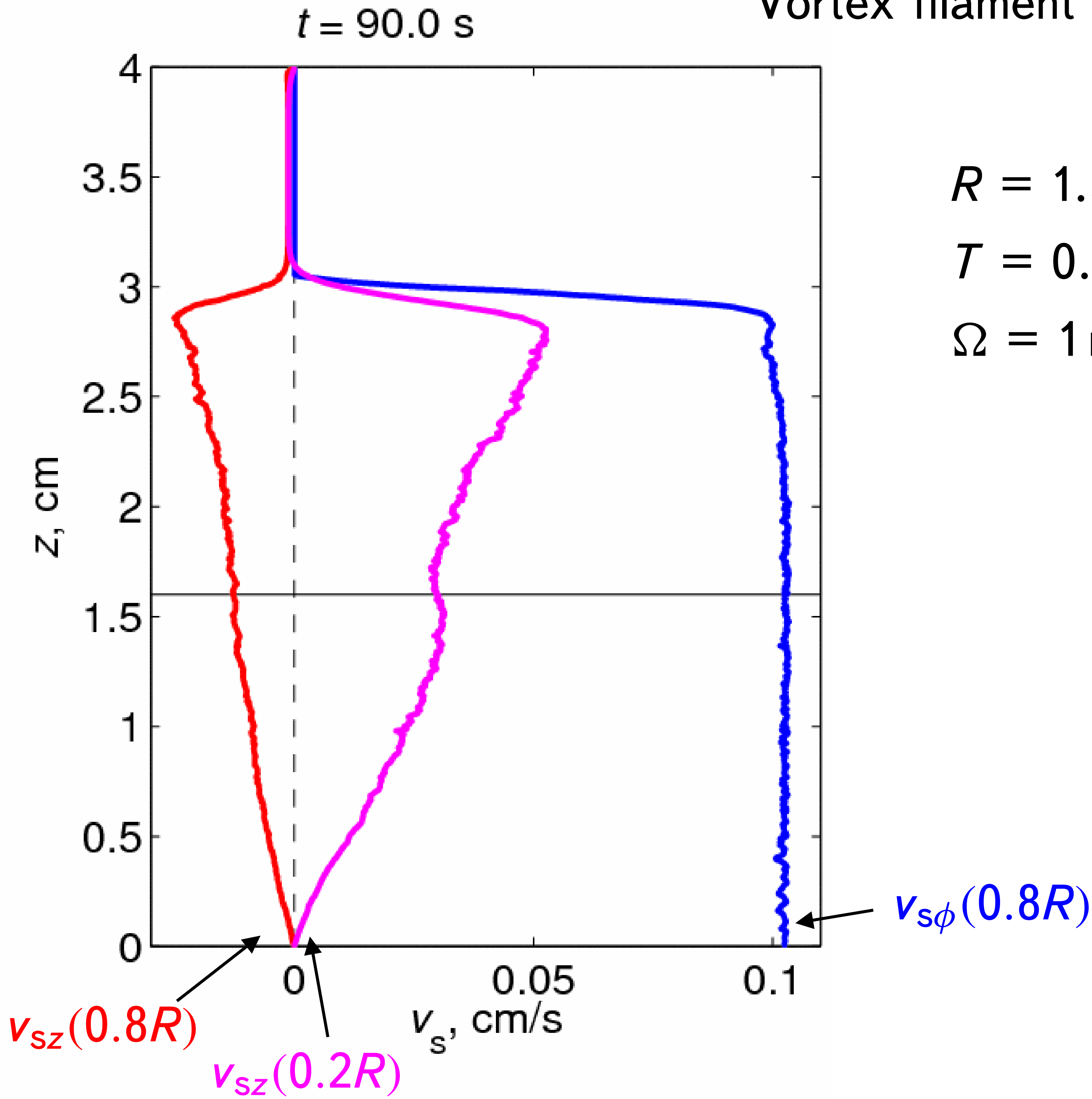

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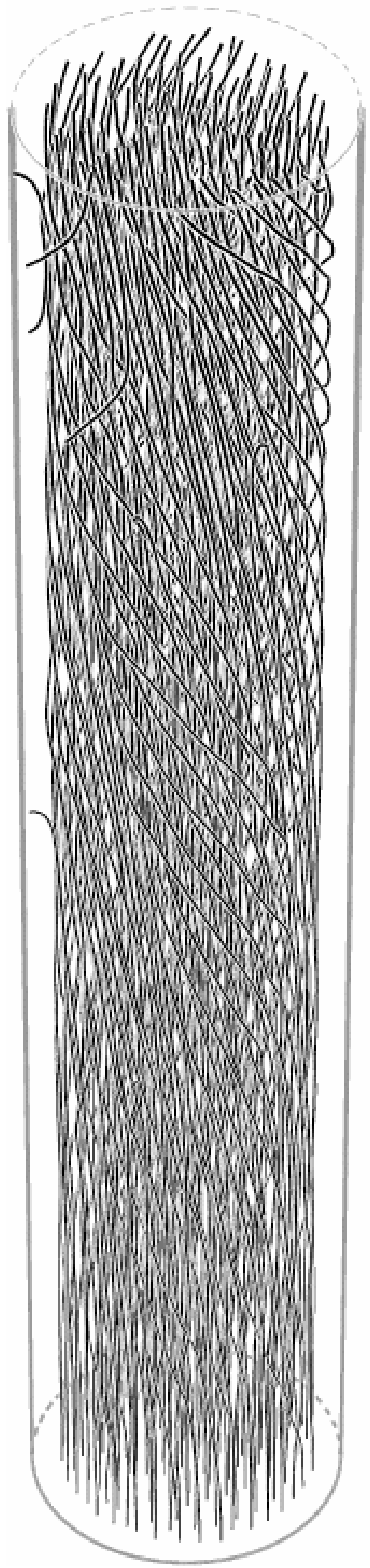

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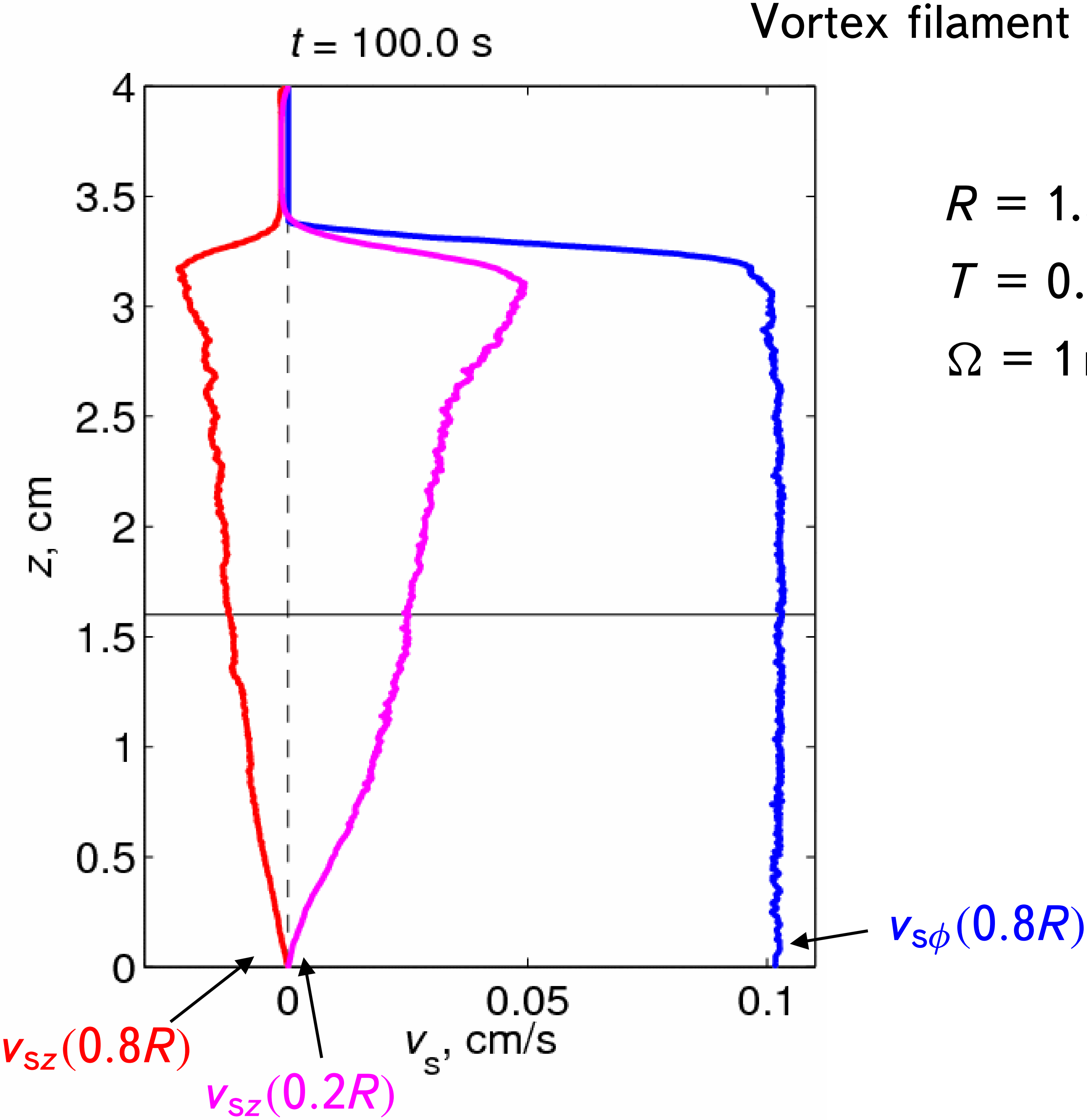

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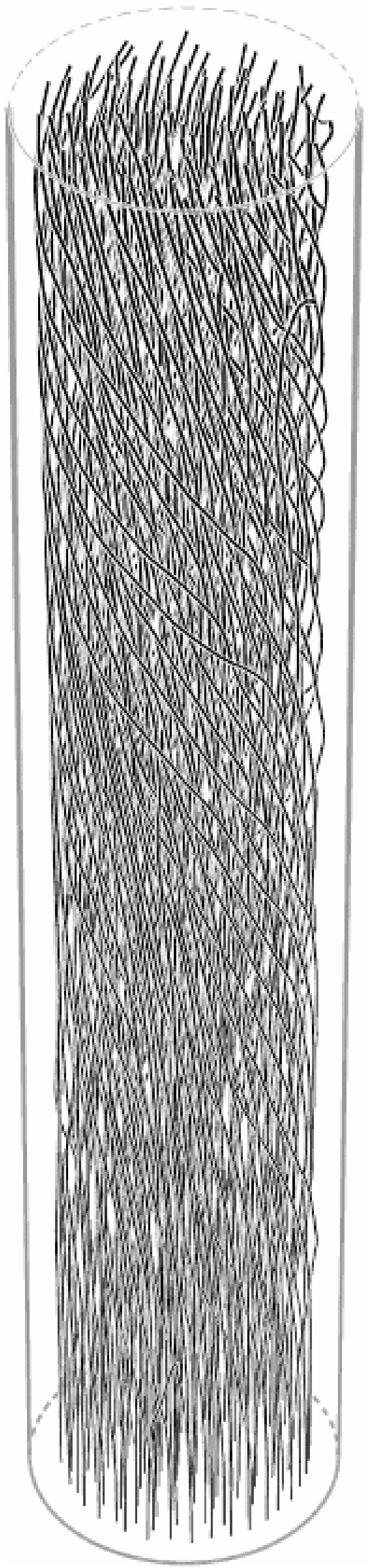

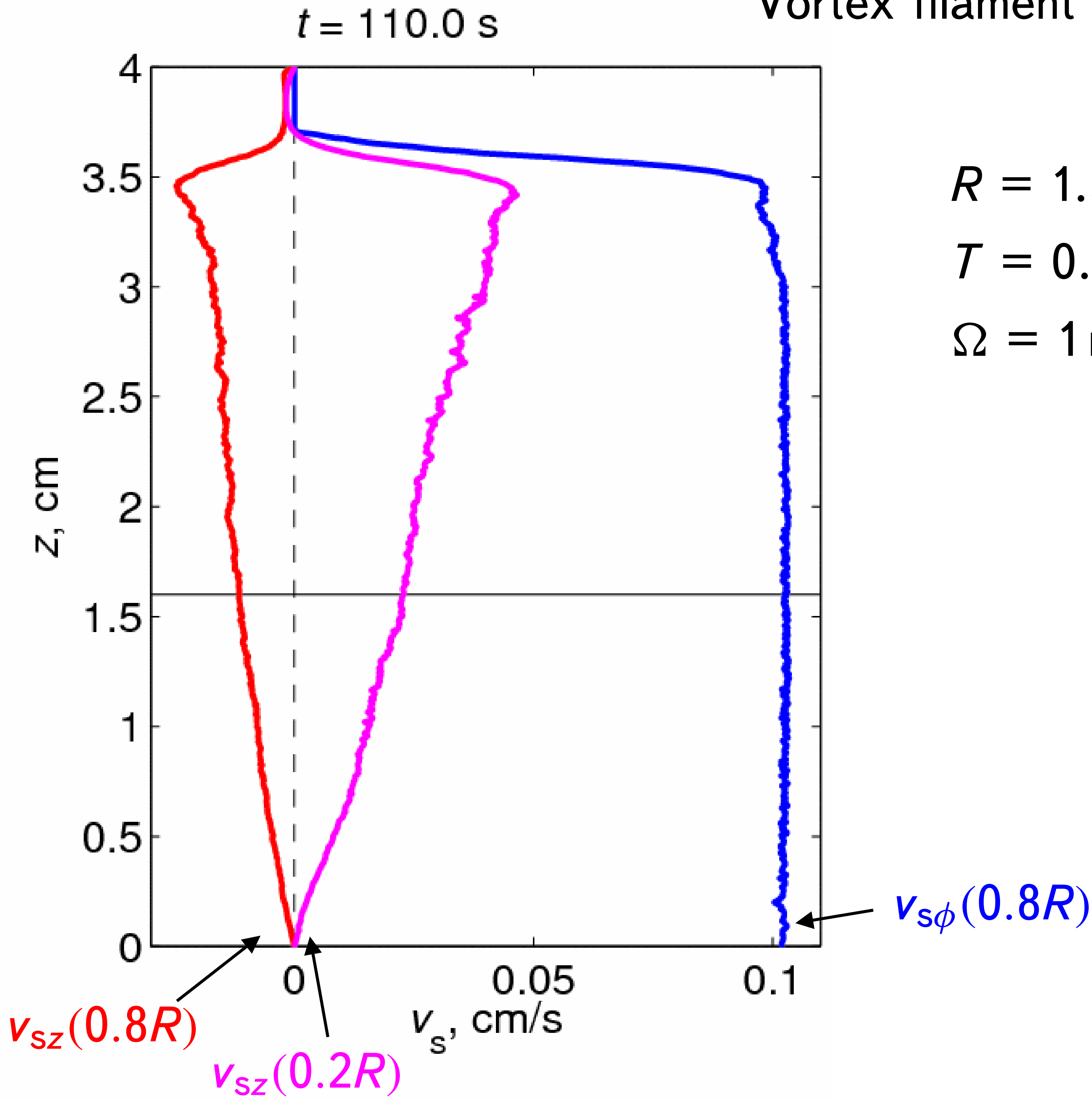

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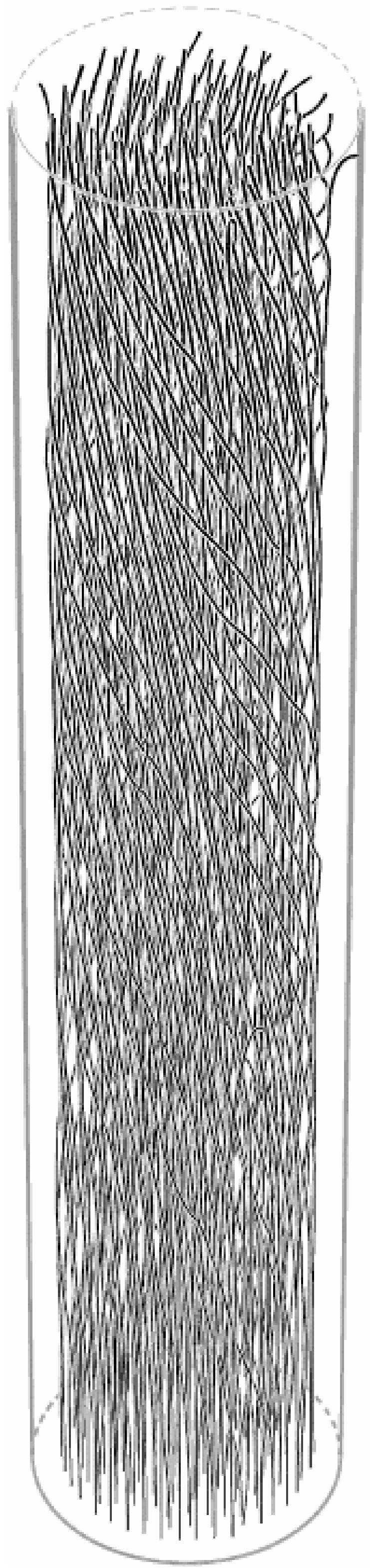

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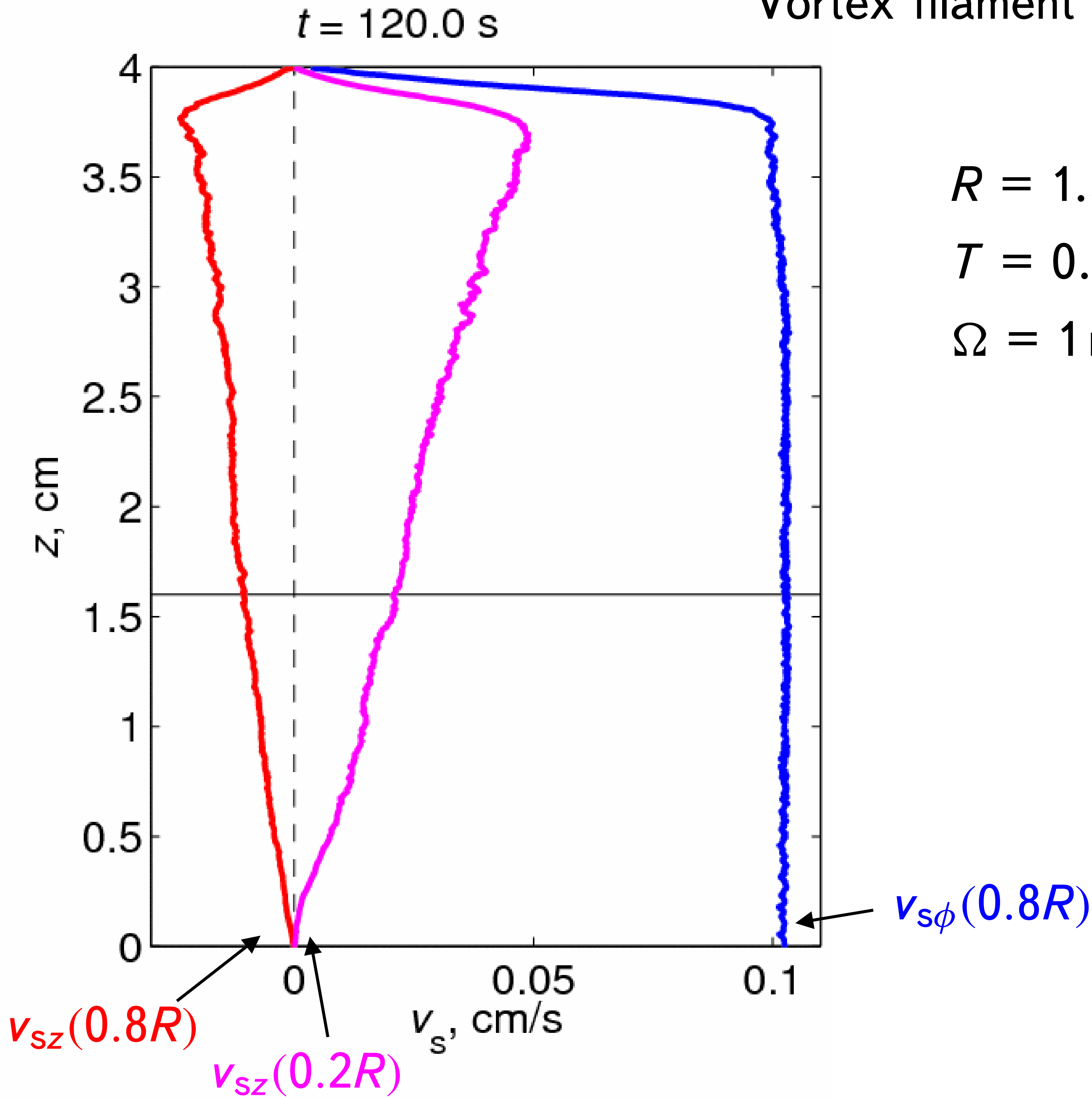

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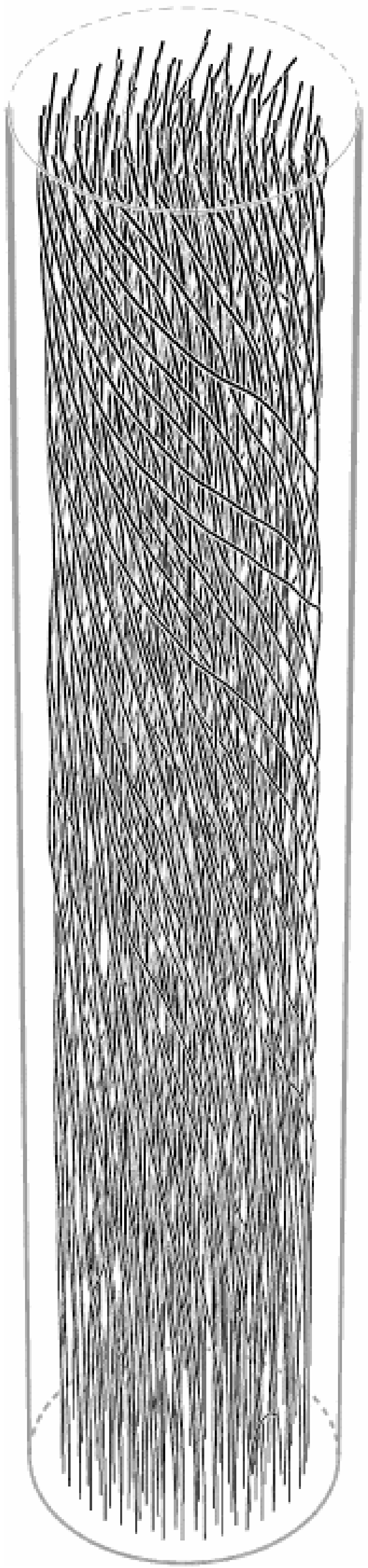

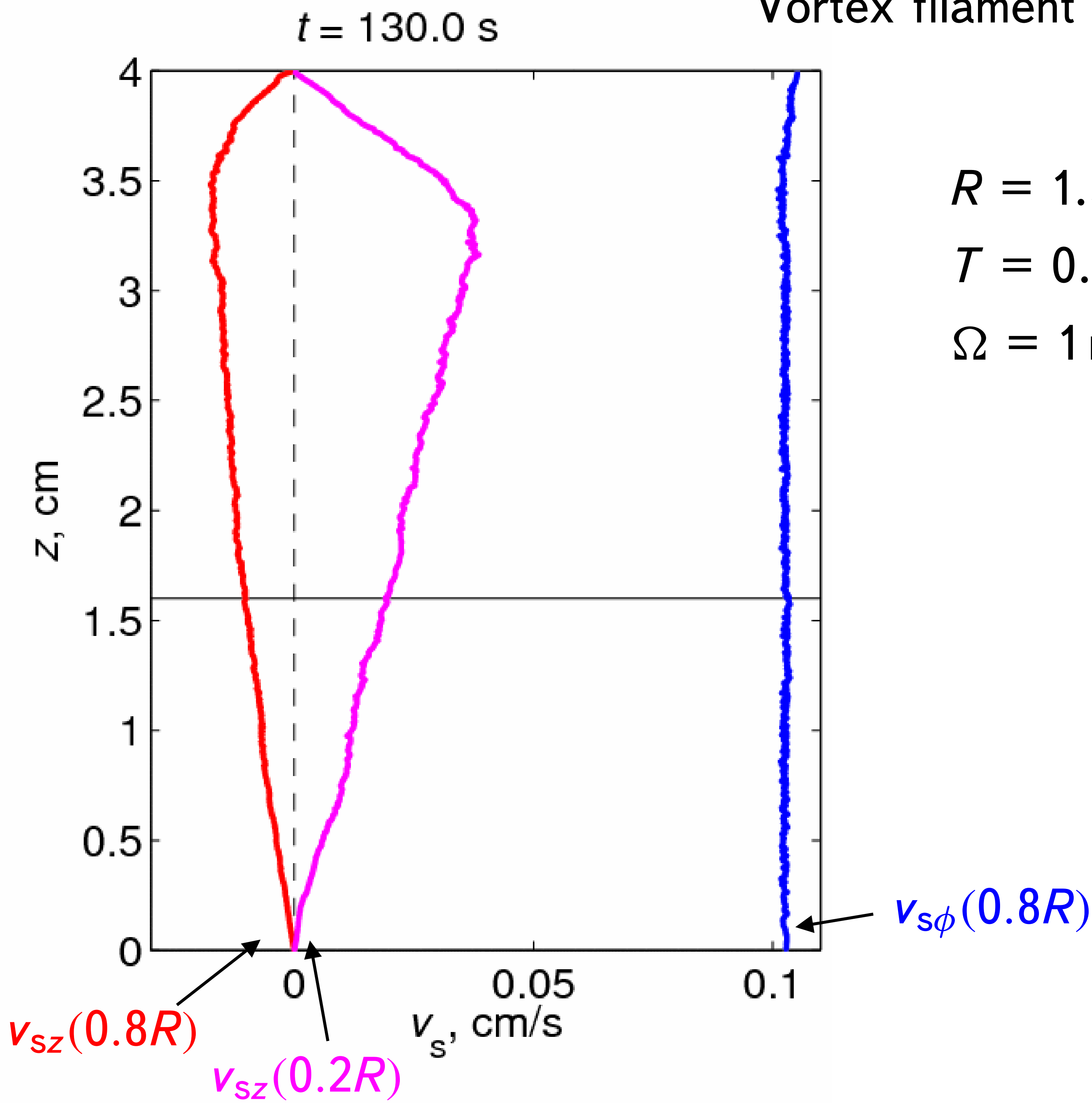

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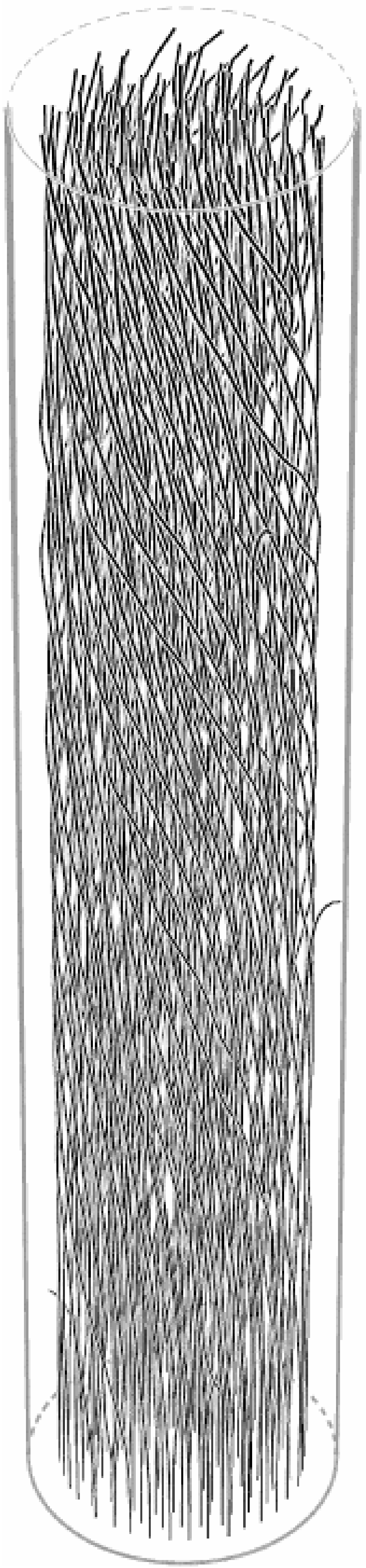

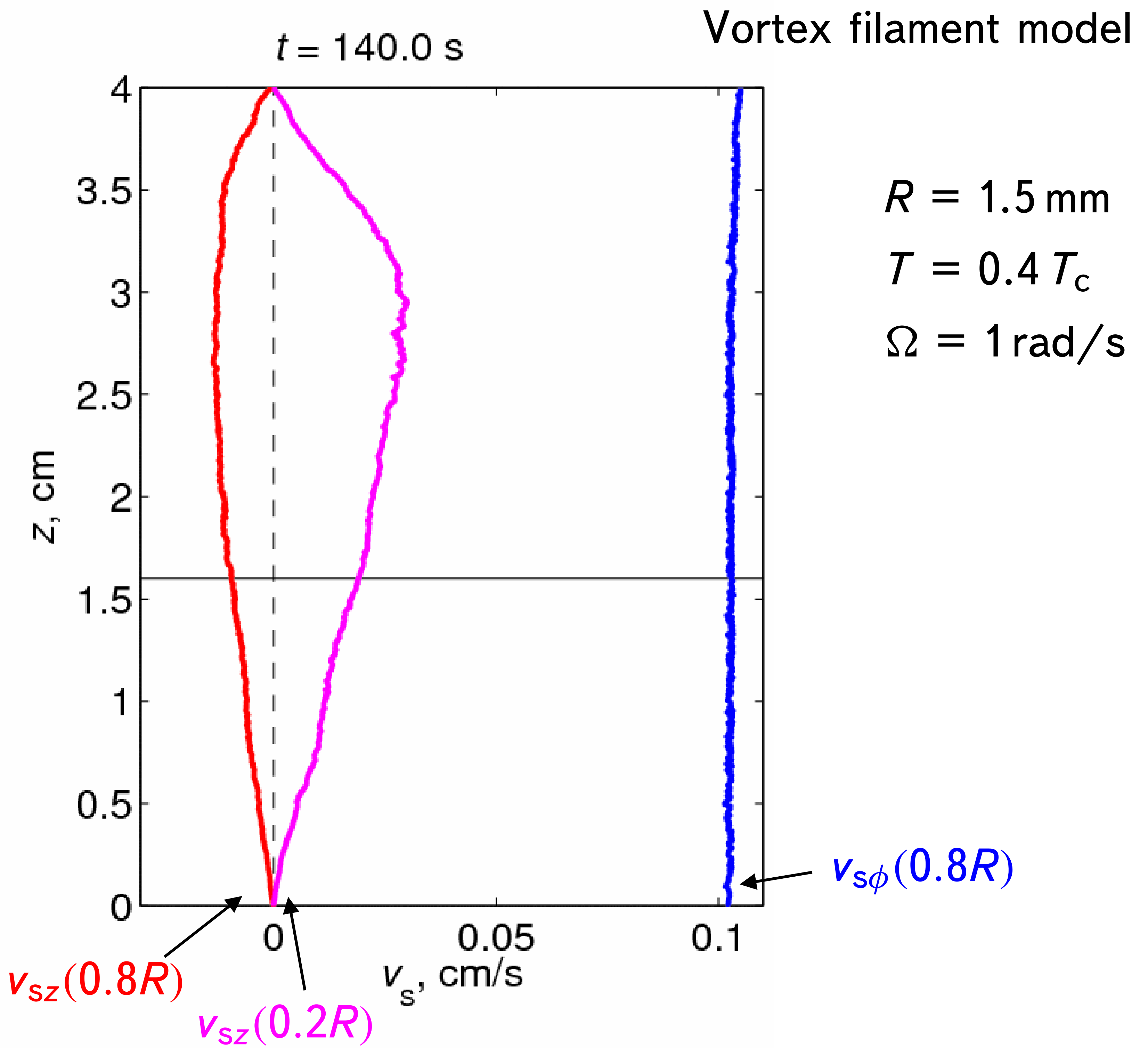

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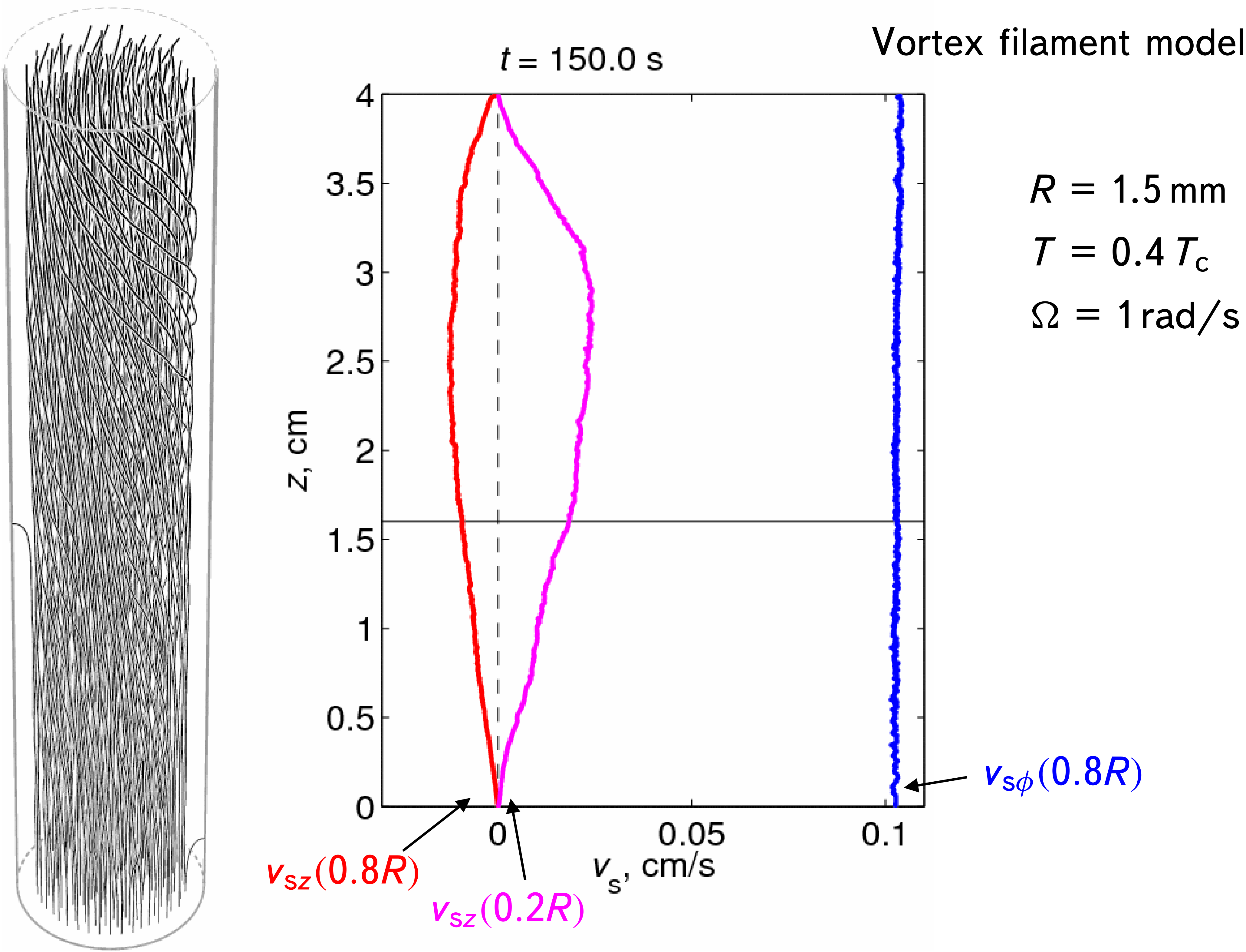

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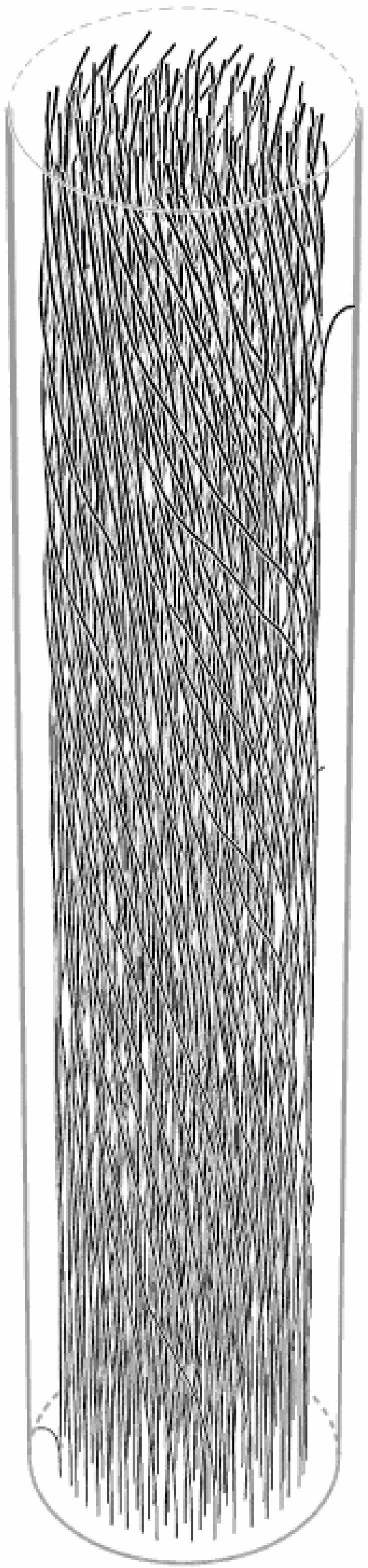

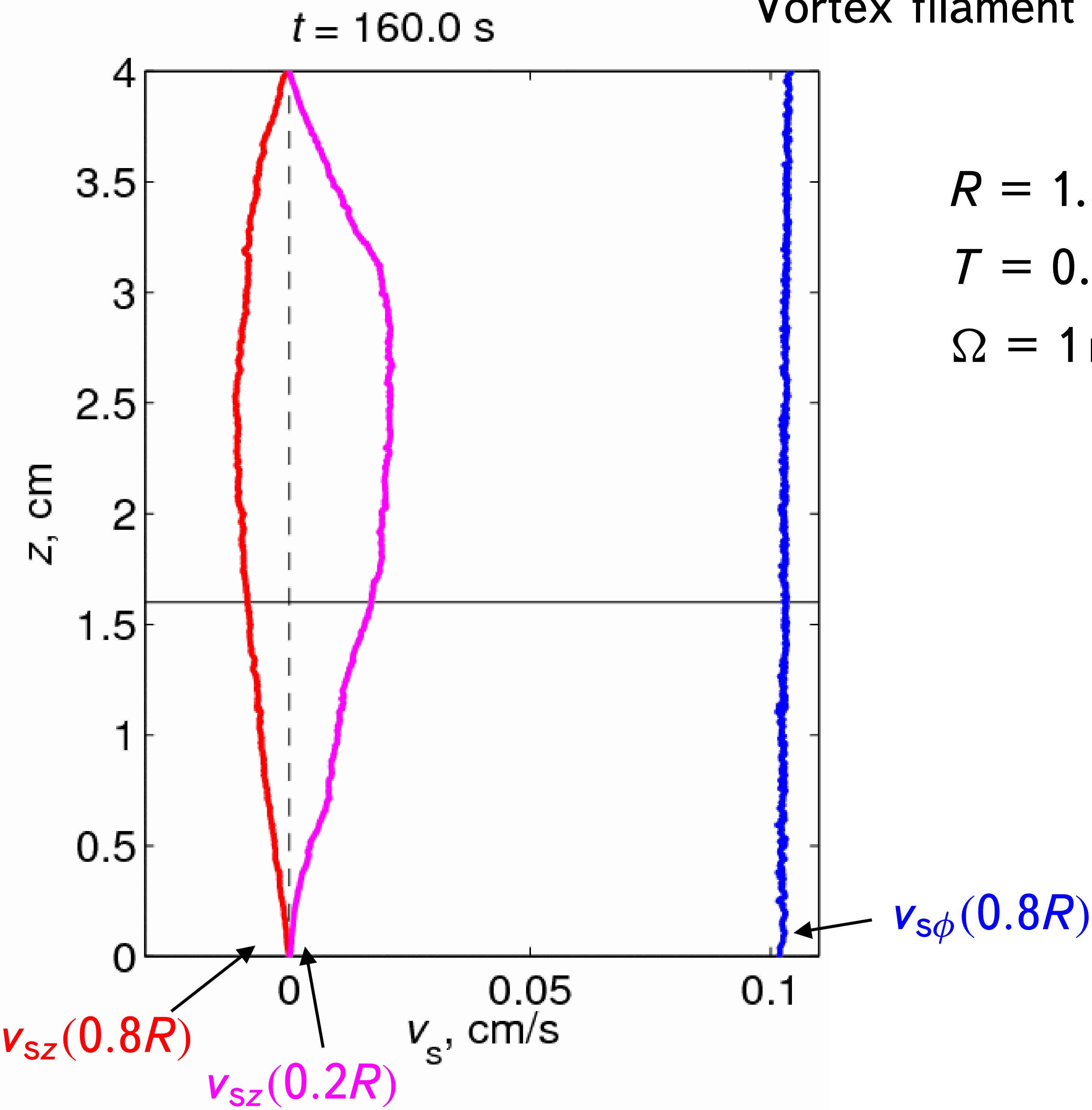

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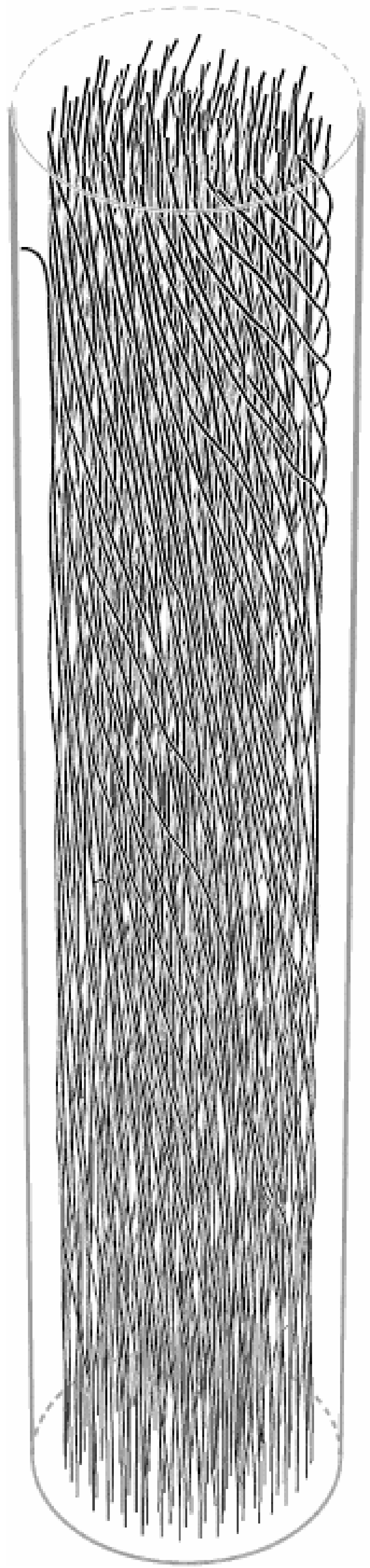

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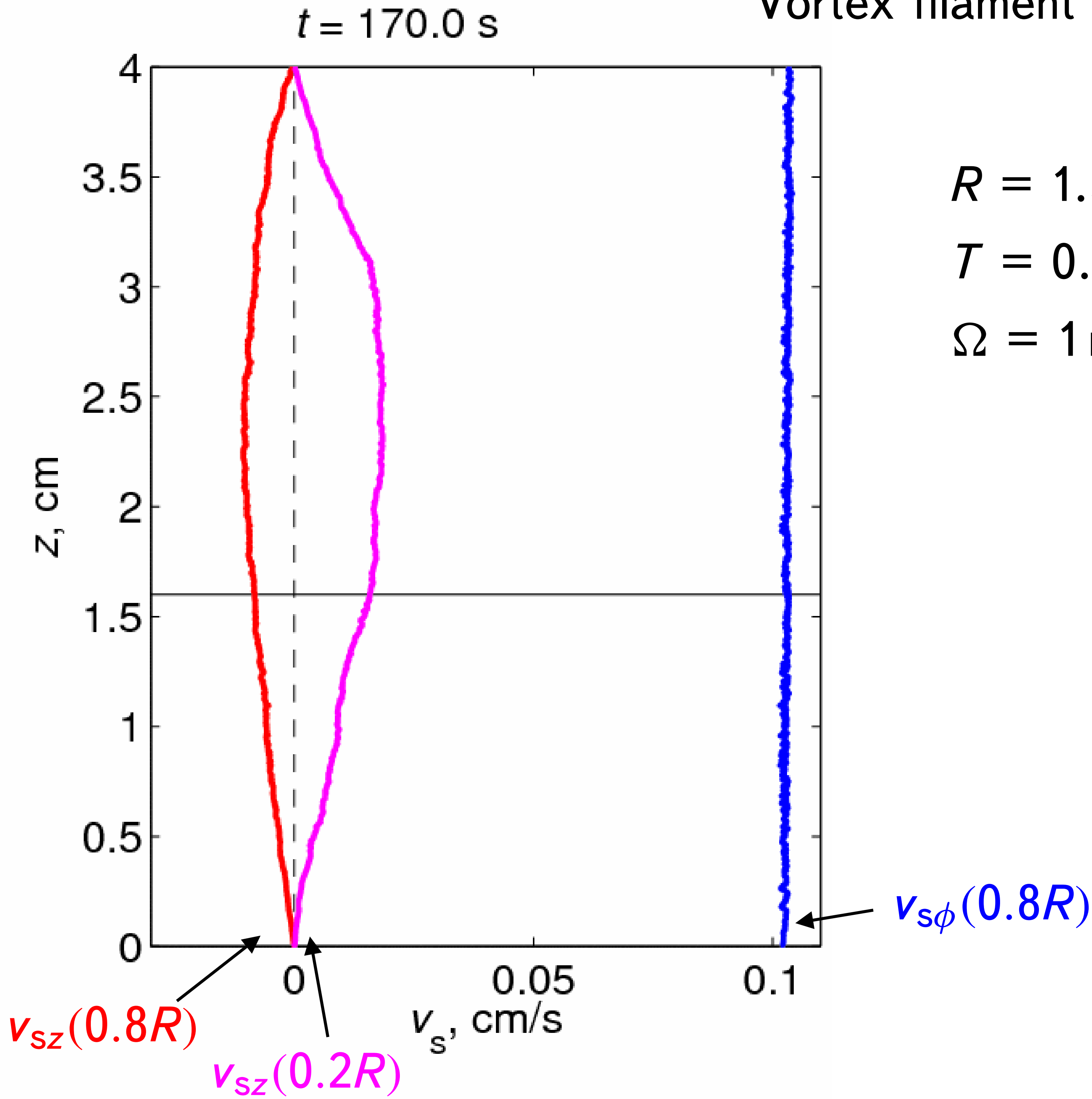

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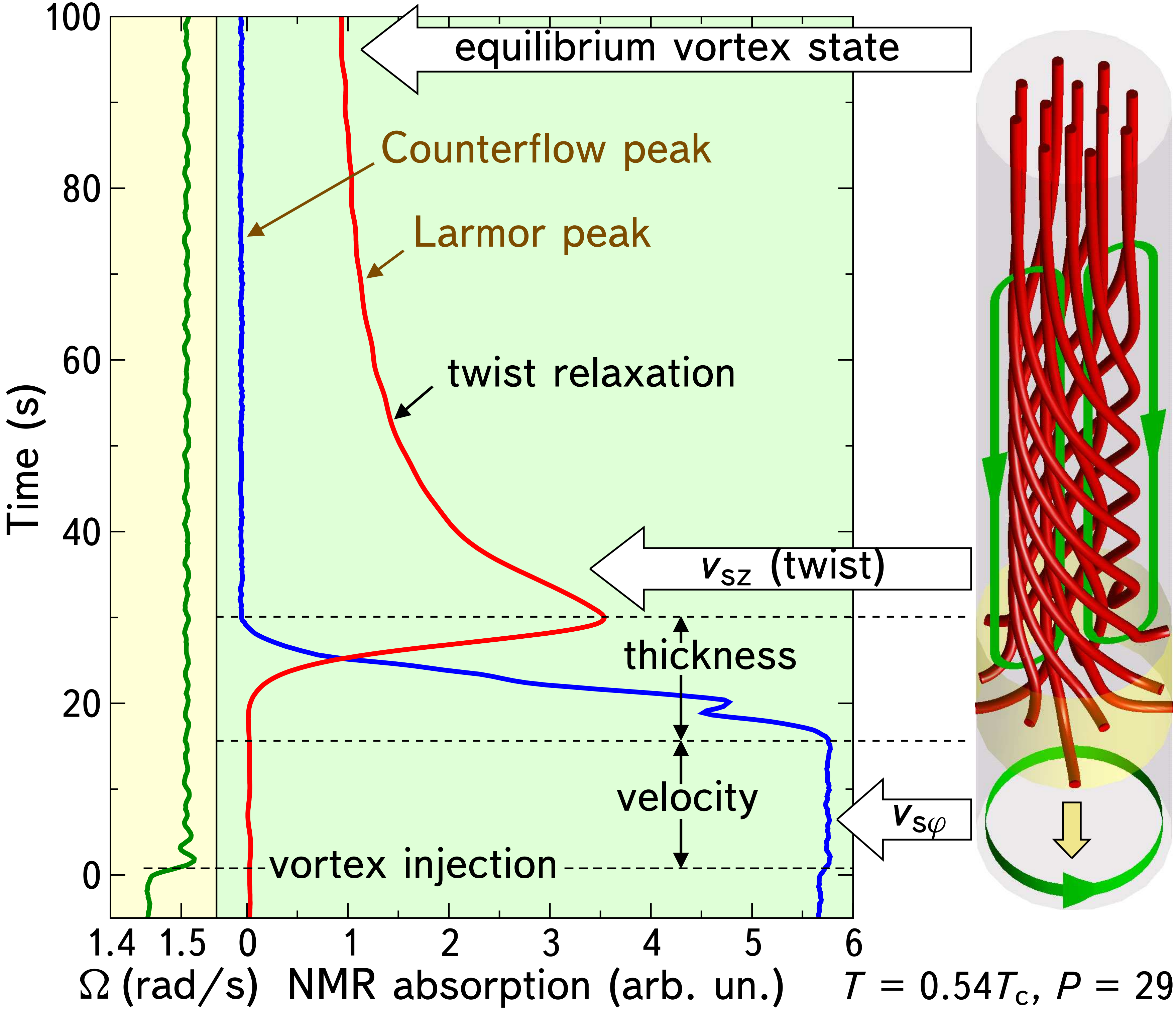
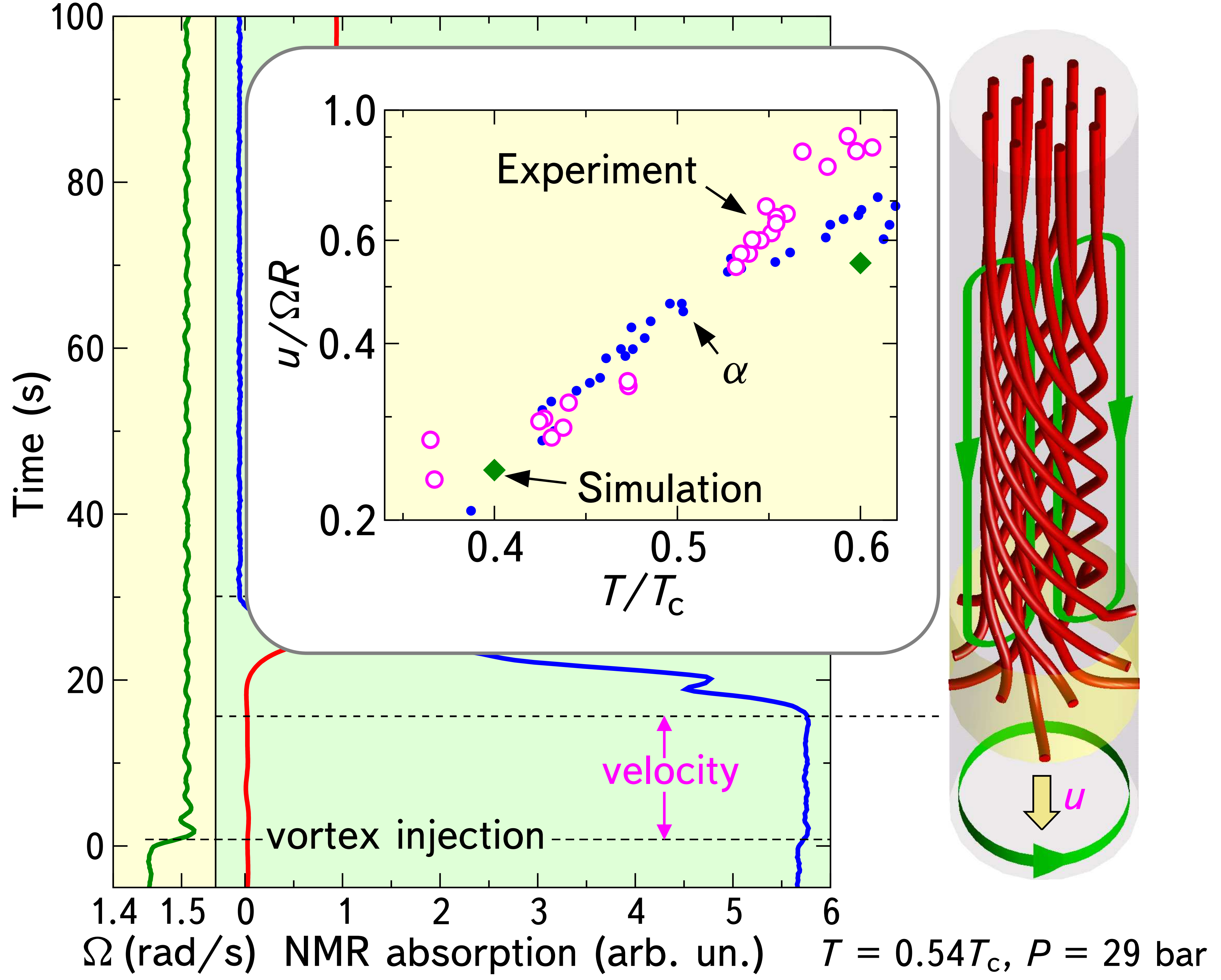
VORTEX FRONT IN NMR MEASUREMENTS



$T = 0.54T_c$, $P = 29$ bar

VORTEX FRONT IN NMR MEASUREMENTS



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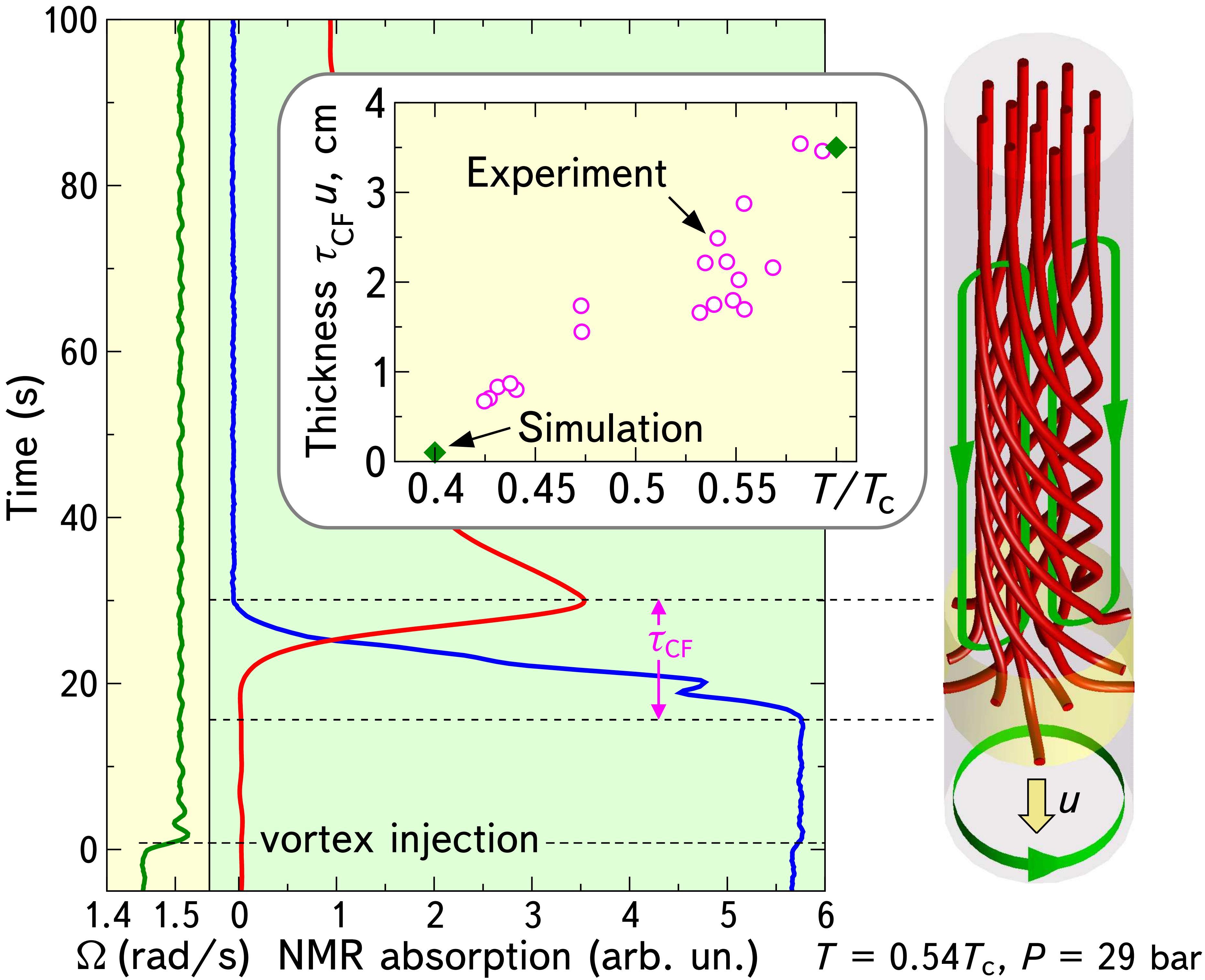

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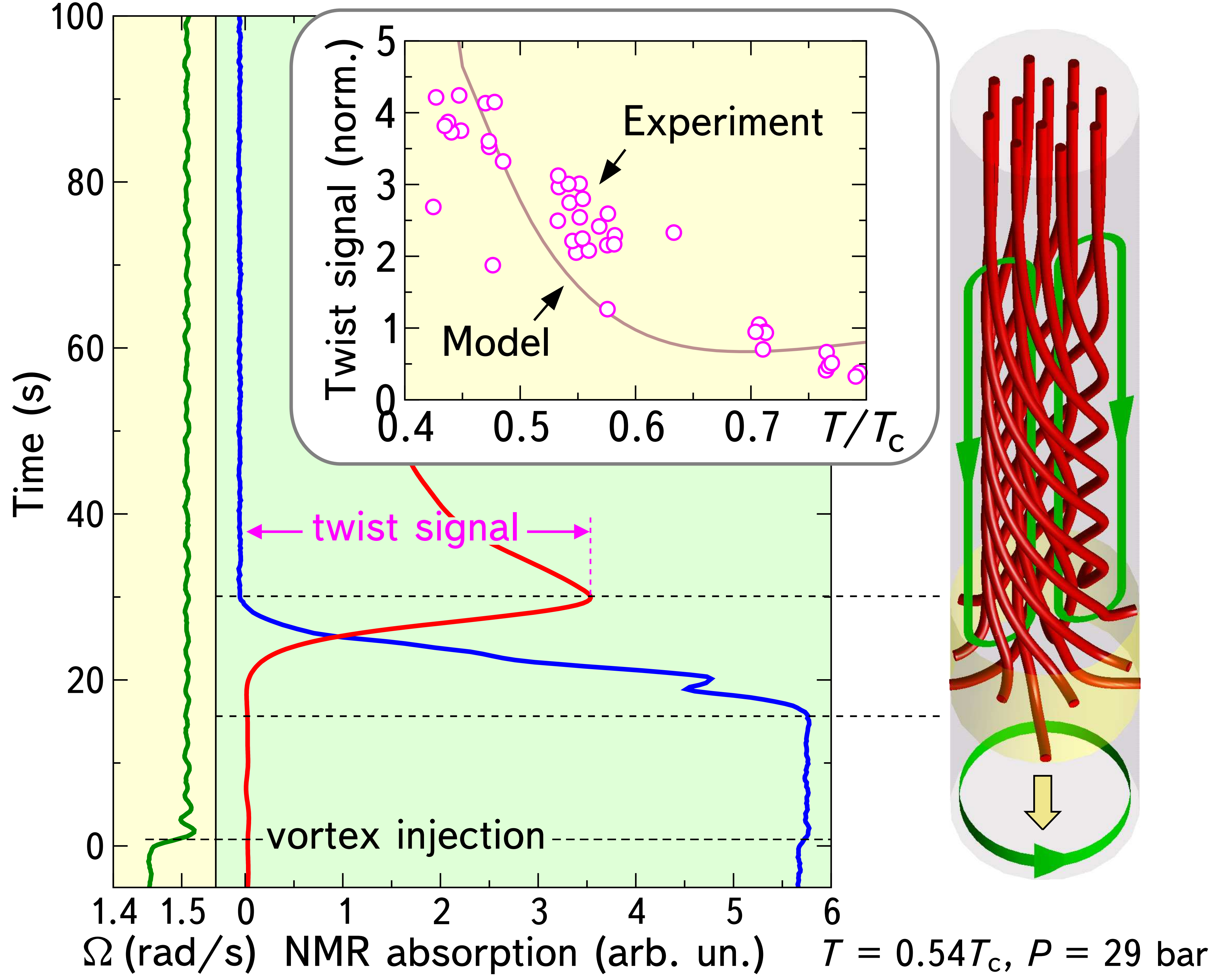

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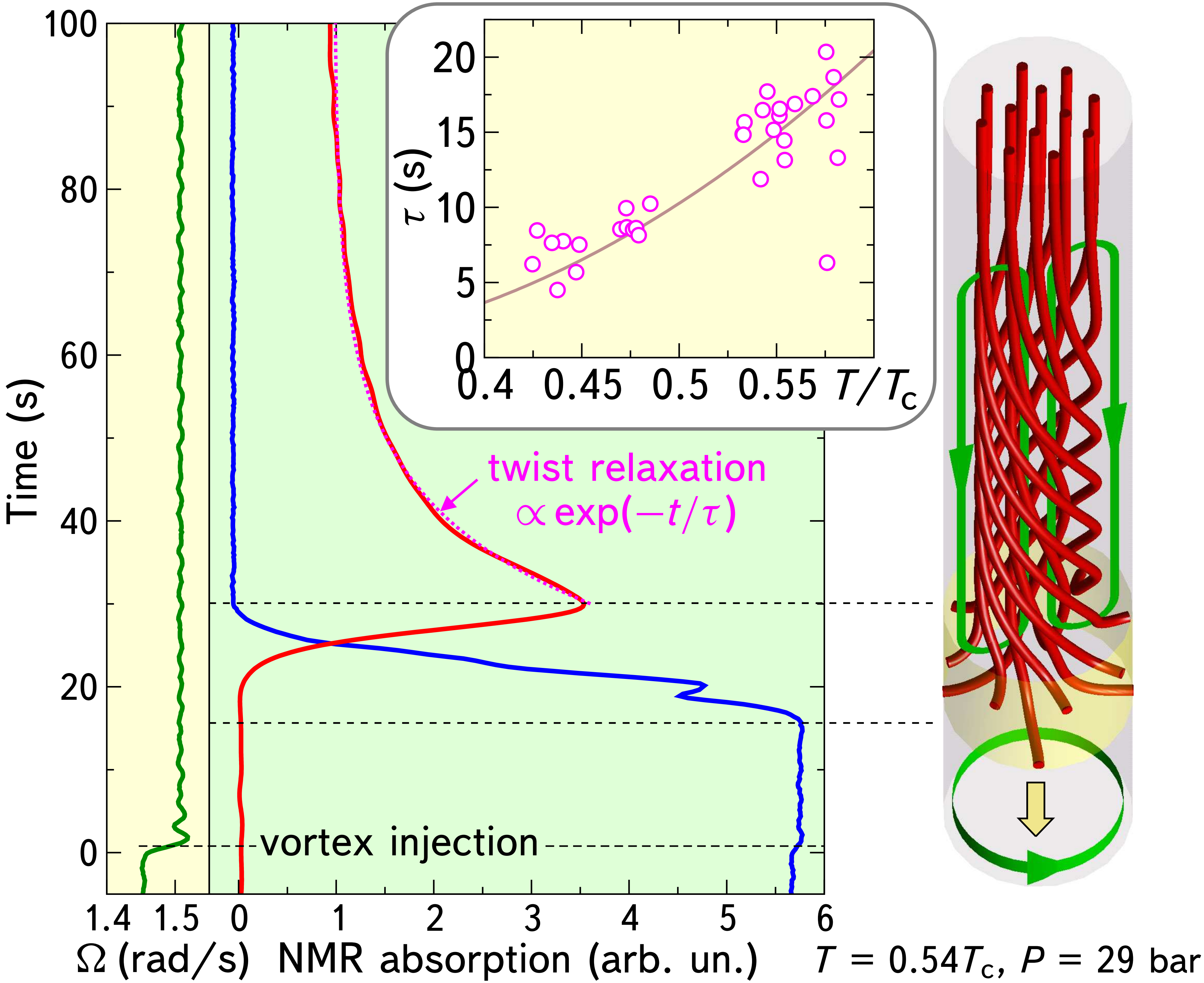
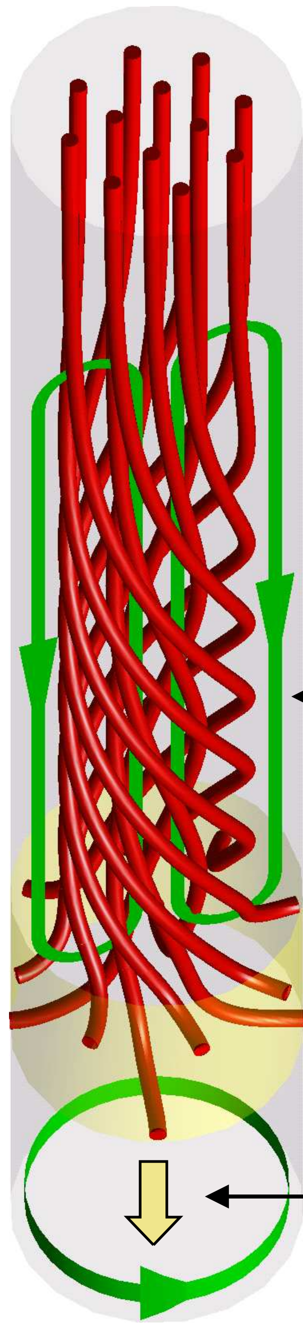


PROPERTIES OF THE VORTEX FRONT



Relaxation to equilibrium vortex state becomes *faster* with decreasing T

Axial flow and twist become stronger with decreasing T

Thickness decreases with decreasing T

$u \approx \alpha \Omega R$, like for a single vortex (*slower* with decreasing T)

COARSE-GRAINED EQUATIONS OF VORTEX DYNAMICS

Equations (HVBK) for the coarse-grained superfluid velocity $\boldsymbol{v}$ if vortex line tension is neglected:

$$\frac{\partial \boldsymbol{v}}{\partial t} + \nabla \tilde{\mu} = \boldsymbol{v} \times \boldsymbol{\omega} + \alpha' \boldsymbol{\omega} \times (\boldsymbol{v} - \boldsymbol{v}_{\mathrm{n}}) + \alpha \hat{\boldsymbol{\omega}} \times [\boldsymbol{\omega} \times (\boldsymbol{v} - \boldsymbol{v}_{\mathrm{n}})],$$

$$\mathrm{div}\, \boldsymbol{v} = 0, \qquad \boldsymbol{\omega} = \mathrm{rot}\, \boldsymbol{v}, \qquad \boldsymbol{v}_{\mathrm{n}} = \boldsymbol{\Omega} \times \boldsymbol{r}.$$

Boundary condition: $\boldsymbol{v} \cdot \boldsymbol{n} = 0$ at the wall ($\boldsymbol{n}$ is a normal to the wall).

Ω dependence is trivial:

Solution $\boldsymbol{v}(\boldsymbol{r}, t)$ for $\boldsymbol{\Omega} \Rightarrow$ solution $\lambda \boldsymbol{v}(\boldsymbol{r}, \lambda t)$ for $\lambda \boldsymbol{\Omega}$.

COARSE-GRAINED EQUATIONS OF VORTEX DYNAMICS

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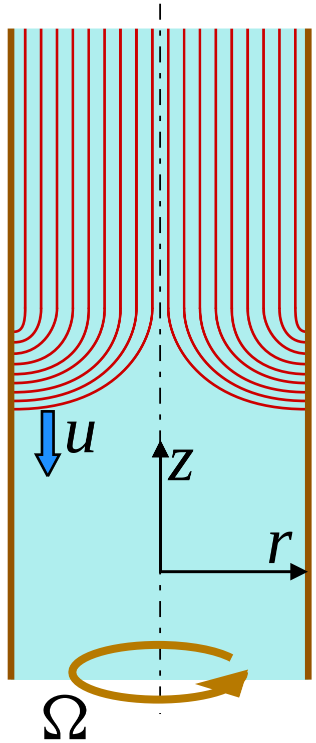
Two cases can be solved analytically:

- Propagating vortex front without the twist ($\alpha \to \infty$).
- Twisted vortex cluster without the front (z-uniform).

ANALYTIC SOLUTION FOR THE FRONT



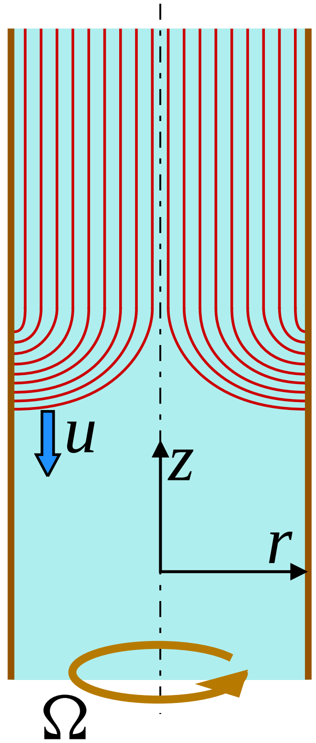
If $T \to T_c$ then $\alpha' \to 1$, $\alpha \to \infty$ and $\omega_\phi \to 0$.

Thus $v_r = v_z = 0$ and the only unknown function is $v_\phi = \Omega r \; f(r, z + ut)$. The equation becomes

$$-\frac{\partial f}{\partial r} + \sqrt{\left[\frac{u}{\alpha \Omega r (1 - f)}\right]^2 - 1}\,\frac{\partial f}{\partial z} = \frac{2f}{r} \,,$$

$$f(r, -\infty) = 0, \quad f(r, +\infty) = 1 \,.$$

ANALYTIC SOLUTION FOR THE FRONT



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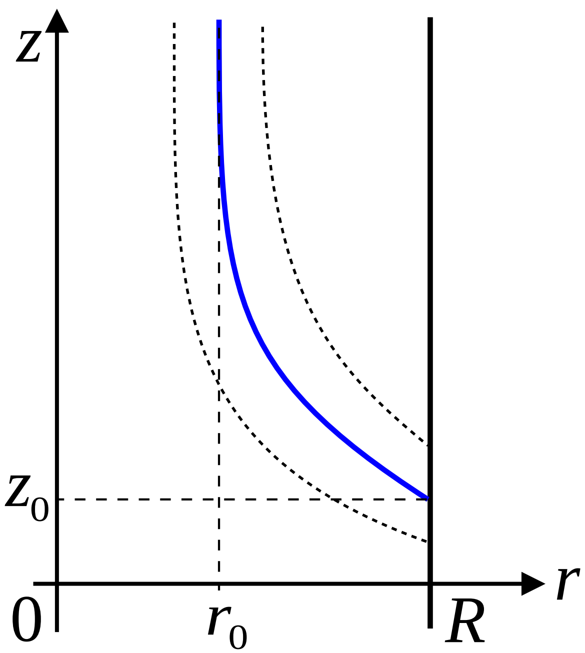
Solution (in parametric form):



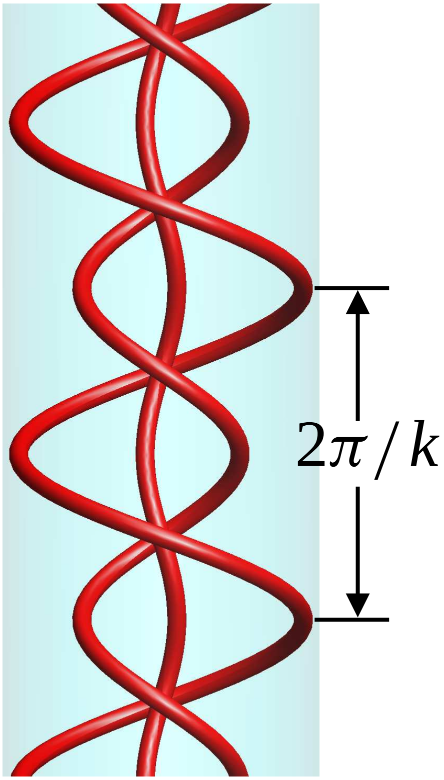
$$f = (r_0/r)^2,$$

$$z = z_0(r_0) + \int_r^R \left(\left[\frac{u\, r'}{\alpha \Omega (r'^2 - r_0^2)} \right]^2 - 1 \right)^{1/2} dr',$$

$$0 \leqslant r_0 \leqslant r \leqslant R, \quad u \geqslant \alpha \Omega R.$$

$\hat{\omega}(R, z) \perp$ wall when $z \to -\infty \quad \Rightarrow \quad u = \alpha \Omega R$

UNIFORM HELICALLY TWISTED CLUSTER



Look for the time-invariant, z-uniform solution for superfluid velocity $\boldsymbol{v}$ where all vortices are helically twisted with the same periodicity:

$$\boldsymbol{v} = v_\phi(r)\,\hat{\boldsymbol{\phi}} + v_z(r)\,\hat{\mathbf{z}}, \qquad \omega_\phi/\omega_z = kr\ .$$

Result:

$$v_\phi(r) = \frac{\Omega r + (kr)v_0}{1+(kr)^2}, \quad v_z(r) = \frac{v_0 - (kr)\Omega r}{1+(kr)^2},$$

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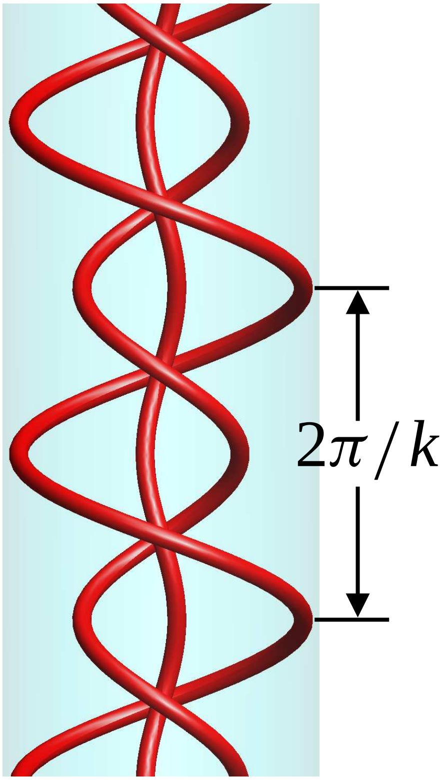

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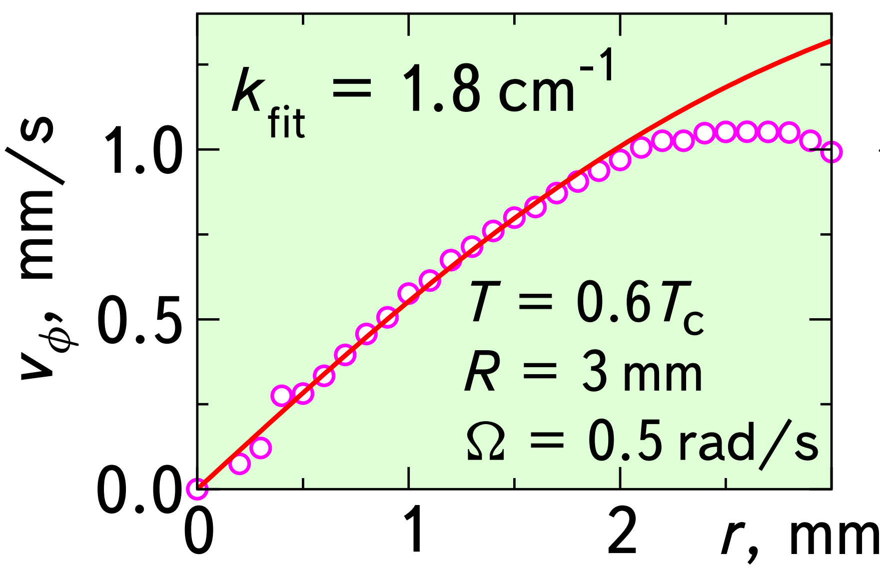
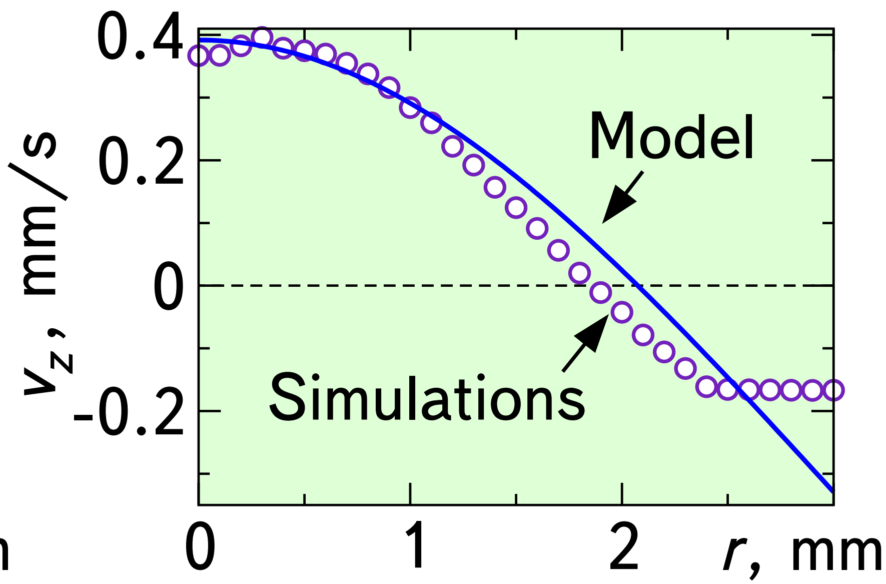


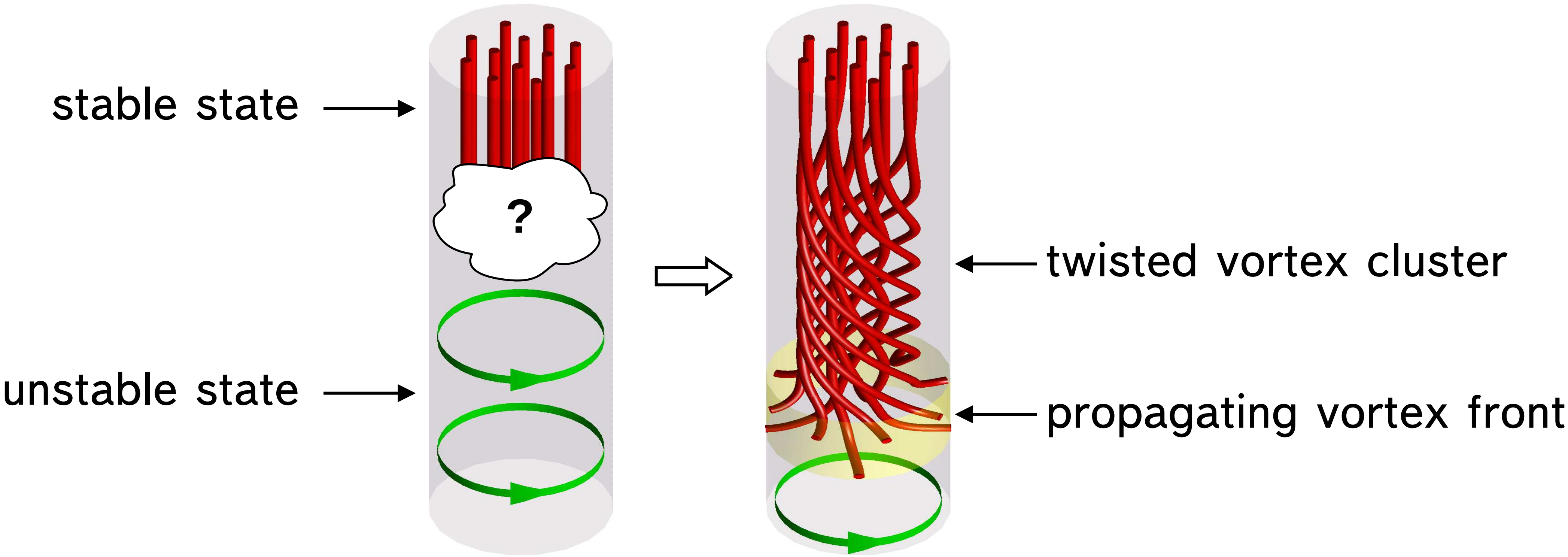


Expected:

$$k \sim \frac{1-\alpha'}{\alpha R}$$

$$\sim 2\ {\rm cm}^{-1}$$

CONCLUSIONS



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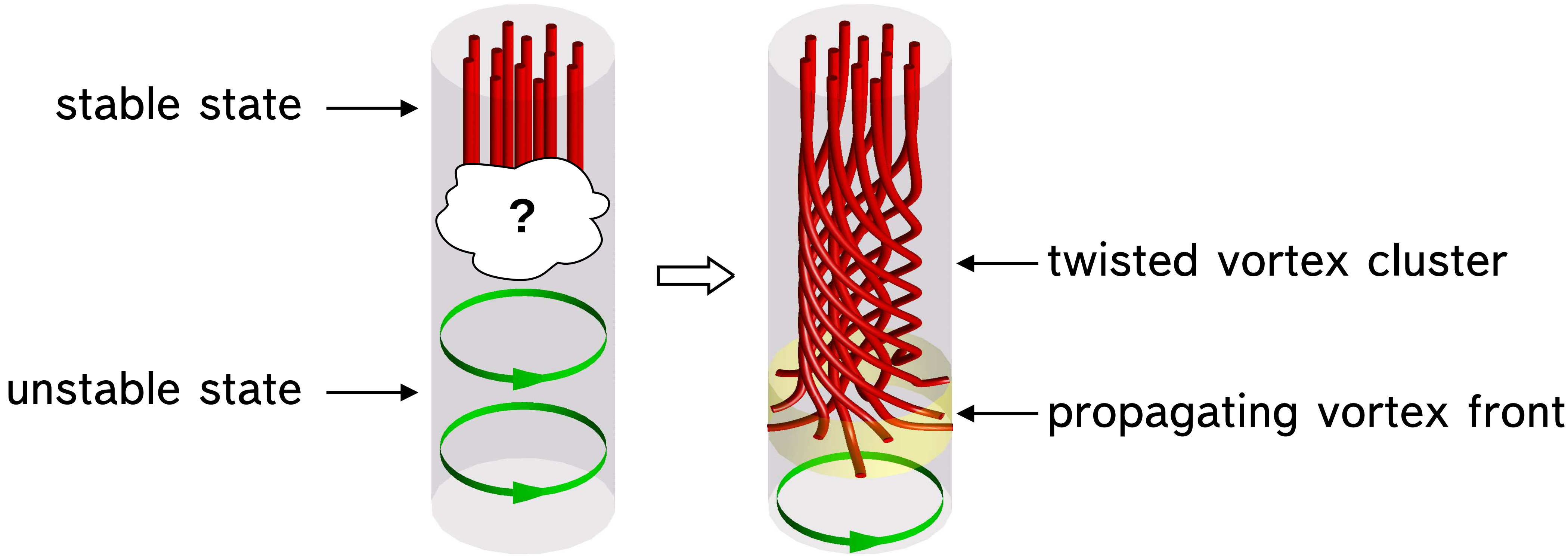

Open questions:

- Relaxation of the twist.
- Role of the instabilities and turbulence.