Magnetic field generation by coherent turbulence structures

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Magnetic field generation...

- The talk is based on the paper:
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  • *D. Kivotides, A. J. Mee, C. F. Barenghi*, Magnetic field generation by coherent turbulence structures, *submitted*
Nonlinear systems of physical mathematics: what is their function within physics?
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- Take advantage of singular/near singular structures that dominate the system’s energetics:
Magnetic field generation...

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  - Shocks, detonations, vortex tubes/sheets,
  - particle-segregation patterns,
  - gravitational collapse structures.
• Turbulent magnetohydrodynamic dynamo $\equiv$ magnetic field amplification by a turbulent flow field.
Magnetic field generation...

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- \( Re = 10^4, \quad Pr_m = \nu/\lambda = 10^{-8} - 0.4; \) use an LES method in vorticity space.
Magnetic field generation...

- Turbulent magnetohydrodynamic dynamo $\equiv$ magnetic field amplification by a turbulent flow field.
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- $Re = 10^4$, $Pr_m = \nu/\lambda = 10^{-8} - 0.4$; use an LES method in vorticity space.
- Is there a turbulent vortex structures dynamo?
• Turbulent magnetohydrodynamic dynamo \equiv \text{magnetic field amplification by a turbulent flow field.}
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  \[ Re = 10^4, \; Pr_m = \nu/\lambda = 10^{-8} - 0.4; \text{ use an LES method in vorticity space.} \]
• Is there a turbulent vortex structures dynamo?
• What are the structure and statistics of the amplified magnetic field?
Magnetic field generation...

- Vortex structures in turbulence (Farge et al, PRL 2001).
Magnetic field generation...

- Vortex structures in turbulence (Farge et al, PRL 2001).
- Vortex structures spectra (Farge et al, PRL 2001).
Magnetic field generation...


\[ \frac{\partial \mathbf{X}}{\partial t} = \mathbf{V}, \]

\[ \mathbf{V}(\mathbf{x}) = -\frac{1}{4\pi} \int_{L} \frac{(\mathbf{x} - \mathbf{X}) \times \omega(\mathbf{X}) d\mathbf{X}}{|\mathbf{x} - \mathbf{X}|^3}, \]

\[ \omega(\mathbf{x}, t) = \gamma \int_{L} \frac{1}{\sigma(s, t)^3} \zeta \left( \frac{|\mathbf{x} - \mathbf{X}(s, t)|}{\sigma(s, t)} \right) \left( \frac{\partial \mathbf{X}}{\partial s} + \frac{\mathbf{x} - \mathbf{X}(s, t)}{\sigma(s, t)} \frac{\partial \sigma}{\partial s} \right) ds, \]

\[ \frac{d\sigma_i^2}{dt} = 2\gamma\nu, \]

\[ T_{L}^{t} \rightarrow T_{L}^{t+\Delta t}. \]
Magnetic field generation...

- Modeling changes in vortex tangle topology (Kivotides & Leonard, EPL 2003).
Magnetic field generation...
Physics of the vortex tube turbulence model:
Magnetic field generation...

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  - Incorporates reconnections.
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Magnetic field generation...

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  - Qualitatively correct alignments between material or vorticity vectors and strain rate eigenvectors.
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- Qualitatively correct alignments between material or vorticity vectors and strain rate eigenvectors.
- Two positive, on average, eigenvalues of the strain rate tensor.
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- Predicted fractal dimension of concentrated vorticity, pdf’s of stretched filament radii, spectra of filament curvature and torsion...
• Schoinoidal model’s spectrum (Kivotides & Leonard, PRL 2003).
Magnetic field generation...

- Schoinoidal model’s spectrum (Kivotides & Leonard, PRL 2003).
- Schoinoidal model’s third order structure function (Kivotides & Leonard, PRL 2003).
Magnetic field generation...

- Magnetic field component, linear magnetic induction equation:

\[
\frac{\partial B}{\partial t} = \lambda \nabla^2 B + \nabla \times (V \times B).
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• In a more familiar fluid dynamic form:

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\frac{\partial B}{\partial t} + V \cdot \nabla B = B \cdot \nabla V + \lambda \nabla^2 B.
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Magnetic field generation...

- Magnetic field component, linear magnetic induction equation:

\[ \frac{\partial \mathbf{B}}{\partial t} = \lambda \nabla^2 \mathbf{B} + \nabla \times (\mathbf{V} \times \mathbf{B}). \]

- In a more familiar fluid dynamic form:

\[ \frac{\partial \mathbf{B}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{V} + \lambda \nabla^2 \mathbf{B}. \]

- The following constraint applies:

\[ \nabla \cdot \mathbf{B} = 0. \]
Magnetic field generation...

- Snapshot of vortex tube turbulence model flow as initial condition.
Magnetic field generation by turbulent vortex structures; $Re = 10^4$. 

![Diagram](image-url)
• Magnetic and fluid spectra; $Re = 10^4$, $Pr_m = 0.4$. 
Magnetic field generation by coherent turbulence structures

- Magnetic field - strain rate alignment cosines
  \[ g_i = \frac{|(\mathbf{B} \cdot \Lambda_i)|}{|\mathbf{B}||\Lambda_i|} \quad (i = 1, 2, 3); \quad Re = 10^4, Pr_m = 0.4. \]
  \[ \langle \Lambda_1 \rangle = 23.284, \langle \Lambda_2 \rangle = 0.482, \text{ whereas } \langle \Lambda_3 \rangle = -23.766. \]
Magnetic field generation...

- Tube to ribbon transition; $Re = 10^4$, $Pr_m = 0.4$. 

![Image of magnetic field generation](image.png)
- Vortex and magnetic structures; $Re = 10^4$, $Pr_m = 0.4$. 
Magnetic field generation...

- Magnetic field spiraling around a vortex;

\[ Re = 10^4, Pr_m = 0.4. \]
Magnetic field generation...

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Magnetic field generation...

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- The thermal superfluid dynamo:
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• The thermal superfluid dynamo:
  • Thermal superfluid $\leftrightarrow$ Electrically conducting fluid.
Magnetic field generation...

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- Thermal superfluid $\equiv$ inviscid superfluid $\oplus$ viscous, Navier-Stokes normal-fluid $\oplus$ mutual friction force.
- The thermal superfluid dynamo:
  - Thermal superfluid $\leftrightarrow$ Electrically conducting fluid.
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  - Superfluid field $\leftrightarrow$ Magnetic field.
  - Mutual friction $\leftrightarrow$ Lorentz force.
Magnetic field generation...

- Superfluid vortex dynamics (Idowu, Kivotides, Barengghi & Samuels, JLTP 2000):

\[
\frac{\partial X_s}{\partial t} = V_s + h_0 V_s + h \times X'_s \times (V_n - V_s) - h \times X'_s \times (X'_s \times V_n),
\]

\[
V_s(x) = -\frac{\kappa}{4\pi} \int_{\mathcal{L}_s} d\xi_s \frac{X'_s \times (X_s - x)}{|X_s - x|^3},
\]

\[
X'_s \equiv \frac{\partial X_s}{\partial \xi_s},
\]

\[
T_{\mathcal{L}_s}^t \rightarrow T_{\mathcal{L}_s}^{t+\Delta t}.
\]
Magnetic field generation...
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- A snapshot of the vortex tube model normal turbulent flow.
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- Is there a dynamo? (Kivotides, PRL 2006).
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- A snapshot of the vortex tube model normal turbulent flow.
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- Is there a dynamo? (Kivotides, PRL 2006).
- \( Re = 40, \gamma \approx 100\kappa. \)
Magnetic field generation...
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- Normal-fluid (green line) and superfluid (red lines) energy spectra.
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Thank you for your attention!