Test — MA3E1 Groups and Reps

Version 1 Monday 12 November 2012

Name: ____________________

Student Number: ____________________

This test covers the material lectured up to and including theorem 107 (orthogonality of characters, page 36). You can bring any notes you like, but no electronic gadgets or library books. Correct answers are worth 1 mark. Everything else is worth 0 marks. Workings or proofs are neither required nor can give marks.

Give your answers on this sheet and hand it in. You are free to make scratchy notes, but please not on this sheet. No name means no marks. *Good luck!*

1. Let $n \in \mathbb{Z}_{>0}$ be even. Find the conjugacy classes in D_{2n} contained in $\{r^k s \mid k \in \mathbb{Z}\}$.

2. Which group is not finitely generated?
   a. $\mathrm{GL}(0, \mathbb{C})$
   b. $\langle a_1, a_2, \dots \mid a_{i+2} = a_i\, a_{i+1} \text{ for all } i \geq 1 \rangle$
   c. $\mathrm{GL}(2, \mathbb{R})$
   d. $\mathrm{GL}(3, \mathbb{Z})$

3. Compute $(\chi, \chi)_G$ if $G = D_6$ and χ is the character of the representation ρ of D_6 defined by

$$\rho(r) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \rho(s) = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

4. How many group homomorphisms $G \to H$ exist if

$$G = \langle a, b \mid - \rangle$$
$$H = \langle c, d \mid c^3, d^3, (cd)^2 \rangle ?$$

5. Find all intertwiners $\rho \to \sigma$ if ρ and σ are the representations of $C_4 = \langle c \mid c^4 \rangle$ defined by

$$\rho(c) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma(c) = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}.$$

6. Which group is isomorphic to none of the others?
   - a. $\langle \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} \rangle \leq \mathrm{GL}(2, \mathbb{Z})$
   - b. $\{z \in \mathbb{C} \mid z^6 = 1\}$
   - c. $\langle a, b \mid a^2, b^3, aba^{-1}b^{-1} \rangle$
   - d. No two of the listed groups are isomorphic.

7. Put $G = \langle x, y \mid x^4 y^{-4} x^5 y^3 \rangle$. Find explicit $a, b \in C_6 = \langle c \mid c^6 \rangle$ such that there exists a surjective homomorphism $f\colon G \to C_6$ with $f(x) = a$, $f(y) = b$.

8. Consider the permutation f of $\mathbb{Z}/8$ defined by $f(x) = 3 - x$. Write f as a word in r and s (our generators of the dihedral group).