# Interview with Caroline Series

#### **Ulf Persson**

Caroline Series and Ulf Persson met as graduate students at Harvard in the autumn of 1972 and have kept in intermittent but sustained contact since. Here, UP interviews CS to find out more about her life and career.

UP: When did you first discover mathematics?

CS: I must have been twelve or thirteen, when we started Euclidean Geometry. One day we were given an especially hard geometry problem for homework. That evening I struggled with it for hours and eventually solved it. The next day it turned out I was the only one to have done it, and I was asked to go to the blackboard and explain. The process of working on it and then the rush of satisfaction at finding the solution made a deep impression on me. At that moment I resolved to solve every maths problem we were given. Not long afterwards, I made it my secret ambition to go to Cambridge and get a first class degree in mathematics.

UP: So you went on to Oxbridge, can you tell me about it?

CS: I applied to both Oxford and Cambridge (you can't do this now) and took the exams in the autumn of 1968, when I was seventeen. Actually I always intended to go to Cambridge – we lived in Oxford where my father was an academic, a physicist, and I wanted to go away. As it turned out, I did very well in the Oxford exams and my interview with the Somerville mathematics tutor Anne Cobbe made a deep impression on me. Somerville (one of the women's colleges at Oxford, colleges were all single sex at that time) offered me their top scholarship, which definitely settled the matter.

I discovered by chance that having won a scholarship I did not need to do any A-levels (the British school leaving exams), so I could skip the last two terms of high school. I left school at a couple of weeks notice and had nine months of freedom. I spend three months as an au pair girl in Heidelberg learning rudimentary German, and after that I worked with a group of young women scanning bubble chamber pictures for some Oxford physicists.



Photo credit: Michelle Tennison

UP: So you grew up in Oxford and your father was a member of the University. Then for you to have gone up to Oxford (or Cambridge for that matter) would not have been such a big deal as it would have been for most students.

CS: Of course that is true. But I had very little to do with the University as a child, and as a student I lived in Somerville College, so it was a new stage of life. I studied very hard and made sure to do all the exercises we were given. The course was tough.

UP: What other interests did you have besides mathematics? And what about social life?

CS: I didn't really have time for anything extra-curricular. As for social life, I met my boyfriend Robin shortly before starting at University. He had been to a boys' school in Oxford and was much more advanced than me in mathematics. He helped me a lot and we spent most of our spare time together. At the end of my second year we got married.

UP: How was life at Oxford? Whom did you meet, who were your teachers?

CS: In the British system, university courses are quite specialised and so I only studied mathematics. At Oxford there are large university lectures for everyone, and then you have tutorials with only a few other students. This is where you get you main contact and feedback. I was fortunate to have wonderful tutors, especially Anne Cobbe whom I have already mentioned. Unfortunately, she developed cancer during my first year. She became too ill to do any teaching, and then sadly died. Because of this, I was taught by some other excellent mathematicians, among them Brian Davies, Peter Neumann and Graeme Segal.

#### UP: What mathematics interested you?

CS: At least in the first two years there was not much latitude for personal choice, but when I had the chance I chose pure mathematics. In the first year I loved semi-philosophical things like the uncountability of the reals, and once I got started I liked analysis. There was very little classical geometry, which I missed, and I do not recall any algebraic geometry. Algebraic topology was very formal, but I did like functional analysis.

UP: How did you come to apply to Harvard for graduate school?

CS: That is a story. While I had always planned to study mathematics as an undergraduate, I had really no plans beyond that. My vision of the future was very vague and high school teaching was my only concrete idea.

UP: Still your father was an academic, at Oxford to boot, the idea of going on to graduate school must have been very natural to you.

CS: Not at all. That we should do well academically was tremendously important to my parents but I don't think they really envisioned an academic career for me. My father probably thought I would mainly be a housewife like my mother.

## UP: So what made you take the step?

CS: It was my husband Robin who was being encouraged to go to graduate school. He had a reputation as a brilliant student and his tutors assumed he would continue to a PhD. Michael Atiyah, who was connected with his college, suggested he might go to Harvard, to work with Raoul Bott. That meant I would have to go to the States also, but it didn't make much sense to train as a school teacher there. I thought the only way I could go to the US would be to do a PhD too. Robin had applied for a scholarship from Harvard so I couldn't very well do so as well. I applied for all the other scholarships I could find, but only to go to Harvard, MIT, or Princeton. The information sent by Princeton seemed to me very elitist, I thought they would never want anyone like me there, and the material from MIT was sent by sea-mail, so by the time I got it, it was much too late. As it turned out, I got a very nice scholarship, a Kennedy scholarship, to study for a year at Harvard.

#### UP: What was your first impression of Harvard?

CS: That our graduate class in mathematics was so small! I had imagined everything in the States would be huge, I was very ill informed. Also that the professors were so informal and wore jeans. That would have been unthinkable at Oxford.

UP: And the mathematics, did you feel lost at first?

CS: Yes indeed. Both Robin and I took the qualifying exam the first thing we did and passed right away. After your qualifying exam, as you know, you are on your own. No more course requirements and you don't really know what you are supposed to do, we weren't given any advice. In the first vacation, we were assigned J. P. Serre's famous article *Faisceaux Algébriques Cohérents* to read. We both struggled with it and Robin began to get depressed as up till then everything had come so easily to him. I just persevered as best I could.

## UP: We hear about Robin, what happened to him?

CS: He gradually lost his interest in mathematics and eventually he dropped out of Harvard. Things between us became very strained: I remained at Harvard but decided to try to get a Masters degree in Statistics, which I thought would be an easier way to make my living. I actually nearly renounced my scholarship but at the last minute my father convinced me to pull back. After a while I settled down and began to work seriously on mathematics again. Robin and I got divorced and we went our separate ways. He did eventually go back to Harvard and finish a thesis, but that was long after we had split up.

## UP: So now you had free sailing?

CS: But I needed to settle on a topic and an advisor. At Harvard at the time, almost all the professors seemed to be doing algebraic geometry or number theory. I was very ill prepared for either, and felt I wanted to do analysis. This narrowed down the choice of an advisor to Loomis, Gleason and Mackey.

UP: Loomis was about to retire ...

 $\mathsf{CS:}\ \ldots$  and for some reason I was terrified of Gleason  $\ldots$ 

UP: ... and that left George Mackey.

CS: Right. Despite people having told me that Mackey was famous for his low opinion of women mathematicians, he was very gracious when I approached him and said he would take me on a trial basis. Nothing more was ever said: later I realised that he must have looked up my records to check me out.

#### UP: And so it worked out.

CS: It worked very well and got me started on independent work in mathematics. Mackey was a good advisor. Group representations, Mackey's subject, seemed to me very interesting. He was writing a book on the many aspects of the subject, drafts of which he gave me to read. I liked it because it gave a broad overview of the subject without getting bogged down in details.

#### UP: Did he give you a problem to work on?

CS: No, he told me he preferred his students to come up with their own problems. But anything I came up with seemed to have already been done. However Mackey was really helpful when it came to suggesting papers to read, so in that way I learned a great deal.

UP: So this was the first time you learned mathematics outside a formal course?

CS: Yes, more or less. In fact I concentrated mainly on dynamical systems, which Mackey was interested in at the time. He had some original ideas about groups acting on measure spaces and the associated unitary representations. However I was getting worried towards the end of my third year that I still didn't have a proper project to work on, while all the other students seems to know what they were doing.

#### UP: So did you worry about not writing a thesis?

CS: Yes of course, a great deal. However, in 1975, the year before I got my PhD, Harvard gave me a small grant to go to a summer school in Kingston, Ontario at which Alain Connes was the main speaker – that was before he won the Fields medal. By chance I had just the right background to understand at least part of his lectures, and I realised that I was following just as well as most other people in the audience. It gave me the self-confidence and determination to come back and finish my thesis, which I did. It was on a problem Mackey had suggested concerning his theory of 'virtual groups', closely related to what is known as orbit equivalence. I spent from September to March working and writing up my thesis. You had

to apply for positions for the following year the previous autumn, so I had to get something on paper quickly. I recall that Mackey rejected the first draft of my thesis because he said it was much too vague. The second time around I started out with a long array of definitions and Mackey did not like that either, he said it was boring. He was quite right. Luckily on the third go he was satisfied.

UP: So now you were about to enter the third stage of your career, but before turning to that let us dwell a little bit more on the social side of graduate school. Were there any fellow graduate students, specifically other students of Mackey, whom you engaged with?

CS: Not really. I had very little contact with his student Bob Zimmer, who was a few years ahead of me, though I did study his thesis. In fact it was one of the BPs (Benjamin Pierce Fellows, or assistant lecturers) Troels Jørgensen who first showed an interest in what I was doing, for which I was very grateful. He also told me about his work, which subsequently became one of the inspirations for Thurston's theory of hyperbolic 3-manifolds. Although I didn't understand much at the time, later I found it was closely connected to my own work.

Some of the main social interactions I had were with other women graduate students. There weren't many of us, at most one in each year. That is how I met Linda Ness, like you a student of Mumford. Then for two years I shared an apartment with Terry Myers whom you certainly remember, she was a graduate student at Boston University and the wife of another Mumford student, Jerry Myers, who had graduated and had a job in Albany in up-state New York. And of course, for a year your then wife Mindy shared with us also, while you had your first job at Columbia in New York.

UP: Indeed, how could I forget that. You must also have met Ragni Piene, you both got your PhD in the spring of 1976.

CS: Yes, I did meet Ragni at about that time, although she was at MIT so I didn't see her often. I think it was through her that I met Dusa McDuff, at that time a lecturer in York but visiting Princeton. Both of them have remained lifelong friends.

UP: Let us talk about the third stage in your mathematical career. The post-doc stage. How did that play out?

CS: I applied for lots of jobs. My first choice was to go to Berkeley, and to my great joy I was offered a two-year lectureship. But I was also offered a Research Fellowship in Newnham College, Cambridge. I felt I couldn't give that up, as I knew I wanted eventually to return to the UK. Newnham kindly allowed me to postpone for a year, and I spent a year at Berkeley which I extended as long as possible by not returning to England until the autumn of 1977.

UP: Why did you want to go to Berkeley? What was it like?



Berkeley campus 1977. C. Series, private collection

CS: What interested me was the group there in dynamical systems. It was a comparatively new subject, and there were great people there – Calvin Moore (another Mackey student), Jack Feldman and Rufus Bowen, not to mention Don Ornstein and his group of ergodic theorists in Stanford.

UP: Dynamical system has origins both in topology and in hard analysis.

CS: I would rather say measure theory and probability than hard analysis. Ergodic theory is about measurable transformations on a measure space, and that is what my thesis was about. Dynamics in terms of topology goes back to Poincaré, while the probabilistic approach was mainly developed by the Russian school led by Kolmogorov. The two strands were just coming together in the 1960s and 70s.

UP: You wrote a paper with Bowen, how did that come about?

CS: After some time working on the abstract parts of ergodic theory, I began to look for more concrete examples and went back to the beautiful geometry and dynamics of the geodesic flow on a surface of constant negative curvature, which had been studied in the 1930s by Hedlund and Hopf. I wanted to reconcile their geometrical ideas with Bowen's more abstract method of constructing what are known as Markov partitions for Anosov flows. Markov partitions had also been developed by Yakov Sinai in the Soviet Union, and ideas of the Russian school were brought to Berkeley by Sinai's student Marina Ratner.

I discussed these ideas with Rufus and we agreed to start a collaboration which turned out to be crucial to my subsequent career. We settled on a joint project to try to make the geometrical approach from the 1930s work for all hyperbolic surfaces. Although by that time I was in Cambridge, I visited in Berkeley again in summer 1978. Not long after I had arrived, I had an idea which I believed should solve our problem. It was the weekend, but I was very excited and called Rufus at home. A voice I didn't recognise answered and when I asked to speak to Rufus the answer was 'I am very sorry, he is dead'. As you can imagine, this came like a thunderbolt. Rufus was only thirty-one, very healthy and athletic, how could this be?

After the initial period of shock and grief, I pulled myself together and checked that my idea did indeed work. I did the only thing I felt I could in the circumstances, and wrote up the solution as a joint paper.

Rufus' influence was also posthumous. Dennis Sullivan was very interested in what Rufus had been doing, he was making connections between dynamics and hyperbolic geometry and hence to the ideas of William Thurston which were just beginning to emerge at that time. So he naturally got in contact with me to see what we had been doing. Not long afterwards Dennis invited me to IHES and his ideas and lectures had a huge influence on me.

#### UP: So what did this lead to?

CS: The geometrical coding I had discovered turned out to be related to many other things, for example the word problem in the fundamental group of a surface, and to what are now called automatic groups, so it became an important tool. I wrote quite a few papers on various aspects and applications.

UP: By the way, you mentioned Cambridge as part of your post-doc years, but you did not collaborate with anyone at Cambridge?

CS: Cambridge didn't really work out for me. I felt quite isolated, as hardly anyone there was at all interested in dynamics and ergodic theory. One person I did talk to was S. J. (Paddy) Patterson. We realised our work had approached the same problem from different angles. I learnt a lot from him and we have remained in contact ever since.

By chance I met Dusa McDuff again, who by this time was a lecturer at Warwick, about to leave for a position in the US. She told me that there were several positions coming up in Warwick. At that time Warwick was the only place in the UK with a substantial group in dynamical systems, led by Bill Parry. I thought they wouldn't want any more dynamicists, but Dusa encouraged me to apply.

UP: And you did and secured one of the jobs. You were in Warwick in the fall of 1979, I recall it very well, I went to England for a short visit that December, just back from the States.

CS: Yes, that's right. You do have a remarkable memory for dates, in fact I started at Warwick in autumn 1978. So in the end I only spent one year in Cambridge.

UP: And you would stay on at Warwick for the rest of your career.

CS: That is true, with some longer breaks for visits of course. I was very happy there.

UP: Tell me about some of your other collaborations.

CS: I had a long and fruitful collaboration with Joan Birman at Columbia, she was known for her work on braids and the mapping class group. She approached me with a problem about simple curves on surfaces and I was able to contribute my expertise. Subsequently, I had a long collaboration with Linda Keen at City University in New York. Linda had been a student of Lipman Bers and was an expert on Teichmüller theory. We decided to try to understand some of the new ideas of Thurston on 3-dimensional hyperbolic geometry which we could approach from different angles. We settled on a problem which involved interpreting the intriguing computer pictures produced by David Mumford and David Wright, at that time a graduate student at Harvard. We spent the best part of a year in fruitless attempts before we began to understand what was going on. It involved going into 3-dimensions and using some of Thurston's wonderful ideas. That led to the discovery of what we called pleating rays, which allow one to understand families of Kleinian groups.

UP: You wrote a book called *Indra's Pearls* with David Wright and David Mumford, when was that?

CS: It came out in 2002, but it took the best part of ten years to write. As I mentioned earlier, David M. and David W. had embarked on a project to explore computer images of the effect of iterating



David Mumford awarded an honorary degree at Warwick, 1983. Miles Reid on left. C. Series, private collection

a pair of Möbius maps in the complex plane. They plotted the places where the orbit of a point under such an iteration accumulated, the so-called limit set of the group generated by the two transformations. These were the pictures that Linda and I had wanted to understand – especially what happens as you vary the group.

The pictures the two Davids created were so spectacular that they wanted to write a coffee-table book about them, similar to books which had appeared on fractals and the Mandelbrot set. The book idea wasn't making much progress. Because of my interest in the pictures, they invited me to join them. In fact Mumford's idea was really more ambitious, he wanted to explain the pictures so that anyone with a good background in high school maths could understand what they were about. He had already drafted the beginning of the book, and Wright had pictures and some text for the last part. So as Mumford said when we got together, all we had to do was to fill in the middle. Easier said than done! It turned out to be a much longer-term project than any of us had bargained for, it spanned over a decade.

#### UP: So how did the collaboration proceed?

CS: We met up periodically in one or other of our home universities. David M. had lots of ideas and David W. was brilliant at making computer pictures. I did a lot of writing, which I have always loved. The main difficulty was finding a way to express what we wanted to say in terms accessible to the intended audience. This meant getting rid of jargon and using as little notation as possible. If you think about it, so much notation is superfluous, for example, mathematicians often write something like 'Take a group G', and then they never actually use the notation G. It is not only laymen who get put off by formulae: even mathematicians can find them daunting and appreciate getting to the basic ideas in simple prose. So it was a very good exercise for me to to write mathematics in this way.

UP: It was not a research project.

CS: Definitely not. This doesn't mean we didn't present new results, but these were mainly experimental and we didn't feel constrained to make anything formal. I learned a lot from writing it, besides getting a much deeper appreciation of the mathematics involved. In the end the book was a lot more ambitious than initially envisioned, but we did get a very good response from all sorts of people, both amateurs and professionals. The beautiful pictures were part of it too, they appealed to many people even if they couldn't follow the maths. We called the book *Indra's Pearls* because the fractal pictures found remarkable analogies with an ancient Buddhist text, and this also sparked a lot of interest.

UP: So what came next after the book was published, did you continue doing mathematical research?

CS: I certainly didn't stop doing mathematics, with a variety of collaborators, but I also got engaged in other aspects of the mathematical community, which I have found very satisfying.

#### UP: Can you tell me some more about this?

CS: In the early 2000s, I organised a big programme on 3-dimensional hyperbolic geometry at the Isaac Newton Institute in Cambridge. Several major breakthroughs had just been made which completed most of Thurston's programme, so the timing was perfect. Then I took my turn at organising a year long symposium at Warwick, although this time I got several other people to run events on somewhat broader themes.

Around the same time I became involved in various national committees, for example of the London Mathematical Society and the Newton Institute. I enjoyed such work, even if it took time away from my research. I met many different people and it gave me insights into how the world works beyond mathematics.

UP: And you became President of the London Mathematical Society (LMS), which was very important to you. How did that come about?

CS: In 2016, just after I had retired, I was elected a Fellow of the Royal Society (FRS) and shortly afterwards I was asked if I would take on being the LMS President. The term is two years, in my case from November 2017 to November 2019. I was the 80th president but only the third woman to hold the post. I really enjoyed the experience and the opportunities it opened up. My involvement



Conference at Warwick 2011. C. Series, private collection



As LMS President, ICM Rio 2018. Left to right: Caucher Birkar, John Hunton, Caroline, Michael Atiyah, June Barrow-Green. Photo credit: LMS

with organisations for women in mathematics had given me some experience with leadership, but this was on a very different scale.

UP: It sounds like a full-time job.

CS: I did spend a great deal of time on it. There was always so much to be done: organising and chairing meetings, discussing and making decisions, initiating new projects, and occasions and travel representing the Society (including leading the LMS delegation to the ICM in Rio in 2018), so different from the regular life of a mathematician. The LMS has a wonderful staff. I got along very well with the Executive Secretary Fiona Nixon which made things much easier for me. One of the first things I did was to get videoconferencing equipment installed so that people didn't always need to come to meetings in person. At the time, although it is only a few years ago, this was something quite new.

UP: I believe Atiyah died on your watch.

CS: Yes, he died in January 2019. He was such a towering figure in British mathematics, I felt that the LMS should make an announcement without delay. Even though I might not have been the best qualified to do it, I spent the weekend writing a short obituary which went onto our official web-page. Later, we set up a big conference and a fellowship in his memory.

UP: So now let us come to the issue of women and mathematics. Has being a minority troubled you? I think that most men would welcome more women in mathematics. Would you actually want to actively promote more women in mathematics?

CS: Of course I am always pleased when a woman wants to study mathematics, I feel I have met a kindred spirit. On the other hand,

I don't think there is any point in pushing unless the person is really interested. Indeed this applies to men also. However, there are still many obstacles for women wishing to become mathematicians.

#### UP: Do you think that women are mistreated in mathematics?

CS: Not any more, although in the past there were some shocking stories. It wasn't so very long ago that some people were wary of appointing a woman. The received wisdom was that she would marry and drop out. I was lucky that by the time I came along most institutions were really rather keen to hire women. When I arrived at Berkeley I felt completely accepted and I had a wonderful time. Cambridge (UK) was very different. Social life in the department was very much male dominated, and in addition I had the impression that no one expected a woman to be a serious mathematician. It seemed I didn't fit in. But as soon as I got to Warwick I felt immediately at home, even though I was for a long time the only woman. My opinions were taken seriously and it was a happy and supportive environment in which to pursue my life and research.

UP: Let us talk about your concrete activities for women and mathematics. How did they start?

CS: My first serious involvement started at the ICM in Berkeley 1986, where I was invited to sit on a panel organised by the Association for Women in Mathematics (AWM). The AWM was largely focused on the USA, and the five Europeans on the panel were inspired to organise something similar in Europe. That's how European Women in Mathematics (EWM) came about. It was an entirely bottom-up organisation. I was closely involved in setting up the basic structures; we set out to have a gathering every year and in 1988 I organised the third meeting in Warwick. It was run on a shoe-string, although we did have some limited funding from the LMS and a few other sources.

I worked extremely hard to make the Warwick meeting happen, from designing a more detailed 'constitution' for EWM, to all the planning of the event, inviting speakers, and so on. Those were the days before email, remember, so everything had to be done with snail-mail. To save money I even arranged for people to stay in the homes of colleagues who were away on vacation. There was a small group of female students helping, but I took on far too much, and had it not been for a very helpful and supportive departmental secretary I don't know how we would have managed.

I am proud that EWM continued its annual meetings, and indeed is still going strong, with a new generation of women behind it.

#### UP: So what did you get out of these meetings?

CS: The main thing for me was getting to know a whole group of women mathematicians, and the feeling of solidarity which we

shared. Being in a mathematical setting with a roomful of other women was a new experience for all of us. I think that is what people valued most – some were much more isolated than I was. I also believe that EWM helped to initiate some quite profound changes, for example we were able to support women in Germany and Switzerland where things were extremely difficult; over time there have been tremendous improvements at the institutional level.

UP: More recently you have been involved in the IMU's Committee for Women in Mathematics (CWM). How did that happen?

CS: When Ingrid Daubechies was President of the IMU (she was its first female president), she had the idea of creating a section of the IMU website as a resource for women mathematicians, with information about all the initiatives and activities internationally. I was asked to be part of a small group gathering up the information, and then I got drawn into organising the material and designing the site. Then, together with Marie-Francoise Roy, one of the other founders of EWM, I approached Ingrid with a proposal that the IMU should create a specific committee for women mathematicians. With Ingrid's support, this is how CWM came about. Backed by generous funding from the IMU, we were able to support and encourage women to form groups in other parts of the world, particularly developing countries. I was the first Vice-Chair with Marie-Françoise as Chair; we worked together very well and I really enjoyed this. It also led me to further involvement with the IMU, for example getting involved with the makeover of the IMU website, which I found very interesting.

UP: Finally, let us turn to issues of so called human interest. What do you do when you are not doing mathematics?

CS: Now that I am retired I spend a lot of time working in my garden, growing herbs and so on, and I like cooking from scratch, which can take a lot of time. I do quite a bit of sewing – altering and what is now called repurposing. I also read a lot, fiction and non-fiction, my reading is very eclectic.

UP: I believe you are also concerned about the environment?

CS: Yes, this is something I care about deeply. I find it completely shocking that our society has been so slow to take effective action. In fact one of the things I did which I am most proud of, is that in 2000 I persuaded Warwick University to set up an Environment Committee to look into things like energy and water conservation, recycling, greener transport and so on on the campus, which is the size of a small town. It was a great struggle to get anything concrete done, but gradually the group became official and better resourced. Now environmental concerns are a major focus of the University.

On personal level, I try to live with as little waste of resources like food, energy, and possessions as is practically possible. It hurts me seeing how our world is steadily being degraded. The cause is both our unbridled consumer society, and also the ever increasing global population which barely seems to get discussed.

UP: The mantra you always hear is about more growth no matter what.

CS: Indeed, although many people are waking up to realise just how serious the situation is, none of us want to give up too much, and besides it is very hard to do this living in the society that we do. The mantra ought to be that we all should be doing everything within our power to alleviate the problems, after all,  $\sum 1/n$  diverges.

UP: I can see this is another issue which could occupy us, but I think it is time to draw to a close. We have touched on many subjects and it has been fascinating to catch up with you again after so many years. Thank you for agreeing to do this interview.

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# **New EMS Press book**



Nalini Anantharaman (Université de Strasbourg and CNRS) Quantum Ergodicity and Delocalization of Schrödinger Eigenfunctions Zurich Lectures in Advanced Mathematics

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This book deals with various topics in quantum chaos, starting with a historical introduction and then focussing on the delocalisation of eigenfunctions of Schrödinger operators for chaotic Hamiltonian systems. It contains a short introduction to microlocal analysis, necessary for proving the Shnirelman theorem and giving an account of the author's work on entropy of eigenfunctions on negatively curved manifolds. In addition, further work by the author on quantum ergodicity of eigenfunctions on large graphs is presented, along with a survey of results on eigenfunctions on the round sphere, as well as a rather detailed exposition of the result by Backhausz and Szegedy on the Gaussian distribution of eigenfunctions on random regular graphs.

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