MA124: Maths by Computer - Week 9

Regulations

- This assignment gives 15% of your final mark.
- The assignment is due on Monday 14 March 2011, to be handed over to the special box in the lobby next to the Mathematics General Office by 3pm the latest.
- Even if you cannot come yourself, you have to arrange for your work to be handed in timely (e.g. ask a friend to bring it). No late work will be accepted.
- Your work should be on paper and stapled (no electronic form, unless you are disabled in a way making this necessary).
- You should put only your student identity number on your work, but not your name.
- Solutions can be by a single author, or by a pair in which case both authors will receive the same mark. In the latter case, do not forget to include both ID numbers.
- If you think you've found a typo, please consult mathstuff to see if has already been corrected before e-mailing me at S.V.Nazarenko@warwick.ac.uk.

A. **Surface plotting** [7 points.]

In class we explored the 3D graphics commands *meshgrid, surf, mesh, contour*. Use the help system to recall the syntax of these commands. The file testsurf.m on mathstuff has a simple example.

(a) The function $f(x, y) = (x^2 - y^2)/(x^2 + y^2)$ is continuous away from the origin but not at the origin. Explore some surface, mesh and contour plots of this function near the orgin. Give a bried description of how the function behaves at zero. Choose one that illustrates its behaviour and attach the plot.

(b) **Lemma** Suppose $f : \mathbf{R} \to \mathbf{R}$ has two continuous derivatives, has only one critical point x_0 and $f''(x_0) < 0$. Then f achieves its global maximum at x_0, that is $f(x) \le f(x_0)$ for all $x \in \mathbf{R}$.

The lemma is sometimes called the 'only critical point in town' lemma. For functions $f : \mathbf{R}^2 \to \mathbf{R}$ the analogue of this lemma fails. Indeed consider the function $f(x, y) = 3xe^y - x^3 - e^{3y}$. Prove that f has only one critical point and that it is a local maximum. What is the global maximum of f? Use surface plotting in matlab to get a view of f that illustrates its behaviour and attach the plot. (Hint: I found it more useful to plot arctan(f) in order to squash down very negtative and very positive values.)

(c) Give a careful proof of the lemma from part (b).

B. **Recursively defined curves** [8 points.]

In class we developed the file koch.m to plot approximations to the Koch curve. A modification of this file will plot approximations to one of Hilbert's versions of a Peano space-filling curve. You might like to have a look at a description and different levels of approximations to the Hilbert curve on Wiki:

http://en.wikipedia.org/wiki/Hilbert_curve

In Analysis III next autumn you will check carefully that these approximations converge, and that the limit is a continuous curve whose image fills up all of the box $[0,1]^2$ - a so called space filling curve.

Part of the modified m-file is on mathstuff, and called hcurve.m, but I have left in only the code that plots level zero. Fill in the rest of the code that will plot the approximation to any level. Attach this m-file and a print out of the approximations at level 5.