

Kinetic equation for point vortices in a shear flow

S.V. Nazarenko and V.E. Zakharov

L.D. Landau Institute for Theoretical Physics, Academy of Sciences of the USSR, GSP-1, Kosygin str. 2, 117940 Moscow, USSR

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The kinetic equation for a set of Stewart point vortices moving on the background of a shear flow with uniform vorticity is derived. Besides the total number of vortices with certain amplitude values, this equation provides conservation of the total vorticity of vortex gas on each shear flow stream line. Such conservation laws prevent, in the general case, relaxation of the system to the Maxwell equilibrium distribution of vortices. In fact, the system is shown to tend to an equilibrium state, in which vortex gas vorticity on each stream line of shear flow is formed by vortices with equal amplitude signs, only the most intense vortices being concentrated in more excited regions, while less intensive vortices are concentrated at the periphery of perturbed domains. Applicability of the kinetic equation obtained for the description of Helmholtz vortices sets is discussed.

1. Introduction

The concept of point vortices has been used for modeling nonlinear dynamics of 2D incompressible fluid motion since Helmholtz. In the last two decades the problem of description of point vortices' motion has acquired a new practical significance. It appears that potential dynamics of charged particles in a strongly magnetized plasma can be described by equations, coinciding with equations for a set of point vortices in an incompressible fluid [1]. Point vortices are also used in the theory of superfluidity in He⁴ films (see e.g. review [2] and references therein).

A lot of processes in meteorology, oceanology [3, 4] and plasma physics [5] are governed by an equation more general than one describing 2D incompressible fluid dynamics. This equation for the stream function $\psi(x, y, t)$ in dimensionless form looks like

$$\frac{d}{dt}(\nabla^2\psi - \psi) = 0, \quad (1.1)$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla), \quad \mathbf{v} = \left(\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x} \right).$$

For small scale motions, $\nabla^2\psi \gg \psi$, eq. (1.1) turns into the equation of 2D dynamics of an incompressible fluid. For modeling of nonlinear dynamics governed by eq. (1.1) point vortices have been introduced by Stewart [3]. In spite of the difference between the Stewart vortex and the vortex in an incompressible fluid (hereafter called Helmholtz vortex) in regions remote from a center, $r \gg 1$, there is certain similarity between dynamics of Stewart and Helmholtz vortex sets. For example, it is easy to show that a system of two Stewart vortices uniformly rotates around the "mass center"; it is similar to the well known behavior of two Helmholtz vortices [6]. For both types of vortices, equations of three-vortex motion turn out to be integrable [7-9]. If the number of vortices $n > 3$, the equations of motion are

not integrable in the general case and the motion of such systems possesses chaotic features [10].

Considerable attention in literature has been paid to the study of ensembles consisting of a large number of point vortices, $n \gg 1$, [1, 2, 7, 11–14]. Such ensembles may be considered a finite-dimensional approximation of turbulent motions. The main progress in this area is due to the construction of thermodynamic theory [2, 12, 13] and direct computer modeling [2, 14] of Helmholtz vortex ensembles. Thermodynamic consideration of point vortices has been introduced by Onsager [11], who also obtained the most important result of such a theory (see also ref. [13]): a point vortex system has been proved to possess negative temperature states, in which vortices with different vorticity signs tend to occupy separate domains, and, hence, to create large scale mean field circulation. In the case of positive temperatures equilibrium states are spatially uniform. The result mentioned above serves as a good explanation [13] of “ergodic boundary” (separating initial conditions starting from which large scale circulation does appear from those which lead to uniform equilibrium) found by Deem and Zabusky [15]. Direct numerical simulations of evolution of point vortex sets are in rather good agreement with conclusions of thermodynamic theory (see e.g. ref. [14]).

It is pointed out by Aref [11] that the description of relaxation of a system may be more important for a theory than the description of its thermodynamical equilibrium states. In addition, in natural turbulence sources and dissipation are always present, while stationary states of a fluid medium are usually remote from thermodynamic equilibrium. In spite of this, few papers [1, 7] are devoted to kinetic theory of point vortices ensembles. Novikov [7] has shown that investigation of a system of n identical vortices can be carried out, based on analysis of Hopf equation for a characteristic functional generating the n -particle distribution function. Vahala and Montgomery [1] have shown that neglecting pair correlations reduces the equation for one-particle distribution function to the one for 2D incompressible fluid. No-

tice that, taking into account the possibility of approximation of fluid vorticity field by a large set of point vortices, this result looks natural.

Intrinsic difficulties arise if one tries to take into account pair correlations of vortex coordinates [1]: for a set of Helmholtz vortices, as well as of Stewart vortices, there exists no proper small parameter which would enable us to express pair correlation function in term of one-particle distribution function and so obtain kinetic equation for vortices of Boltzmann type.

Constructing kinetic theory seems more simple for those systems, in which point vortices move on the background of regular flows. Such flows can represent global oceanic flows, zonal flows in atmospheres of major rotating planets of the solar system (Jupiter, Uranus, Saturn, Neptune), and – in the case of magnetized plasma – azimuthal drift motions in magnetic traps driven by the radial component of an electric field.

The pattern of point vortex motion on the background of a regular flow is most simple when the flow is characterized by uniform vorticity. In such cases the flow is steady, and its influence on the dynamics of vortices can be considered as the appearance of “kinetic energy” of vortices (see section 2). The problem of interaction of two Helmholtz vortices with uniform vorticity was solved by Bogomolov [16], and an analogous problem for two Stewart vortices was considered by Gryanik [17]. It will be demonstrated in section 3 that pair interaction of Stewart vortices, in contrast to Helmholtz vortices, looks like collision which transfers vortices from one state of unperturbed motion to another one (i.e. from one stream line of regular flow to another one). Accordingly, the kinetic equation for the ensemble of Stewart vortices on the background of constant vorticity flow can be obtained by means of a standard procedure of kinetic theory of gases, which operates with cross-section of pair collisions.

The derivation and analysis of such an equation are the main purposes of the present paper (see sections 4–6). The results obtained are discussed in section 7. In addition, in section 7 the

arguments are presented in favor of applicability of the kinetic equation obtained in the present paper, also for the description of Helmholtz vortices kinetics.

2. Equations for motion of Stewart vortices in a flow of constant vorticity

Let us consider dynamics of n Stewart vortices on the background of regular flow ψ_r , which is characterized by the spatially homogeneous value of potential vorticity:

$$\nabla^2 \psi_r - \psi_r = C = \text{const.} \quad (2.1)$$

The equations for vortices' coordinates $x_i(t), y_i(t)$ ($i = 1, 2, \dots, n$) in such a system have the Hamiltonian form:

$$\Gamma_i \dot{x}_i = \frac{\partial H}{\partial y_i}, \quad \Gamma_i \dot{y}_i = -\frac{\partial H}{\partial x_i}, \quad (2.2)$$

where Γ_i is the intensity of vortex with number i , and Hamiltonian H looks as follows:

$$H = \sum_{i=1}^n \Gamma_i \psi_r(r_i) + \sum_{i<j}^n \Gamma_i \Gamma_j K_0(r_i - r_j), \quad (2.3)$$

where $K_0(z)$ is the modified Bessel function of zero order. The first term in expression (2.3) corresponds to transfer of vortices by velocity field of regular flow; it can be interpreted as "kinetic energy" of vortices. The second term in (2.3) corresponds to the sum of pair interactions of vortices; these interactions are small on large distances between vortices because of exponential decrease of $K_0(z)$ for large values of the argument z .

The general solution of eq. (2.1) for the case of shear flows has the following form:

$$\psi_r = A \text{ch}(y) + B \text{sh}(y) + C,$$

where A, B, C are constants. However, we shall

consider for simplicity flows with stream function

$$\psi_r = A \text{ch}(y) \quad (2.4)$$

only, since the value of constant C does not influence the flow's velocity field, while analysis of the case $B \neq 0$ can be carried out similarly to the consideration below, and would not lead to any new effects. In addition, let us assume $A > 0$; the case $A < 0$ comes to the case $A > 0$ by the transformation $t \rightarrow -t, \Gamma_i \rightarrow -\Gamma_i$ ($i = 1, n$).

In the case of regular flow depending on coordinate y only, the Hamiltonian (2.3) is invariant with respect to translations along the x -axis, and hence the x -momentum of the system of vortices is conserved:

$$\sum_{i=1}^n \Gamma_i y_i = \text{const.} \quad (2.5)$$

3. Pair collisions of vortices

The interaction of Stewart vortices on the background of constant vorticity flow [17] bears some resemblance to the interaction of two Helmholtz vortices in a constant shear flow [16]. We shall demonstrate below, however, that there are some essential differences between these two cases, which allow us to consider directly kinetics of a rare enough gas of Stewart vortices as a sequence of pair collisions (contrary to a gas of Helmholtz vortices).

Using the conservation law (2.5) it is not hard to reduce eqs. (2.2) in the case $n = 2$ to a set of 1D Hamilton equations. Let us consider for simplicity the case of strong enough flow:

$$A \text{ch}(y_{1,2}) \gg \Gamma_i. \quad (3.1)$$

Under this condition, as will be demonstrated below, only collisions of vortices having close values of coordinates y_1 and y_2 should be considered:

$$|y_1 - y_2| \ll 1. \quad (3.2)$$

So, the motion of a two-vortex system on the background of a flow of type (2.4) is described by the following equations:

$$\Gamma_{12} \dot{x}_{12} = \frac{\partial H^{12}}{\partial y_{12}}, \quad \Gamma_{12} \dot{y}_{12} = -\frac{\partial H^{12}}{\partial x_{12}}, \quad (3.3)$$

where

$$r_{12} = (x_{12}, y_{12}) = (x_1 - x_2, y_1 - y_2), \quad (3.4)$$

$$\Gamma_{12} = \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2}, \quad (3.5)$$

and Hamiltonian H^{12} under condition (3.2) has the following form:

$$H^{12} = -\Gamma_{12} A \operatorname{ch}(y) y_{12}^2 / 2 + \Gamma_1 \Gamma_2 K_0(r_{12}), \quad (3.6)$$

where y is the constant corresponding to coordinate of the "mass center" of the vortices:

$$(\Gamma_1 + \Gamma_2) y = \Gamma_1 y_1 + \Gamma_2 y_2.$$

Notice that the expression (3.6) is similar to the expression for relative motion energy of two ordinary particles having velocities y_1, y_2 and masses $-\Gamma_1 A \operatorname{ch}(y)$ and $-\Gamma_2 A \operatorname{ch}(y)$; the first term in (3.6) corresponds to the kinetic energy of such particles, while the second corresponds to the energy of their interaction.

The system dynamics described by Hamiltonian (3.6) is essentially different in the following two cases (cf. refs. [16, 17]):

$$\Gamma_1 + \Gamma_2 > 0, \quad (3.7)$$

$$\Gamma_1 + \Gamma_2 < 0. \quad (3.8)$$

(We shall not consider the case $\Gamma_1 = -\Gamma_2$ because the fraction of such pairs in general situation is negligible.) The phase plots for eqs. (3.3) in cases (3.7), (3.8) are shown in figs. 1, 2, respectively. In both cases Hamiltonian H^{12} tends to a function independent of x when $x \rightarrow \pm\infty$, and phase trajectories tend to asymptotes parallel to the x_{12} -axis. (Notice that the assertion of paper [17], that in the case (3.7) all phase trajectories

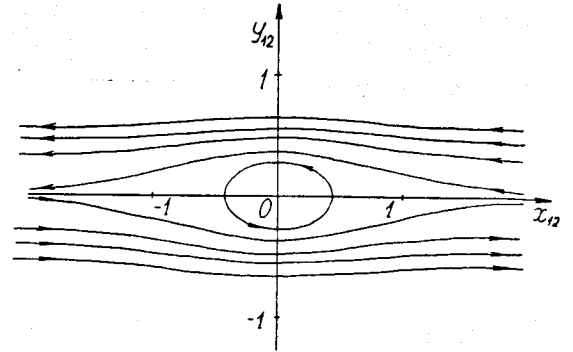


Fig. 1. The phase curves for two vortex system in the case $\Gamma_1 + \Gamma_2 > 0$.

are closed, is false.) Therefore, the motion of two vortices lying far enough from each other along the x -axis may be considered independent and defined by shear flow only, i.e. by "kinetic energy" of vortices. Hence, the process of two-vortex interaction in this situation can be considered as collision; it is the process of pair collisions that governs the kinetics of rarified gas of such vortices.

An essentially different situation occurs when two Helmholtz vortices interact with each other on the background of a shear flow with constant vorticity [16] (this situation corresponds to the limit $r_{1,2} \rightarrow 0$ in the expression (3.6)). In this case under the condition (3.7) all phase trajectories are closed, while under the condition (3.8) they move away logarithmically from the x_{12} -axis. So, the motions of Helmholtz vortices are not inde-

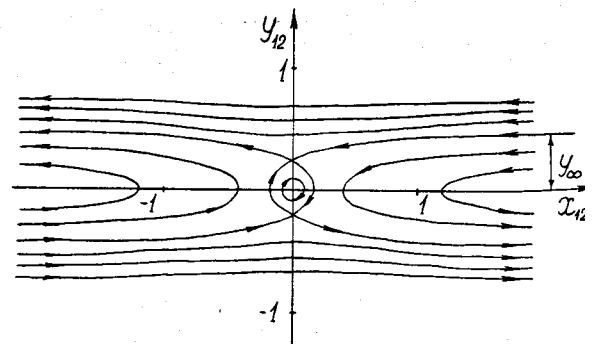


Fig. 2. The phase curves for two vortex system in the case $\Gamma_1 + \Gamma_2 < 0$.

pendent even for large distance between vortices, and the interaction of such vortices has collective character. Nevertheless, in some systems with collective interactions (e.g. in a gas of particles with Coulomb interactions) the particles' collisions can be considered as pair collisions of dynamically shielded particles. The consequence of such an approach for the case of Helmholtz vortices ensemble in a constant shear flow is presented in section 7.

4. Derivation of the kinetic equation

We shall start the derivation of the kinetic equation for point vortices from finding of a cross-section of the pair collisions considered in section 3. According to fig. 1, in the case (3.7), whatever the value of y_{12} before the collision (i.e. the value of y_{12} for $x_{12} \rightarrow +\infty$ or $x_{12} \rightarrow -\infty$), it will restore its initial value after the collision (when $x_{12} \rightarrow -\infty$ or $x_{12} \rightarrow +\infty$, respectively). So, taking into account (2.5), the values of the x -momenta of each of colliding vortices will not be changed as a result of the collision. In other words, vortices' collisions in the case (3.7) do not lead to their transfer along the y -axis. A similar situation occurs in soliton turbulence governed by the KdV-equation [18]: after the collision solitons restore the initial values of their velocities.

The same situation takes place in the case (3.8), if initial values of y_{12} lie out of the region (see fig. 2)

$$|y_{12}| < y_{\infty}, \tag{4.1}$$

where

$$y_{\infty} = \left[\frac{\Gamma_1 + \Gamma_2}{A \operatorname{ch}(y)} \left(1 - \ln \frac{\Gamma_1 + \Gamma_2}{A \operatorname{ch}(y)} \right) \right]^{1/2}. \tag{4.2}$$

If the initial value of y_{12} hits the region (4.1), then, according to fig. 2, after collision its value will be $y'_{12} = -y_{12}$, i.e. according to formula (2.5), vortices will exchange some finite value of

x -momentum:

$$\Gamma_1 y'_1 - \Gamma_1 y_1 = \Gamma_2 y_2 - \Gamma_2 y'_2 = -2\Gamma_{12} y_{12}. \tag{4.3}$$

Under the condition $\Gamma_1 \neq \Gamma_2$ such collisions will provide finite transport of vortices across the regular flow.

Therefore, the cross-section of pair collisions σ is equal to unity if the conditions (3.8), (4.1) are satisfied and is equal to zero if these conditions are violated:

$$\sigma = \theta(-\Gamma_1 - \Gamma_2) \theta(y_{\infty} - |y_{12}|), \tag{4.4}$$

where $\theta(z)$ is the theta-function. If condition (3.1) is satisfied, the quantity y_{∞} (which restricts the region of values y_{12} leading to a transversal transport) is small, $y_{\infty} \ll 1$; the latter proves the statement about the importance of small values y_{12} only (3.2).

Consider now a system with a large number of vortices. Let us take the vortices' distribution to be statistically homogeneous along the x -axis and be described by some distribution function $f(y, \Gamma, t)$, so that the number of vortices in the phase volume $dy d\Gamma$ at a moment t is $f(y, \Gamma, t) dy d\Gamma$. Following the standard procedure of the derivation of kinetic equations by means of a cross-section of pair collisions, it is easy to obtain the equation for evolution of the distribution function $f(y, \Gamma, t)$:

$$\frac{\partial}{\partial t} f(y_1, \Gamma_1, t) = \operatorname{St}[f], \tag{4.5}$$

where the integral of collisions St is defined by the following relation:

$$\operatorname{St}[f] = \int (f'_1 f'_2 v' \sigma' - f_1 f_2 v \sigma) dy_2 d\Gamma_2, \tag{4.6}$$

where

$$f_{1,2} = f(y_{1,2}, \Gamma_{1,2}, t), \quad f'_{1,2} = f(y'_{1,2}, \Gamma'_{1,2}, t),$$

v, σ and v', σ' are the initial values of relative vortices' velocity and cross-section of their collisions, when initial positions of these vortices are

y_1, y_2 and y'_1, y'_2 , respectively. Velocity v is defined only by the difference of shear flow velocities on the stream lines y_1, y_2 ; taking into account (2.4), (3.2), we have for this velocity the following formula:

$$v = A \operatorname{ch}\left(\frac{y_1 + y_2}{2}\right) y_{12}. \quad (4.7)$$

Substituting the quantities $y'_{1,2}, v', v, \sigma, \sigma'$ obtained from the relations (4.3), (4.4), (4.7) in formula (4.6) and taking into account that the value y_∞ is small, let us expand the expression (4.6) over the value y_{12} . As a result of integration over y_2 in the expression obtained we shall have the following final relation for the collision integral St of the kinetic equation (4.5):

$$\begin{aligned} St = & \frac{\partial}{\partial y_1} \left[\frac{1}{A \operatorname{ch}(y_1)} \right. \\ & \times \int_{\Gamma_1 + \Gamma_2 > 0} \left(\Gamma_2 f(y_1, \Gamma_2) \frac{\partial f(y_1, \Gamma_1)}{\partial y_1} \right. \\ & \left. \left. - \Gamma_1 f(y_1, \Gamma_1) \frac{\partial f(y_1, \Gamma_2)}{\partial y_1} \right) \right. \\ & \left. \times \left(1 - \ln \frac{\Gamma_1 + \Gamma_2}{A \operatorname{ch}(y_1)} \right)^2 \Gamma_2 d\Gamma_2 \right]. \quad (4.8) \end{aligned}$$

5. Conservation laws

It is obvious that the kinetic equation (4.5) with the collision integral (4.8) provides conservation of the total number of vortices having definite intensity of vorticity Γ :

$$\int_{-\infty}^{+\infty} f(y, \Gamma, t) dy = N(\Gamma), \quad (5.1)$$

where $N(\Gamma)$ is a function independent of time. In addition, it is not difficult to see that the medium vorticity is conserved at each point y :

$$\int_{-\infty}^{+\infty} f(y, \Gamma, t) d\Gamma = R(y), \quad (5.2)$$

where $R(y)$ is a function independent of t .

The kinetic equation (4.5) has a stationary solution in the form of a Maxwell distribution:

$$f(y, \Gamma, t) = g(\Gamma) \exp(-\Gamma y^2/T), \quad (5.3)$$

where $g(\Gamma)$ is an arbitrary function, T is a constant which can be interpreted as temperature (it will be recalled that the quantities Γ and y are the analogs for mass and velocity of particles). However (as distinct from a gas of particles), an arbitrary initial distribution of vortices does not relax to a Maxwell distribution (5.3) because of the infinite number of integrals of motion (5.2). What are the stationary states to which the system does relax? We will answer this question in the next section in the particular case of a set of vortices having only two vorticity values.

6. Set of vortices with two values of vorticity

Let us consider a vortex ensemble having distribution function of the following type:

$$\begin{aligned} f(y, \Gamma, t) = & n_1(y, t) \delta(\Gamma - \Gamma_1) \\ & + n_2(y, t) \delta(\Gamma - \Gamma_2). \quad (6.1) \end{aligned}$$

In other words, some vortices in such a system have intensities Γ_1 , while other vortices are characterized by the intensity Γ_2 . Substituting this distribution function in the expression for the collision integral (4.8) and carrying out the integration in this expression, in place of kinetic equation (4.5) we shall obtain the following set of equations for the vortices' densities n_1, n_2 :

$$\begin{aligned} \frac{\partial n_1}{\partial t} = & \Gamma_2 \frac{\partial}{\partial y} \left[S \left(\Gamma_2 n_2 \frac{\partial n_1}{\partial y} - \Gamma_1 n_1 \frac{\partial n_2}{\partial y} \right) \right], \\ \frac{\partial n_2}{\partial t} = & \Gamma_1 \frac{\partial}{\partial y} \left[S \left(\Gamma_1 n_1 \frac{\partial n_2}{\partial y} - \Gamma_2 n_2 \frac{\partial n_1}{\partial y} \right) \right], \quad (6.2) \end{aligned}$$

where

$$S = \frac{1}{A \operatorname{ch}(y)} \left(1 - \ln \frac{\Gamma_1 + \Gamma_2}{A \operatorname{ch}(y)} \right)^2. \quad (6.3)$$

The vorticity conservation laws (5.2) now take the form:

$$\Gamma_1 n_1 + \Gamma_2 n_2 = R(y), \quad (6.4)$$

where $R(y)$ is the function of variable y being defined by the initial data of the problem. If the mean vorticity of a vortex system at any point is equal to zero, $R(y) = 0$, then the set of equations (6.2) comes to the following nonlinear diffusion equation:

$$\frac{\partial n_1}{\partial t} = \Gamma_1 \Gamma_2^2 (\Gamma_1 - \Gamma_2) \frac{\partial}{\partial y} \left(S n_1 \frac{\partial n_1}{\partial y} \right).$$

For any initial distribution of vortices in this case the system evolves to the equilibrium state $n_1 \equiv n_2 \equiv 0$ (all the vortices move to infinity).

It is not hard to find an equilibrium state also in case of an arbitrary vorticity distribution $R(y)$; such a state is defined by the condition

$$\Gamma_2 n_2 \frac{\partial n_1}{\partial y} - \Gamma_1 n_1 \frac{\partial n_2}{\partial y} = 0. \quad (6.5)$$

(If a source and dissipation of vortices are present in the system, then the stationary state between the source and dissipation will correspond to a constant flux of vortices P across the shear flow: $\Gamma_2 n_2 \partial n_1 / \partial y - \Gamma_1 n_1 \partial n_2 / \partial y = P = \text{const.}$) Integration of eq. (6.5) leads to the following relationship of quantities n_1, n_2 :

$$n_2 n_1^{-\Gamma_1/\Gamma_2} = C, \quad (6.6)$$

where C is some positive constant.

Let the system possess a finite number of vortices, i.e. the values n_1, n_2 are decreasing rapidly enough while $y \rightarrow \infty$. In this case, if the amplitudes Γ_1, Γ_2 are of the same sign, $\Gamma_1 \Gamma_2 > 0$, then, due to the equality (6.4), the values of quantities n_1, n_2 are small in those places where the total vorticity value is small; in such places, according to relation (6.6), the density of vortices with less amplitudes is much higher than the density of

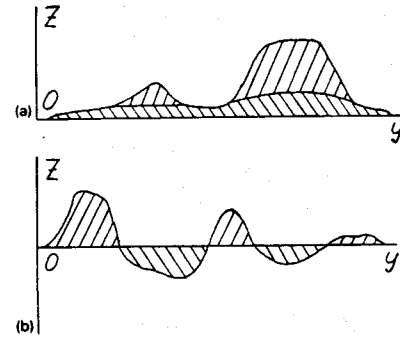


Fig. 3. Vorticity distribution in a system with two vortex species. The bold line is the total vorticity curve $z = R(y)$. The shadings $////$ and $\\\\\\$ are the contributions to the total vorticity of vortices having intensities Γ_1 and Γ_2 , respectively: (a) the case $\text{sgn}(\Gamma_1 \Gamma_2) > 0$, (b) the case $\text{sgn}(\Gamma_1 \Gamma_2) < 0$.

vortices with greater amplitudes. In the other words, in the case $\Gamma_1 \Gamma_2 > 0$ scales of turbulence in most excited regions tend to enlarge, while in outlying regions (where the total vorticity of point vortices is small) scales of turbulence tend to diminish; see fig. 3a.

If the amplitudes Γ_1, Γ_2 have different signs, $\Gamma_1 \Gamma_2 < 0$, then the constant C is equal to zero, since (according to the expression (6.6)) only under this constant it is possible for the values n_1 and n_2 to be small simultaneously in equilibrium state. This fact and the equality (6.4) lead to the following equilibrium state in the case $\Gamma_1 \Gamma_2 < 0$:

$$\begin{aligned} n_1 = 0, \quad n_2 = R(y) & \text{ if } \Gamma_1 R(y) > 0, \\ n_2 = 0, \quad n_1 = R(y) & \text{ if } \Gamma_2 R(y) > 0. \end{aligned} \quad (6.7)$$

Expressions (6.7) signify, that in equilibrium state the vorticity at each space point (the latter, according to the expression (6.4) is conserved in the process of evolution) is created only by vortices with equal signs, see fig. 3b; i.e. vorticity is formed by the minimal possible number of vortices (the superfluous vortices pass away to infinity). One may interpret the process of evolution of the system to such a state as an enlargement of the turbulence scales.

7. Summary and discussion

Dynamics of a rare Stewart vortex gas on the background of shear flow with constant vorticity may be considered as a sequence of pair collisions. Only collisions with small enough aiming parameter (4.1) lead to momentum exchange between vortices. To describe evolution of such a system, kinetic equation (4.8) was derived in the present paper. Kinetic equation (4.8) conserves not only the full number of vortices of definite species (5.1), but also the mean vorticity of the vortex gas on each shear flow stream line (5.2). This equation has the equilibrium solution of Maxwell distribution form (5.3), but due to vorticity conservation law (5.2), this distribution can be reached by the system only for a very narrow class of initial conditions.

On the basis of results obtained by the study of a set of vortices with two vorticity values (6.1), one can draw some general conclusions concerning the equilibrium state: the system evolves to the equilibrium state in which total vorticity of the vortex gas on each shear flow stream line is constituted by vortices with one sign of vorticity only, the most intensive vortices being concentrated in the regions of large total vorticity, while less intensive vortices are in small vorticity regions (i.e. at the periphery of perturbed regions).

As we have shown in section 3, the interaction of two Helmholtz vortices on the background of a constant vorticity flow cannot be considered a collision because of the increase of vortices' interaction energy with the distance between vortices. Therefore the interaction of Helmholtz vortices has collective nature. Such interaction seems to lead to dynamic shielding of each vortex field by the mean field created by the rest of vortices [1]; this is similar to the situation for a gas of parti-

cles with Coulomb interaction. The kinetics of the vortices ensemble in this case can be considered as pair collisions of dynamically shielded particles. It is not hard to make sure that, independently of the particular form of dynamic shielding, the asymptotic behavior of such a system for large values of variable x coincides with the asymptotic behavior of a system of Stewart vortices (since the value y_∞ depends only on vortex field in the unshielded region, $r \rightarrow 0$). Hence such an approach will also lead to the kinetic equation (4.8); the only difference is that the values $ch(y), ch(y_1)$ in this equation should be replaced by unity.

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