MA124: Maths by Computer - Week 5

Regulations

- This assignment gives 15% of your final mark.
- The assignment is due on Monday 14 of February 2011, to be handed over to the drop in box by the Mathematics General Office by 3pm the latest.
- Even if you cannot come yourself, you have to arrange for your work to be handed in timely (e.g. ask a friend to bring it). No late work will be accepted.
- Your work should be on paper and stapled (no electronic form, unless you are disabled in a way making this necessary).
- You should put only your student identity number on your work, but not your name.
- Solutions can be by a single author, or by a pair in which case both authors will receive the same mark. In the latter case, do not forget to include both ID numbers.
- If you think you've found a typo, please consult mathstuff to see if has already been corrected before e-mailing me at S.V.Nazarenko@warwick.ac.uk.

Question 1: How big is a matrix? [7points.]

One measure of the size of A is to look at $\|Ax\|/\|x\|$. But how to choose x? The idea is to find the x which makes this largest. Because the ratio is invariant to scaling of x by a constant it suffices to consider only x of length 1.

The following MATLAB script will approximate the x which maximizes the ratio in the case of 2x2 matrices:

```
>> for i=1:1000
theta=2.0*pi*rand;
x=[cos(theta); sin(theta)]; s(i)=norm(A*x)/norm(x);
end
>> size=max(s)
```

- Generate a random symmetric 2×2 matrix A ($A^T = A$) and apply the sript to it. Then use the MATLAB command **eig** to find the eigenvalues of A. What relationship, if any, can you see between the output of the script and the use of **eig**?

  Your solution to this part should contain the matrix you used, and the output **size** of the script applied to that matrix. You should also describe the relationship between this number and the output of `eig`, and explain this relationship.

- Write and present a similar script for 3x3 matrices. **Hint:** To characterize vectors which take values on the unit sphere in 3D space, you will need two angles (think of spherical coordinates). Thus, you will now need two for-loops for coding all possible directions in the 3D space.

Question 2: Lissajous figures and knots. [8points.]

A Lissajous curve (Lissajous figure or Bowditch curve) is the graph of the system of parametric equations

$$x = \sin(at + \delta), \quad y = \sin(bt),$$

which describes complex harmonic motion. This family of curves was investigated by Nathaniel Bowditch in 1815, and later in more detail by Jules Antoine Lissajous in 1857.

The appearance of the figure is highly sensitive to the ratio a/b. It ranges from simple shapes, like straight line, parabola, circles and ellipses, to more involved *Chebyshev polynomials*, to complicated fascinated closed figures suggestive of a three-dimensional knot, to space filling curves.

- Write an m-file for Lissajous figures. Plot results for the following special values, describe what curve do you see and why.
  1. $a = b = 1$ and $\delta = 0$.
  2. $a = b = 1$ and $\delta = \pi/2$.
  3. $a = b = 1$ and $\delta = \pi/3$.
  4. $a = 1$, $b = 2$ and $\delta = \pi/2$.
  5. Try now $a = 1$, $b = N$ (N is a natural number) and $\delta = \frac{N-1}{N}\frac{\pi}{2}$. These curves are Chebyshev polynomials of the first kind of degree N (here, you do not need to explain why).
  6. Try irrational values for a/b, e.g. $a = 1$ and $b = \pi$. Try different different sizes for the range of parameter t to get a better idea of what is happening. Describe what you see.
- 3D Lissajous knots are defined parametrically as

  $$x = \sin(at + \delta_1), \quad y = \sin(bt + \delta_2), \quad z = \sin(ct + \delta_3).$$

  Write an m-file and plot Lissajous knots by choosing various sets of parameters, e.g. $a = 1, b = 7, c = 5, \delta_1 = 0, \delta_2 = \pi/2, \delta_3 = 0.5$. What do you see? Present your m-file and a couple of plots, - not too many please. (If this part has excited your curiosity, you can chose 3D knots as a subject for your essay next year).