

Clique complexes of cycle powers

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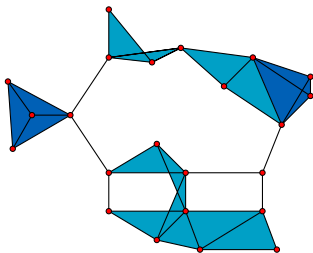


DIMAP Workshop on Combinatorics and Graph Theory
April 2011

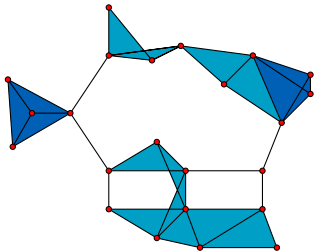
- Graphs

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 - $V, \mathcal{F} \subset 2^V$ – faces $B \subset A \in \mathcal{F}$ then $B \in \mathcal{F}$

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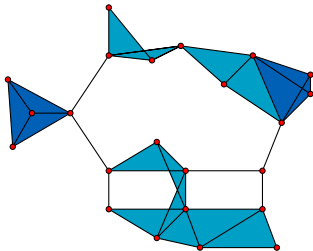


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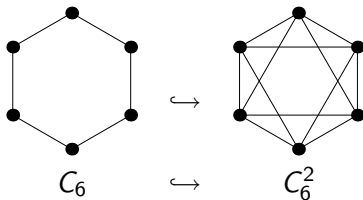


- $\text{Cl}(G)$: \mathcal{F} = cliques of G
- $\text{Ind}(G)$: \mathcal{F} = independent sets of G

Graph powers vs. clique complexes

- The r -th power G^r has edges $uv \in E(G^r)$ whenever

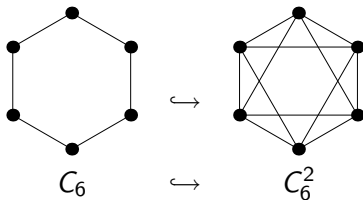
$$\text{dist}_G(u, v) \leq r$$



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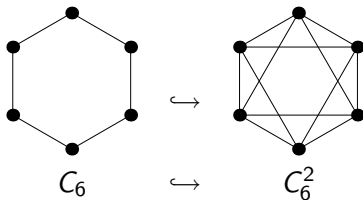


- $G \hookrightarrow G^2 \hookrightarrow G^3 \hookrightarrow \dots \hookrightarrow K_n$

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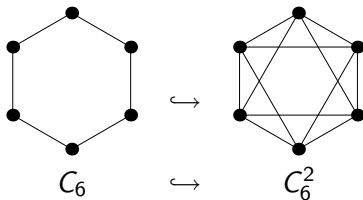


- $G \hookrightarrow G^2 \hookrightarrow G^3 \hookrightarrow \dots \hookrightarrow K_n$
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Graph powers vs. clique complexes

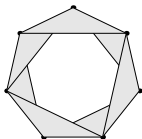
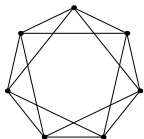
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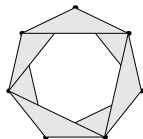
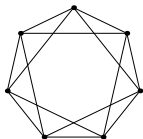


- $G \hookrightarrow G^2 \hookrightarrow G^3 \hookrightarrow \dots \hookrightarrow K_n$
- $\text{Cl}(G) \hookrightarrow \text{Cl}(G^2) \hookrightarrow \text{Cl}(G^3) \hookrightarrow \dots \hookrightarrow \Delta_n$
- $\text{Cl}(G^r)$ is the Vietoris-Rips complex of G

- $G = C_7^2$

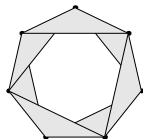
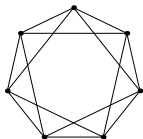


- $G = C_7^2$



Möbius band \mathcal{M}

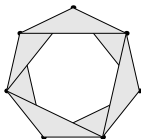
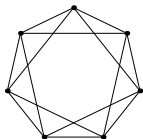
- $G = C_7^2$



Möbius band $\mathcal{M} \simeq S^1$

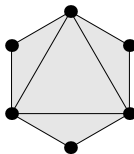
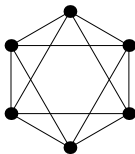
Examples

- $G = C_7^2$

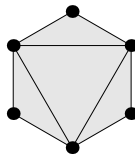


Möbius band $\mathcal{M} \simeq S^1$

- $G = C_6^2$

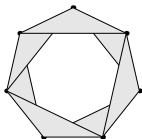
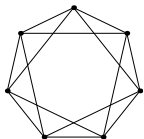


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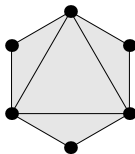
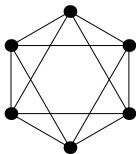
Examples

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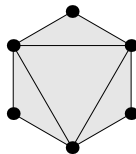


Möbius band $\mathcal{M} \simeq S^1$

- $G = C_6^2$



\cup



$= S^2$

Theorem (A.)

For any r and any simplicial complex K there is a graph G with a *homotopy equivalence*

$$\text{Cl}(G^r) \simeq K$$

When is the inclusion $\text{Cl}(G) \hookrightarrow \text{Cl}(G^2)$ a homotopy equivalence?

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Theorem (A.)

It is the case when every maximal clique of G^2 is of the form

$$N_G[v].$$

Stability (for $r = 2$)

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Theorem (A.)

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Theorem (A.)

The above condition is satisfied when G is



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,



,



– free

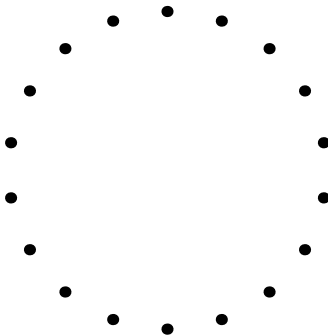
*(it does not have either of the four graphs as an **induced** subgraph.)*

$$\text{Cl}(C_n^r)$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_8	$V^7 S^0$	S^1	S^1	S^3	*	*	*	*	*	*	*
C_9	$V^8 S^0$	S^1	S^1	$V^2 S^2$	*	*	*	*	*	*	*
C_{10}	$V^9 S^0$	S^1	S^1	S^1	S^4	*	*	*	*	*	*
C_{11}	$V^{10} S^0$	S^1	S^1	S^1	S^3	*	*	*	*	*	*
C_{12}	$V^{11} S^0$	S^1	S^1	S^1	$V^3 S^2$	S^5	*	*	*	*	*
C_{13}	$V^{12} S^0$	S^1	S^1	S^1	S^1	S^3	*	*	*	*	*
C_{14}	$V^{13} S^0$	S^1	S^1	S^1	S^1	S^3	S^6	*	*	*	*
C_{15}	$V^{14} S^0$	S^1	S^1	S^1	S^1	$V^4 S^2$	$V^2 S^4$	*	*	*	*
C_{16}	$V^{15} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^7	*	*	*
C_{17}	$V^{16} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^5	*	*	*
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*
C_{19}	$V^{18} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	*	*
C_{20}	$V^{19} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^3 S^4$	S^9	*
C_{21}	$V^{20} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	$V^6 S^2$	S^3	$V^2 S^6$	*
C_{22}	$V^{21} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	S^{10}
C_{23}	$V^{22} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^3	S^7
C_{24}	$V^{23} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	$V^7 S^2$	S^3	S^5
C_{25}	$V^{24} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^4 S^2$

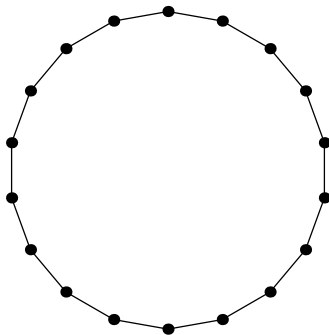
$$(C_{18})^0$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*



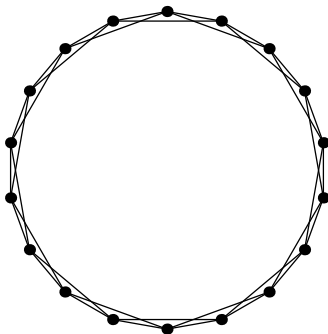
$$(C_{18})^1 \rightarrow S^1$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*



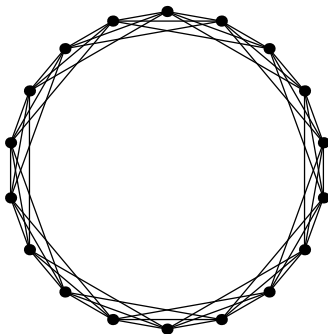
$$(C_{18})^2 \rightarrow S^1$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*



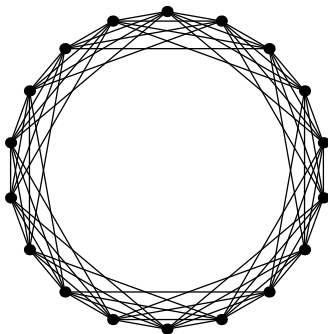
$$(C_{18})^3 \rightarrow S^1$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*



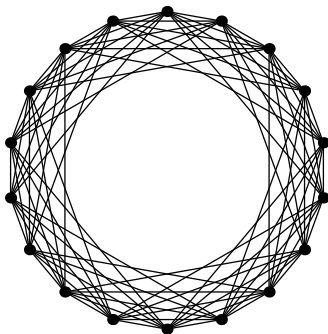
$$(C_{18})^4 \rightarrow S^1$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*



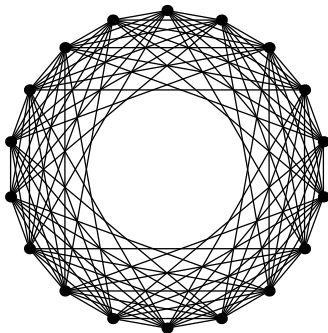
$$(C_{18})^5 \rightarrow S^1$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*



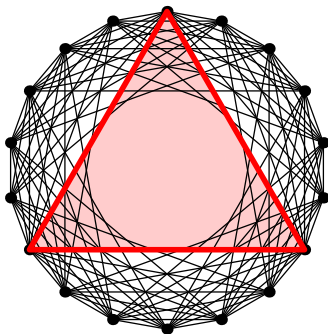
$$(C_{18})^6 \rightarrow V^5 S^2$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*



$$(C_{18})^6 \rightarrow V^5 S^2$$

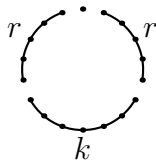
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Cycles and their complements

With the parameter

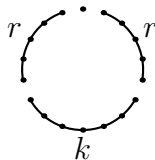
$$k = n - 2r - 1$$



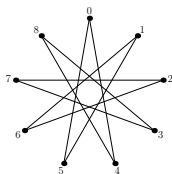
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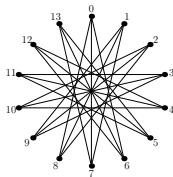
$$k = n - 2r - 1$$



consider the complements $T_{n,k} := \overline{C_n^r}$ known as the *circular complete graphs*.



$$T_{9,2} = \overline{C_9^2}$$

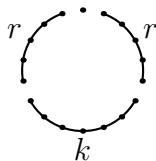


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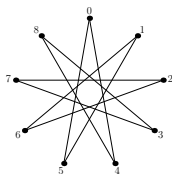
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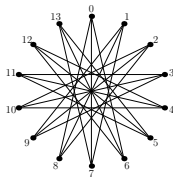
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$$T_{9,2} = \overline{C_9^2}$$



$$T_{14,3} = \overline{C_{14}^3}$$

$$\text{Cl}(\mathbf{C}_n^r) = \text{Ind}(\mathbf{T}_{n,k})$$

Theorem (A.)

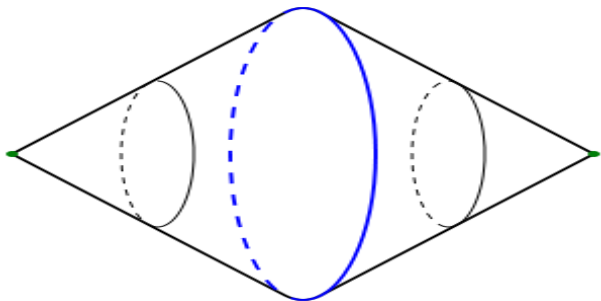
For reasonable values of n, k the independence complexes of *circular complete graphs* satisfy

$$\text{Ind}(\mathcal{T}_{n,k}) \simeq \Sigma^2 \text{Ind}(\mathcal{T}_{n-2(k+1),k})$$

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- If v is in no triangle both summands are contractible, so

$$\text{Ind}(T_{n,k}) \simeq \Sigma(\text{st}(v) \cap \bigcup_{w \in N(v)} \text{st}(w))$$

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- These constraints can be encoded in a graph

$$\text{Ind}(T_{n,k}) \simeq \Sigma \text{Ind}(S_{n,k}).$$

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- These constraints can be encoded in a graph

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- The procedure can be repeated for $S_{n,k}$ leading to $T_{n-2(k+1),k}$.

Conclusion

$$\text{Cl}(C_n^r) \simeq \Sigma^2 \text{Cl}(C_{4r-n}^{3r-n}) = \Sigma^2 \text{Cl}(C_{n-2}^{r-1 \cdot (2r-n)})$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_8	$V^7 S^0$	S^1	S^1	S^3	*	*	*	*	*	*	*
C_9	$V^8 S^0$	S^1	S^1	$V^2 S^2$	*	*	*	*	*	*	*
C_{10}	$V^9 S^0$	S^1	S^1	S^1	S^4	*	*	*	*	*	*
C_{11}	$V^{10} S^0$	S^1	S^1	S^1	S^3	*	*	*	*	*	*
C_{12}	$V^{11} S^0$	S^1	S^1	S^1	$V^3 S^2$	S^5	*	*	*	*	*
C_{13}	$V^{12} S^0$	S^1	S^1	S^1	S^1	S^3	*	*	*	*	*
C_{14}	$V^{13} S^0$	S^1	S^1	S^1	S^1	S^3	S^6	*	*	*	*
C_{15}	$V^{14} S^0$	S^1	S^1	S^1	S^1	$V^4 S^2$	$V^2 S^4$	*	*	*	*
C_{16}	$V^{15} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^7	*	*	*
C_{17}	$V^{16} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^5	*	*	*
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*
C_{19}	$V^{18} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	*	*
C_{20}	$V^{19} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^3 S^4$	S^9	*
C_{21}	$V^{20} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	$V^6 S^2$	S^3	$V^2 S^6$	*
C_{22}	$V^{21} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	S^{10}
C_{23}	$V^{22} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^3	S^7
C_{24}	$V^{23} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	$V^7 S^2$	S^3	S^5
C_{25}	$V^{24} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^4 S^2$

Conclusion

$$\text{Cl}(C_n^r) \simeq \Sigma^2 \text{Cl}(C_{4r-n}^{3r-n}) = \Sigma^2 \text{Cl}(C_{n-2}^{r-1 \cdot (2r-n)})$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_8	$V^7 S^0$	S^1	S^1	S^3	*	*	*	*	*	*	*
C_9	$V^8 S^0$	S^1	S^1	$V^2 S^2$	*	*	*	*	*	*	*
C_{10}	$V^9 S^0$	S^1	S^1	S^1	S^4	*	*	*	*	*	*
C_{11}	$V^{10} S^0$	S^1	S^1	S^1	S^3	*	*	*	*	*	*
C_{12}	$V^{11} S^0$	S^1	S^1	S^1	$V^3 S^2$	S^5	*	*	*	*	*
C_{13}	$V^{12} S^0$	S^1	S^1	S^1	S^1	S^3	S^6	*	*	*	*
C_{14}	$V^{13} S^0$	S^1	S^1	S^1	S^1	S^3	S^4	*	*	*	*
C_{15}	$V^{14} S^0$	S^1	S^1	S^1	S^1	$V^4 S^2$	$V^2 S^4$	*	*	*	*
C_{16}	$V^{15} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^7	*	*	*
C_{17}	$V^{16} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^5	*	*	*
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*
C_{19}	$V^{18} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	*	*
C_{20}	$V^{19} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^3 S^4$	S^9	*
C_{21}	$V^{20} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	$V^6 S^2$	S^3	$V^2 S^6$	*
C_{22}	$V^{21} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	S^{10}
C_{23}	$V^{22} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^3	S^7
C_{24}	$V^{23} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	$V^7 S^2$	S^3	S^5
C_{25}	$V^{24} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^4 S^4$

Conclusion

$$\text{Cl}(C_n^r) \simeq \Sigma^2 \text{Cl}(C_{4r-n}^{3r-n}) = \Sigma^2 \text{Cl}(C_{n-2}^{r-1 \cdot (2r-n)})$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_8	$V^7 S^0$	S^1		S^3	*	*	*	*	*	*	*
C_9	$V^8 S^0$	S^1	S^1	$V^2 S^2$	*	*	*	*	*	*	*
C_{10}	$V^9 S^0$	S^1	S^1	S^1	S^4	*	*	*	*	*	*
C_{11}	$V^{10} S^0$	S^1	S^1	S^1	S^3	*	*	*	*	*	*
C_{12}	$V^{11} S^0$	S^1	S^1	S^1	$V^3 S^2$	S^5	*	*	*	*	*
C_{13}	$V^{12} S^0$	S^1	S^1	S^1	S^1	S^3	S^6	*	*	*	*
C_{14}	$V^{13} S^0$	S^1	S^1	S^1	S^1	S^3	S^5	S^7	*	*	*
C_{15}	$V^{14} S^0$	S^1	S^1	S^1	S^1	$V^4 S^2$	$V^2 S^4$	*	*	*	*
C_{16}	$V^{15} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^7	*	*	*
C_{17}	$V^{16} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^5	*	*	*
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*
C_{19}	$V^{18} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^5	*	*	*
C_{20}	$V^{19} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^3 S^4$	S^9	*
C_{21}	$V^{20} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	$V^6 S^2$	S^3	$V^2 S^6$	*
C_{22}	$V^{21} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	S^{10}
C_{23}	$V^{22} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^3	S^7
C_{24}	$V^{23} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	$V^7 S^2$	S^3	S^5
C_{25}	$V^{24} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^4 S^4$

Conclusion

$$\text{Cl}(C_n^r) \simeq \Sigma^2 \text{Cl}(C_{4r-n}^{3r-n}) = \Sigma^2 \text{Cl}(C_{n-2 \cdot (2r-n)}^{r-1 \cdot (2r-n)})$$

$r =$	0	1	2	3	4	5	6	7	8	9	10
C_8	$V^7 S^0$	S^1	S^1	S^3	*	*	*	*	*	*	*
C_9	$V^8 S^0$	S^1	S^1	$V^2 S^2$	*	*	*	*	*	*	*
C_{10}	$V^9 S^0$	S^1	S^1	S^1	S^4	*	*	*	*	*	*
C_{11}	$V^{10} S^0$	S^1	S^1	S^1	S^3	*	*	*	*	*	*
C_{12}	$V^{11} S^0$	S^1	S^1	S^1	$V^3 S^2$	S^5	*	*	*	*	*
C_{13}	$V^{12} S^0$	S^1	S^1	S^1	S^1	S^3	S^6	*	*	*	*
C_{14}	$V^{13} S^0$	S^1	S^1	S^1	S^1	S^3	S^5	S^7	*	*	*
C_{15}	$V^{14} S^0$	S^1	S^1	S^1	S^1	$V^4 S^2$	$V^2 S^4$	*	*	*	*
C_{16}	$V^{15} S^0$	S^1	S^1	S^1	S^1	S^1	S^3	S^7	*	*	*
C_{17}	$V^{16} S^0$	S^1	S^1	S^1	S^1	S^1	S^5	S^8	*	*	*
C_{18}	$V^{17} S^0$	S^1	S^1	S^1	S^1	S^1	$V^5 S^2$	S^3	S^8	*	*
C_{19}	$V^{18} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^5	*	*	*
C_{20}	$V^{19} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^3 S^4$	S^9	*
C_{21}	$V^{20} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	$V^6 S^2$	S^3	$V^2 S^6$	*
C_{22}	$V^{21} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	S^{10}
C_{23}	$V^{22} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^3	S^7
C_{24}	$V^{23} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	$V^7 S^2$	S^3	S^5
C_{25}	$V^{24} S^0$	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$V^4 S^4$