PRACTICAL, THEORY BASED PRINCIPLES FOR TEACHING MATHEMATICS IN HIGHER EDUCATION

W. Cox
Aston University

INTRODUCTION

With the increasing professionalisation of teaching in HE it is timely to consider how we might establish it on a more formal, principled, perhaps even scientific, basis. We have to remember that the professionalisation of teaching in schools has only taken place in the last couple of centuries. Since modern UK higher education has evolved primarily through the research route and has until recently recruited students from the top few percent of ability (or income), there was never much need to concern oneself too much with the quality of the teaching. So it is no surprise that we are a long way behind school-teaching in the training and education of our HE teachers. However, things have changed rapidly in the last couple of decades or so. Specifically:

- Universities now recruit over 40% of the eligible age group
- The world economy is now highly 'information – based' requiring high levels of good education at every level to ensure international competitiveness
- Students are now paying directly for their higher education, accumulating vast debts in the process
- HE teaching is under increasing scrutiny from government and the taxpayer

For such reasons formal teacher training for academics is essential if they are to retain credibility as a profession. The question is, what form should it take? One could argue that it should be every bit as lengthy, rigorous and theoretically founded as school-teaching, but this is neither affordable nor necessary. But we do need something that is rigorous, theoretically informed and discipline-led. In Mathematics we are fortunate in that we have a long established discipline of Mathematical Education on which to draw – a great deal of work has been done at the HE level. But there are a number of problems. Practical HE mathematics teachers have little time for the study of pedagogy, and find it difficult enough just keeping up with the demands of day to day class contact partly because the pressure of the RAE. On the other hand Mathematical Educators are obliged, again by the RAE, to focus their efforts on theoretical questions that are often too far removed from the chalk face to appear useful to the practitioner. And of course the language of the typical mathematics education paper will of necessity be almost impenetrable for the average lecturer. There may also be a deep-seated distinction between Mathematics and Mathematics Education (Weber, 20007) purely in the way they approach problems and communicate ideas. Mathematicians tend to be analytical, focused, convergent thinkers, constantly aiming for 'the solution'. Mathematics Educators on the other hand tend to be empirical, broad-based, divergent thinkers, aiming more for ranges of solutions to meet differing circumstances and often satisfied with less than perfect evidence-base for their actions. But in fact the latter is more the nature of the teaching activity itself.

None of this is new. The same applies to schoolteachers in relation to mathematics education at their level, but because of their longer training they have more opportunity to digest the theoretical foundations which are made more accessible for them during their training. And as mathematicians we should be used to the distance that often exists between the theoretical and practical aspects of a discipline. For example there are whole swathes of Mathematical Physics that have little actual use in Physics. But eventually theory does catch up with practice, and can eventually lead it. In the early days of the industrial revolution theoretical fluid dynamics was often far removed from (but admittedly driven by) practical ship building and civil engineering which were largely craft based. But eventually advances in theory began to make an impact on the practical aspects of engineering and now no one would entertain launching a new vessel or building a substantial bridge without extensive prior mathematical modelling and simulation It will be the same in Mathematics Education. Certainly, much that has been done so far has limited relevance to day to day teaching, but increasingly there will be a greater need for theoretical underpinning of teaching and, combined with the craft basis existing amongst practicing teachers, Mathematics Education has probably already produced most of this that will be required. In order for all of our graduates, whatever their innate ability, to compete effectively at the international level the quality of our teaching needs to be first class – there is no room for a minimalist approach to teaching in HE. The use of IT, web, and other technological advances now provides unprecedented opportunities for investigating and delivering education, not just as a mode of delivery, but as an investigative tool for advancing our knowledge of how students learn. One can imagine that in a decade or so incoming students will take some kind of psychological test that will be used to design the curriculum to suit each of them and enable them to fulfil their individual potential. And in future lecturers will learn, as part of their training, ideas about student learning that are scientifically evidence-based.

So, how can we train, educate and support lecturers for their teaching, while recognising at the same time the practical and political realities (RAE, etc) within which we all have to work? How would I do such a thing in my mathematics teaching – that is how would I give the students the best education in a limited time? Would I give them a list of quick 'top tips' on group theory – the top ten theorems? Would I show them rules of thumb designed to get the job out of the way quickly, thereby confirming its lowly status? Would I give them a recipe book, a troubleshooting manual? Encourage them to learn by rote and blindly follow algorithmic processes? Do we train them as we would call-centre personnel?

Or would I teach them the fundamental principles, illustrate how one can build on these using one's own independent intellectual resources, how to use and apply them to inform practice? Would I teach them the basic ideas of reasoned and rigorous argument – with examples from the key proofs? Would I teach them to adapt theorems and techniques to new circumstances? And would I try to instil attitudes commensurate with their professional duties and the values of their discipline and scientific enquiry, generating a love and respect for the subject, an intellectual honesty and integrity that will guide them through their careers?

I think the latter would be better. So I reject the 'top tips for teaching' approach. Such teaching approaches, often adopted by generic courses teach teaching in exactly the opposite way to how one would teach anything properly. Top tips, teaching tips, 1000 tips, etc try to get the teacher to accept bite-sized chunks without underlying unity and coherence – would we teach algebra like that? Certainly, we use menomics such as BODMAS, but these are just reminders that trigger what we hope is fundamental understanding that the student can apply to particular circumstances. I believe that practical teaching should be underpinned by sound evidence-based theoretical frameworks. These will only evolve gradually over extensive and intensive debate, controversy and practice. Since we have to start somewhere, in this paper I suggest eleven basic principles of teaching that are evidence-based and can be used to build most good teaching practice. My claim is that any set of '2000 Teaching Tips' can be derived as applications or consequences of these by any thoughtful teacher. The term 'principle' is used loosely here. These are intended to be used as guidance for actual practice in the classroom, and so they are phrased precisely at this level. Also, they are not meant to be 'principles' in the sense of say Newton's Laws of motion, experimentally founded to high orders of accuracy within their context. Nor are they meant to be 'axioms' as we might understand them in the mathematics sense, immutable and divorced from requirements of external reality, satisfying only logical consistency. They are however meant to be founded on convincing evidence which will be a composite of findings from Mathematics Education theory, widespread practitioner consensus and plain common sense. Their main function will simply be as a start for continual critique, improvement, replacement and so on. The object is that if the lecturer learns and continually applies these principles then they will be a good practical guide for effective teaching. Many authors on pedagogy already attempt such a thing – Krantzs' 'Guiding principles' (1999) or Baumslag's Rules of Good Teaching (2000). In the eleven given here I have tried to be more theoretically–based, more comprehensive and more rigorous. Some of them may be wrong, some may be unnecessary, they may be incomplete, they may be ill-informed, they may be unfashionable. But, they are there for anyone so inclined to critique and hopefully improve. There is always the danger that such principles will appear bland, trite, vacuous, anodyne. But this is often the case with fundamental principles of any kind. Such principles only really come to life when they are put together, their consequences explored, and they are applied to particular situations.

BASIC PRINCIPLES OF TEACHING MATHEMATICS IN HIGHER EDUCATION

Our main problem is to find ways in which deep learning about teaching can be facilitated within the competing resources and priorities that lecturers have available these days. Rather than overload the lecturer with copious disconnected and unsupported 'tips' we adopt the approach of providing a small number of basic principles that underpin everything we do in teaching maths in HE. The hope is that in observing and embedding these principles in all that they do, the lecturers will have a firm foundation on which to build their day to day practice.

Current ideas in theories of how we learn mathematics are too much for most people to digest and assimilate into their day to day teaching practice. As a compromise, the principles suggested here are distilled from such theory. Combined with other sources, that can be used to underpin practice in a fairly common sense way. These are by no means definitive or final, and the greatest service that the reader can do is to examine them critically and suggest improvements. They are intended to provide useful guidance for teaching that will best support the students' learning. If they do not, then they need to be changed. If they are incomplete they need to be supplemented. If they are inconsistent, they need to be rationalised

The suggestions below have been grouped under the practicalities of setting up the learning environment, how we thing students learn and the main teacher's tasks in helping them to do so.

Practicalities of providing the learning environment

These principles relate to the practical aspects of setting up the sort of learning environment and situations that are most conducive to learning. Of course this involves a large range of considerations from using accommodation and equipment effectively to building up beneficial relationships and rapport with the students. We have to think about resources available and how to use them, we need to be prepared for the human aspects of teaching, everyone involved needs to be clear about what it is we are trying to achieve, and the teaching, learning and assessment activity must be designed to do this.

P1. **Resources:** Teaching and learning is constrained by limited resources (the most important of which is time) available to you and your students.

While obvious as a principle, this is often forgotten in practice. How much time do the students have for this coursework? Do I have the time to prepare the material for this mini-project? What are the hardware and software implications of introducing some computer aided assessment? Do I have a budget for these handouts, or do I charge the students? Such questions arise all the time, and so we need to have a good knowledge of our resources from the start.

But resources should be interpreted in the broadest sense including the intellectual resources of our students – and ourselves. We are constantly bombarded by innovations in teaching, usually in the form of some new application of IT in teaching. Such things, whatever their educational benefits are almost invariably labour intensive in the development stage and also require a lot of intellectual input, from designing or adapting the materials to the students having to learn the associated IT skills.

P2. **Professional and human:** Teaching and learning has both professional and human/pastoral dimensions and management of the curriculum, the student group and the associated interpersonal interactions requires a mix of both.

Teaching and learning involves continual human interaction with all that brings, and yet it denies us many of the normal coping mechanisms that we employ in everyday human interaction. How we teach and interact with the students can have an emotional and psychological impact on them to which they may respond as any adolescents might. In normal human interactions, such as with our family and friends we can often let off steam with anger or aggression or tears! None of these is allowed the teacher, who has to have the detachment to rise above such things and respond in a professional manner. We have to do as good a job for those students who we don't like as for those we do. We must judge students on their merits as people who want to learn, not on their innate mathematical ability.

And, as in most human situations, we have to realise that in order to do a good job, we may have to adapt our personality, put on an act, control our own behaviour. While the stereotype of the muddle-headed, disorganised mathematics professor may be overworked there are those of us who are a bit chaotic in our ways and this is not always good in teaching situations. It is part of the professionalism to realize this and to take action to minimize the effects it has on our students. Some of us are forgetful – we forget meetings, deadlines, what we are supposed to be talking about. We cannot afford to do this in teaching. Some of us are arrogant and disdainful of slower thinkers than ourselves. Again, such behaviour is out of place in the classroom and has to be put aside – if you don't really care for your students then you at least have to be able to pretend that you do. Indeed the first of Krantz's (1999) 'Guiding principles' is to respect your students, and that is one of the key human aspects of the activity of teaching.

Why is it so important to empasize this human aspect? In our normal mathematics activities our personalities, human frailties, and emotions are not really that important – indeed they may be an advantage. For the absent-minded professor the forgetfulness of intense concentration may be just what they need to crack a difficult problem. The arrogant, self-opinionated, chap may have just the confidence to press on into the unknown to find the solutions necessary. In these examples there is one key point. The problem and its solution will never go away, will never change and is not the slightest bit influenced by your personal emotions and characteristics. If impatience causes you to throw away your calculations in disgust the problem will still be there, unmoved, when you calm down. Not so with your students – your behaviour can have a lasting influence on them and seriously affect your ability to help them to learn. A mistake in this respect may not only change the problem, but also destroy any chance of a solution. Conversely, if you get it right and produce the right rapport and relationship with your students then you can have a powerful positive effect on their learning, which can often transcend your professional and technical skills.

P3. **Clarity and precision:** There need to be clarity and precision about what is expected of the students and how that will be measured.

No academic likes to read a paper in which the objectives are not clear, or submit to a journal or apply for a grant when criteria for acceptance are not clear. Think then how it feels for the student, powerless, timid and maybe overawed by the environment they have just entered. We cannot get the best out of them without being clear and precise about what we expect of them. So, from statements about course content and objectives, through instructions for learning activities, to criteria for student assessment, there needs to be clarity and precision about what is expected of the students. This is not the same as being over informative on assessment or limiting their learning to low level easily prescribed activities. Also, in lectures and tutorials the teacher's communication needs to be clear and precise, ditto for the learning materials. No one can learn or absorb information and ideas efficiently if the message is not clear and unambiguous. Sometimes of course interpretation under such circumstances is actually part of the process of learning or assessing learning, but by and large people learn most effectively if they have clear objectives, criteria, and instructions. And one should always aim to help students to learn in the most efficient way, so they can move more quickly onto new things. There is no merit in making things artificially difficult for the learner.

P4. **Allignment:** The teaching, learning and assessment strategies need to be aligned with what is expected of the students.

For given learning objectives, the teaching and learning strategies and activities must be designed to achieve them. Quite simply students must be assessed on what and on how they are taught. The assessment strategy and tasks must be designed to measure achievement of those objectives. This linkage of the objectives, the teaching activities and the assessment is called **alignment**. This is a well-known and basic tenet of good teaching. It is also common sense that if you have to teach somebody something then you have to fit the method of teaching to the thing that is to be learned, and you have to only assess that which they were expected to learn. Most practitioners do all this automatically with little thought, yet it is one of the most researched aspects of teaching (Anderson and Krathwohl, 2001). It involves classifying cognitive skills by some taxonomy, designing teaching methods appropriate to these skills and assessment methods to assess them. Most such taxonomies are too far removed from how practitioners operate, although there are user-friendly versions for the mathematician (Cox, 2003).

How students learn

These might be the most contentious of the principles proposed here. There are those that think that we do not know how students learn, and that there are no convincing theories that tell us much about this area. Far from it, we know enormous amounts about student learning, the real problem is that there is too much for the busy academic to assimilate and incorporate into their day to day teaching. Viewed as an emerging science teaching is perhaps in the periodic table stage – we are certainly past the alchemy stage, but we don't yet have an atomic theory, let alone a Grand Unified Theory. But what is certain is that we will get there and what is already available can be of real practical use in the classroom. The difficulty is in filtering it all out to achieve a palatable form that can be used by the practitioner. Now this is in fact more difficult when talking to the experienced academic than it is when presenting it to the new lecturer. The former often already has preconceived notions developed over long experience. These may or may not accord with the principles we present here, indeed it may make them appear obvious and vacuous. It is sometimes forgotten that when a craft metamorphoses into a science, much of the principles and practice 'discovered' by the science are no more than explanations for and validation of previous folklore. For example, modern medicine can explain down to its molecular structure why drugs derived from digitalis are so effective in heart disease, but this does not invalidate the empirical work of William Withering in isolating it from the purple foxglove, or indeed alter the efficacy of the folk remedies based on the plant known to witchdoctors down the ages! And of course the experienced lecturer has lots of data to draw from to fit into any new theories and cause possible conflicts. The new lecturer on the other hand has fewer preconceptions, is likely to be more flexible in their thinking and perhaps be grateful for some anchors to guide them through their teaching.

What I have done here is to try to identify some key implications of current thinking on student learning of mathematics to the actual practice of teaching mathematics. I have tried to encapsulate these in just a few principles, at the risk of trivializing such work, but with the intention of encouraging lecturers to embed at least some of the key ideas from Mathematics Education into their everyday teaching practice.

P5. **Intellectual load:** The workload on students, in terms of intellectual progression, must be appropriate to the level and standards of the course, and the background of the students.

Even with the crudest 'tape recorder' model of learning, it is obvious that any learner has a limit to what they can reasonably assimilate in a given time. And teaching should progress from a starting point and at a rate that is well within those for most of the students in the class. Even if this were not obvious, there is abundant evidence of the demotivating and counter effective results of intellectual overload in learning Skemp ref). To measure the appropriate load we need an accurate assessment of where the students start from – and this is where we as teachers often fall down – we don't always have a good knowledge of the students' backgrounds. But we can certainly make the effort to find out, whether it is by simply talking to the students in class or something more formal such as an initial assessment test. This idea of first determining what students know and teaching from that is often referred to as 'Ausubel's principle' (Ausubel, et al, 1978), but of course it predates this, forming the underlying motivation behind Socratic dialogue and probably utilized automatically by any sensible caveperson teaching their offspring to hunt. But to the lecturer, faced with increasingly wide variations in student background, it now takes on a much more technical aspect.

There is another implication of this principle about intellectual load, which has a less politically correct tone to it. The appropriateness of workload is relative to the student. It is pointless to pretend that top grade students at the best institutions are not capable of a much higher workload than those at average institutions. And of course the converse can be said of less well qualified students, or non-specialist students such as engineers and scientist who have to take mathematics as a part of their studies – they are not capable of the same workload as the average student. Deciding on the appropriate workload for different cohorts of students, and detaching that consideration from confusions over 'standards' is one of the most difficult jobs in teaching.

P6. **Process to product:** Mathematics is most effectively studied in the away it is done, rather than in the way it is finally presented.

It is now widely realised that the polished 'Definitions, theorem, proof' format in which some books and lecturers present mathematics is not how we actually do mathematics, in which the development is more exploratory, less systematic and less inexorable. It is also not how we as lecturers and researchers actually learn mathematics. What is perhaps so surprising about this principle is that it should be so necessary to state it at all – yet it is. It is the most repeatedly rediscovered idea in mathematical education. It goes back at least to Archimede's '*Method'* (Boyer, 1991) and can be traced through generations of distinguished mathematicians and mathematical educators (Poincare, 1913; Hadamard, 1945; Skemp, 1971; Kline 1977; Tall, 1991) and yet we still find it in recent mathematics education papers once again rediscovered as if it were something new and surprising. Worse still, we still find many lecturers teaching as if the principle had never occurred to them, churning out bite-sized definition, theorem, proof with no semblance of sensible explanation, continually pulling rabbits out of a hat, the word 'why' never having occurred to them. This principle emphasizes that by presenting mathematics to students in a more 'sensible', properly explained way, rather than as a rigorous polished product, we both make it easier for them to learn and also induct them to the ethos and mathematical ways of thinking.

P7. **Reconstruction of ideas:** Mathematics is learned most effectively if the student reconstructs the ideas involved and blends them with their current (corrected!) understanding.

Again, the evidence for this (in any subject) is clear (Skemp, 1971, Tall, 1991). Any learner absorbs the information they receive only as the first step in internalising it. It becomes material in an internal dialogue through which the learner reorganises the input and fits it into what they already know, modifying both to arrive at their own understanding. The teacher can help them in this by doing more than just transmitting the information. They can help the learner as they develop the internal dialogue by providing appropriate exercises and problems, by intervening in external expressions of the internal dialogue, by providing motivation, encouraging effective engagement, and explaining well. There is something especially distinct about mathematics in this respect. We hear a great deal these days about group and discussion methods. These are aimed at helping students to learn through discussion amongst themselves, the discussion facilitating the reconstruction process. Of course, this is very beneficial and in many subjects such as the arts it is perhaps the prime method of learning. But in mathematics it is probably the internal dialogue referred to above that is most important. Mathematics is too detailed, intricate and hierarchical to progress far by external communication with others alone. Much of the work has to be done by the learner literally talking to themselves (Not always silently!!), and this self critical, rigorous and high integrity dialogue is something that students need to develop to a high order, with our help. It is precisely why we give them lots of drill to do – it is not only to consolidate specific skills, but to give them training in internalizing their own arguments.

The 'corrected' in parentheses is a reminder of the fact that the reconstruction may involve a correction or revision of previous ideas, which sometimes has to be addressed before progressing. An example here is when we begin complex numbers. The student may well have had it drummed into them that you cannot take the square root of a negative number. So we may have to spend a little time explaining why it is that that is just what we are about to do. As another example, we might be teaching the Baker-Campbell-Hausdorff identity for non-commuting matrices A, B:

$$e^A e^B = e^{A+B} e^{[A, B]/2}$$

which holds provided the commutator [A, B] = AB – BA commutes with A and B. Now we would naturally expect the students to connect this with their previous knowledge of the exponential function where we have the result

$$e^x e^y = e^{x+y}$$

for numbers x, y. But in fact the very notation in the new identity can bewilder students. And there may be some students who actually believe that $e^x e^y = e^x + e^y$. It is first necessary to correct this impression if it is present. Similarly, if we are teaching completing the square we may need to be aware that some students don't see $(a + b)^2$ as $a^2 + 2ab + b^2$ but as $a^2 + b^2$, which again must be corrected before proceeding. All these examples illustrate that when we invite students to build on and into their previous knowledge, we have to ensure that that knowledge is itself secure before we start. In the literature such commonly occurring errors are called 'mal-rules' (Martin Greenhow, 20007), and a useful Mathematics Education project would be to categorise these rules so that they may be anticipated and dealt with.

P8. **Learning how to learn:** The specific skills of learning mathematics may need to be explicitly taught.

By this we mean the skills that the student employs to monitor, adapt and apply their learning processes. That is we need to help them learn how to learn. This does not mean routine study methods, but self-critical examination of their learning as it progresses. For example they need to recognize when they need to increase their rigour, dig deeper into an issue, or check what they have done so far. The well trained mathematician takes such things for granted, but the novice may need proactive help to develop such skills. This is even more essential these days with first year students particularly. The nature of school level assessment is now such that students arrive with little experience of higher order skills, being used to single step, objective exercises requiring little more than memorization and recall and routine recipe following. We need to help them realise that there is more to mathematics (or any form of learning) than this and support them in developing the more advanced skills they need. Not only that but we need to help them to develop the skills of recognising when they need particular skills and how to set about acquiring them themselves. Even for the very best students we need to train them in such skills as research skills and communicating their ideas.

Teacher's tasks (Explain, Engage, Enthuse)

Armed with ideas about how students learn the teacher can then provide the sorts of experiences and activities to support the students in learning mathematics. This of course includes the usual sorts of things such as lecturing, tutoring and assessing. The technical details of such things will be learnt from training courses, mentoring, experience, and so on. But underpinning all this we suggest there are three underlying principles the teacher needs to remember. Whether it is in the material we write for the students, in how we present our lectures and tutorials, or in how we assess we have to pay particular attention to how we **explain** things, how we **engage** the students in fruitful activities for learning mathematics, and how we generate **enthusiasm** and motivation in the students. These principles require little explanation as most experienced practitioners would take them for granted, but there is ample evidence in the literature to support them should it be needed.

P9. **Explaining:** Good skills in explanation are essential in supporting students in learning efficiently and effectively.

How well a teacher explains a topic can have a dramatic effect on how easily the student learns it. There is no merit in presenting students with unnecessary challenges as a result of poor explanation, this just wastes time and discourages them. The art of good explanation is developed by practice, but the initial skills can be learnt.

P10. **Engaging:** Students must be actively engaged in the process of doing mathematics in order to learn mathematics.

That mathematics is a 'doing subject' is obvious to anyone who studies mathematics, but consideration is not always given to how we get the students engaged, and with what. If we engage then with routine, repetitive, one-step exercises, then that is what they will learn to cope with, no matter how long they spend at it. If we want to develop higher order skills in mathematics, solution of major problems, etc then that is what we have to get them working on, while at the same time providing them with the ladders to reach such levels.

P11. **Enthusing:** High levels of motivation and enthusiasm are required for effective learning of mathematics.

No one learns anything well if their heart is not in it! And this is particularly so in mathematics. Devlin (2000) argues that one of the major reasons that the lay-person finds mathematics so difficult is simply because they don't want to do it, but often have to. In effect, many students who have to do mathematics are press-ganged onto a voyage they don't relish. And even if students start off interested in mathematics, it is very easy to turn them off by not paying enough attention to keeping them motivated. Generating enthusiasm is therefore another skill for the teacher to master (Cornish, et al, 2007).

AN EXAMPLE – PROVIDING FEEDBACK ON COURSEWORK

To illustrate how the above principles might inform a typical everyday teaching activity we will consider the task of providing feedback on student coursework. We can of course find plenty of advice on providing feedback to students on their work, but here we aim to illustrate how most of it arises from simply applying the principles we have outlined.

Preparing the coursework and the feedback requires a consideration of resources (P1) of course, in terms of materials, handouts, etc. But there is also the question of time and scheduling – we have to consider how much time it is reasonable to expect the students to put into the work, and how much time we have to provide feedback. Certainly, the feedback must be provided in the most efficient and effective way possible to avoid wasteful use of resources. The feedback needs to be provided as soon as possible while the work is still fresh in the students' minds, helping reconstruction of their ideas (P7), so the scheduling of the feedback needs to be managed to fit in with our other duties (P2).

The nature of the feedback also needs to be considered. This is one of the areas of teaching where human aspects (P2) come in strongly. Students, like everyone, are usually sensitive to comments and criticism of their work and how we actually comment on the work can have an influence on how they learn from it and also on their confidence. Of course, students differ on how they react and if you know them well you can use this in your feedback – some will react better to encouragement, some to a challenging 'You can do better than this!' If we don't know the students that well it is better, as in all human interactions, to take an encouraging, helpful line, making it clear that you are not judging them, but trying to help them.

The tasks set for the coursework should be clear and precise (P3), and the feedback linked to those tasks in a clear way. Suppose for example we asked them to solve the differential equation

$$x\frac{dy}{dx}+y=x$$

with the intention that this is an exercise in solving linear differential equations by integrating factor. Unless we make it clear exactly what is intended, some students might treat this as a homogenous equation, or may immediately recognise that the left hand side is the derivative of a product *xy* and integrate directly. It is then no good providing feedback of the kind 'You were supposed to do this by integrating factor', and docking marks for not doing so. And this relates to the need for allignment of the coursework with the course objectives and the teaching strategy (P4). In this case if we are forced to give full marks for a student who presents a solution as a homogeneous equation because we didn't specify the method to be used then we have no evidence that the student knows anything at all about the integrating factor. Our feedback will have to point this out if it is to help the student to achieve the course objectives.

The nature of the feedback should also be designed to help students to learn – that is the prime purpose of coursework. For this, the workload must be reasonable (P5). In terms of the feedback this means that it must be concise and to the point. A barrage of comments and criticisms, with little indication of what is really important will simply overload the student and possibly demoralise them – indeed good positive feedback on our work is one of the most motivating influences there can be in learning (P11). Far better to identify the few major points that the student needs to focus on and aim comments at how they may address these. So far as P6 is concerned the feedback can place a high premium on how the student themselves explains what they are doing – is it verbatim repetition of material and approaches used in the notes, or can we follow their independent thinking, can we see them working as mathematicians? If they miss out steps or explanation do they really understand the omissions? Is there any evidence for the finished product of their work that they understand the processes required to get there?

Since students learn by reconstructing their current knowledge (P7) the feedback should include not only 'specimen solutions' in terms of how we the teachers think the work should be approached, but also reactions to and possibly corrections of how the students have tackled the work. This feedback takes more effort of course – it is so easy to simply issue a pre-prepared solution sheet and indeed this satisfies the need for prompt feedback if students are to learn from it. But this is transmission teaching – it doesn't take account of the students' thinking – it doesn't build on their preconceptions or their misconceptions and doesn't directly help them in the reconstruction of their ideas. It is therefore more helpful if feedback is provided after marking, in the light of the students' attempts, that picks up and addresses their errors and misconceptions. Usually there will only be only a few such issues that can be gone through with the whole class, and after a few years experience we will be able to anticipate what these issues are likely to be (that doesn't mean we will be able to prevent them!).

Coursework is ideal for helping students to learn how to learn (P8). We can help by not only providing corrections and comments in our feedback, but also setting new questions, occasionally asking why students took particular steps, refer them to appropriate reading, etc. In our solutions issued for the students, or our feedback in class, we can use the fact that they will be particularly attuned to our comments and primed to learn from them, since unlike during a lecture they will have already wrestled with the problems. So we can take advantage of what we have learned from their work to make our explanations particularly effective (P9) and engage them actively in working through the solutions together (P10). But don't expect **all** students to participate and benefit from the exercise – that some won't is simply human nature, which takes us back to P2!

A MNENOMIC FOR TEACHING MATHEMATICS

The principles of the previous section are intended to underpin everything we do in teaching mathematics. For practical day to day activities we can use **MATHEMATICS** (Cox, 2004). This use of **MATHEMATICS** might be thought of as equivalent to the use of such things as BODMAS for the axioms of a field, or CAST for the principles of general angles. It is simply an aide memoire to help lecturers quickly run through the things they need to think about in their teaching duties. But in thinking about these things they will all the time have the basic principles in mind. So, whether we are preparing a whole degree programme, a single course, or an individual lecture or tutorial class, the large number of things we have to think about can be summarised in the apt acronym **MATHEMATICS** below.

Mathematical content

Aims and objectives of the curriculum

Teaching and learning activities to meet these aims and objectives

Help to be provided to the students - support and guidance

Evaluation, management and administration of the curriculum and its delivery

Materials to support the curriculum

Assessment of the students

Time considerations and scheduling

Initial position of the students – their background in mathematics

Coherence of the curriculum – how the different components fit together

Students

This covers most of the main areas one has to think about. If used with the eleven underlying principles in mind **MATHEMATICS** should provide a good initial guide for effective practical teaching. Of course, once sufficient experience has been developed, such things as the principles and the mnemonic become redundant, in the same way that an experienced algebraist would not give a second thought to BODMAS or the axioms of a field.

The major challenges facing the Mathematics Education community

If the Mathematics Education community is to help the practitioner in the lecture or tutorial room then there are a number of pressing issues to address:

- What is **known**, or at least widely accepted, about student learning of mathematics?
- What of this is of practical usefulness and readily transferable to the classroom?
- How do we measure the effectiveness of our mathematics teaching?
- How do we enthuse and inspire students as well as teaching them?
- How do we most efficiently prepare lecturers for high quality teaching while not encroaching inappropriately on their other duties?

Central to all of these is the provision of an easily applicable underpinning foundation of pedagogical principles on which the teacher can base their classroom activity. I believe this is the only way to establish teaching practice on a sound theoretical background. The shower of 'teaching tips' is certainly **not** the way to do this. For this reason I have proposed the eleven principles in this paper. A severe critique of such an approach, from both the mathematicians and the mathematics education people, would be valuable. Is such an approach useful in practice? Are the principles sufficient, is there something missing? Are they all acceptable, what is the evidence to support them?

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