1. The voting power approach: Response to a philosophical reproach

Dan S. Felsenthal and Moshé Machover

Introduction

Despite its title, ‘The Voting Power Approach: Measurement without Theory’, Albert’s (2003) philosophical critique, which appeared in a previous issue of this journal, is actually directed against the theory of the measure of a priori voting power, based on the intuition of voting power as I-power. This theory,
founded by Penrose (1946, 1952), is presented in detail in our book (Felsenthal and Machover, 1998) and briefly outlined in Felsenthal and Machover (2000). As we shall see, Albert has his own reasons for avoiding the terms ‘theory’ and ‘measure’ in this connection; but we have no such reason and we shall speak of the ‘Penrose measure’ (using this term to refer also to its derivatives and refinements) and the ‘Penrose theory’.

A priori voting power is the component of actual (or a posteriori) voting power that voters derive solely from the decision rule itself – computed without regard to (or in ignorance of) all information about the personality of the voters (their specific interests and preferences, relations of affinity or disaffinity between them) – and the nature of the bills to be voted upon. Coleman (1971: 297) aptly describes it as ‘formal power as given by the constitutional rules of a collectivity’.

I-power is the notion of voting power as a voter’s degree of influence over the outcome – under a specified decision rule – of a division of a decision-making body: whether a proposed bill is approved or rejected. Albert does not seem to have any philosophical objection to the alternative, P-power, notion of voting power, which regards a decision rule as a simple cooperative game with transferable utility and conceptualizes voting power as a voter’s relative share in a fixed total payoff.

Albert has two fundamental philosophical objections to the Penrose theory. First, he claims that this theory is inapplicable to the real world because it cannot be used for purposes of prediction or explanation. Second, he alleges that the Principle of Insufficient Reason, which underlies the Penrose measure, is unsound. We shall rebut these two objections in the next two sections.

Is the Penrose theory applicable?

Albert spends considerable space arguing that the Penrose theory (or, as he insists on calling it, ‘the VP approach’) is not empirical. He could have saved himself the trouble: the theory is avowedly about a priori voting power; and, if ‘a priori’ means anything, it means ‘prior to or independent of experience; contrasted with “a posteriori” (empirical)’ (Cambridge Dictionary of Philosophy, 1995: 29). So much is uncontroversial. Nevertheless, as we shall show, this theory is applicable to the real world and does lead to empirically testable predictions.

What is somewhat eyebrow-raising is Albert’s rather extreme disparagement of, not to say hostility towards, non-empirical theories. This is made quite evident by his choice of terminology. On page 356 he tells us that ‘among the sciences, one must distinguish between formal sciences, such as
logic and mathematics, on the one hand, and factual sciences, such as physics or economics, on the other.' This choice of terminology is very tendentious: is the proposition $2 + 2 = 4$ or Fermat’s last Theorem any less factual than Gresham’s Law or the Law of Diminishing Marginal Utility? It seems that for Albert only empirical facts are really factual; the truths of mathematics, ‘even truths about numbers’, are, he asserts, ‘empty’, because they ‘do not tell us anything about the physical or social world. . . . At least according to the prevailing view, mathematics provides only a language for (some of) the factual sciences’ (pp. 356–7). Perhaps this is the prevailing view in certain doctrinaire philosophical circles; but most scientists are aware that mathematics provides not merely a language but also, at the very least, an indispensable deductive apparatus for various sciences.

The status of mathematics is relevant to Albert’s polemic, because of the following assertion he makes about the Penrose theory: ‘Viewed as a scientific theory, it is a branch of probability theory and can safely be ignored by political scientists (p. 351)’. The first half of this assertion is arguable. But the second half – if it has any connection at all to the first half – implies that probability theory as a whole, of which the Penrose theory is (allegedly) but a branch, can safely be ignored by political scientists. In our view, prudent political scientists should ignore this fundamentalist philosophical advice.

Because the Penrose theory is non-empirical, Albert not only prefers to refer to it as a mere ‘approach’ but would even deny it the right to speak of ‘measuring’: ‘Felsenthal and Machover . . . talk as if they were using a positive theory.6 For instance, . . . [they] repeatedly speak of “measuring” voting power. But this is a paradigmatic case of measurement without theory’ (p. 359). Measuring is presumably a prerogative of empirical science; hence the title of his paper. Yet pure mathematics – that non-factual science of empty truths – abounds with talk of ‘measuring’: one of Archimedes’ best-known works is On the Measurement of the Circle; and modern pure mathematics has an important branch (which, as it happens, encompasses probability theory) called ‘measure theory’.

Albert is much occupied with categorizing the Penrose theory: is it part of political science (he thinks it isn’t), or a branch of probability theory (he thinks it is), or perhaps political philosophy (he thinks that under an ‘alternative interpretation’ it may be). We think that it may partake of all three branches of knowledge – depending, of course, on how their boundaries are defined. But we are not really worried about this kind of demarcation dispute, beloved of certain taxonomically-minded philosophers of science. What we do wish to argue is that the Penrose theory is applicable and useful in a political and constitutional context.

Contrary to the impression Albert wishes to create, the Penrose theory
can and does lead to empirically testable predictions. Here is an example. One of the concepts of the theory is the a priori probability $A$ that a decision-making body acting under a given decision rule will adopt a bill rather than blocking it. $A$ is not directly observable, because in the real world decision-making is mediated by voters’ preferences and other behavioural factors. However, $A$ does have a definite effect on the actual propensity of the decision-making body to adopt proposed bills. We have shown (Felsenthal and Machover, 2001b) that the decision rule known as ‘qualified majority vote’ (QMV) prescribed by the Treaty of Nice for an enlarged 27-member EU Council of Ministers has reduced the value of $A$ – in other words, lengthened the a priori odds against a bill being adopted – to such drastic extent, compared with its past and current values, that the engine of diplomacy will have great difficulty overcoming this hidden but very real obstacle. On these grounds we predict in that, if the quota of the QMV rule for the enlarged Council of Ministers is not considerably lowered, the body will tend to get bogged down in immobilism. This is a definite prediction of an observable phenomenon, made on the basis of the Penrose theory.8

However, the main application of the Penrose theory – certainly its intended aim – is not as a predictive or descriptive tool but as a prescriptive normative one. Here it may be noted, that the main aim and intended application of game theory – a theory that Albert holds up for praise and emulation as truly scientific, in contrast to the ‘VP approach’ – is also normative. Although game-theoretic models are now used for explaining various empirical phenomena (such as evolutionary equilibria), the main purpose for which game theory was invented is as a normative guide for ‘rational behaviour’ in certain situations of conflict.9

Although the Penrose theory can also be used to prescribe rational behaviour in precisely the game-theoretic sense,10 its main prescriptive application is in the analysis and design of decision rules, especially as part of the constitutional design of a decision-making body. In this connection it is vital to focus on ‘formal [voting] power, as given by the constitutional rules of a collectivity’ (Coleman, 1971: 297), rather than on actual voting power. Thus, when the designers of QMV for the EU Council of Ministers assigned equal voting weights to member states with roughly equal population size – for example, France and Italy – they could not thereby equalize these members’ actual voting power; nor was this their intention. What they did, and evidently intended to do, was to equalize the component of these members’ voting power that derives solely from the decision rule itself – their a priori voting power.11 Similarly, when they assigned Italy greater voting weight than Spain (on the grounds that the former is more populous), they could not thereby guarantee that Italy would have greater actual power than Spain; nor could
this be their intention. What they were doing was to give Italy greater a priori voting power than Spain. But how much greater? Another important question is how to allocate weights under QMV so as to equalize the indirect a priori voting powers of all citizens of the EU (exercised through the political representatives whom the citizens elect). To answer questions such as these, one needs a sophisticated mathematical theory of a priori voting power. This is where the Penrose theory comes in.

Here we must correct a grossly mistaken allegation made by Albert. For some reason that is not clear to us, he attributes to the Penrose theory (generally or as expounded by us) the ‘premise . . . that fairness requires the adoption of voting rules that equalize the probability of each voter being decisive’. Against this he protests that, in joint-stock companies, ‘[i]t is usually not considered unfair if each stockholder has one vote per share, although this means that, under simple random voting, the voting power of shareholders with different shares in the company is different.’ Moreover, ‘[o]ne might argue . . . that . . . different stakes of voters must be taken into account. But this is problematic, because in a political context it could be argued that, for instance, land owners, tax payers, younger people, or parents have higher stakes than others’ (p. 362).

All this is a red herring. The Penrose theory is not concerned with laying down principles of fairness (nor are we, qua its exponents). Such value judgements are a matter for legal and political ethics. For example, it is generally accepted in parliamentary democracies that the a priori voting power – the voting power derived from the constitution – of all citizens ought to be as nearly equal as possible. This is what is normally meant by the slogan ‘One person, One vote’ (Note, by the way, that this does not imply equality of actual voting powers!). What the Penrose theory can prescribe is how to implement this equality. But it can also prescribe how to implement, as precisely as possible, a given unequal distribution of a priori voting powers, if that is desired.

The Principle of Insufficient Reason

The Penrose measure of voter \( v \)'s voting power (under a given decision rule) is the a priori probability of \( v \) being decisive; that is, the probability of the other voters being so divided that \( v \) is in a position to determine the outcome of the division.\(^{12}\)

Since we are concerned with a priori rather than actual voting power, we must go 'behind a veil of ignorance' and disregard any information about the personality of the voters (their specific interests and preferences, relations of affinity or disaffinity between them) and the nature of the bills to be voted
upon. The normal practice in the absence of all such behavioural and substantive information is to assign equal a priori probability to all possible ‘atomic events’ – in the present case, to all possible divisions of the set of voters into ‘yes’ and ‘no’ camps (If there are $n$ voters, the number of possible divisions is $2^n$). This is an application of the Principle of Insufficient Reason (PIR) of classic probability theory, which many authors (including Albert) attribute to Laplace (1749–1827), but which in fact goes back to Jacob Bernoulli’s (1654–1705) posthumous classic *Ars conjectandi* (1713).13

Citing Howson and Urbach (1993: Ch. 4), Albert claims that PIR ‘has been devastatingly criticized since the 19th century’ and must be rejected. This would of course undermine the Penrose measure and with it the whole of the Penrose theory. But Albert’s claim is, at the very least, quite misleading. PIR is indeed incoherent and may lead to contradiction when applied to infinite probability spaces.14 But it is quite safe and unobjectionable when applied to a finite probability space consisting of finitely many clearly distinguished indivisible ‘atomic’ events. In this special case, PIR is not rejected by the best critical authorities on the subject, including Keynes (1921) and the one cited by Albert himself, Howson and Urbach (1993).15 It is precisely such a safe application of PIR that is required for justifying the Penrose measure.

Apart from his misguided appeal to authority, Albert has one substantive argument against this use of PIR:

The example of voting nicely illustrates the inherent difficulties of the principle. The basic VP approach assumes simple random voting, which leads, for large constituencies, to an approximately normal distribution of yes votes. However, one could apply the principle of insufficient reason directly to the distribution of yes votes, assuming that this distribution is uniform. Which application of the principle is correct? If nothing is known about voter behavior, this is a matter of taste. (2003: 361)

This argument is fallacious because it turns the whole issue upside down. In order to discredit an application of PIR to a finite probability space (of $2^n$ atomic events), Albert first takes us to the infinite limit, where he is faced with the normal limit distribution. Then he invites us to wonder: why use PIR to choose the normal distribution? Why not choose a uniform distribution?

But in the Penrose theory PIR is applied not to the infinite limit distribution but to the finite case. Here the choice is quite clear. Suppose there are 20 voters. A uniform limit distribution would result if the event that exactly $k$ of them vote ‘yes’ had a priori probability $1/21$, independently of $k$. For example, the probability that all 20 vote ‘yes’ would be the same as the probability that only half vote ‘yes’ and the other half ‘no’. But the former event can happen in only one way, whereas the latter can occur in 184,756 different
ways, because the 10 ‘yes’ voters can be chosen in 184,756 different ways from among the 20. Suppose we had no behavioural information whatsoever about the 20 voters and knew nothing about the issue on which they are going to divide. If we had to bet, would it be rational to assign the same a priori odds to the event that they all vote ‘yes’ as to the event that exactly half vote ‘yes’? Of course it wouldn’t. The most rational choice would be to assign equal probability to all $2^{20} (= 1,048,576)$ possible divisions (atomic events). This is what PIR would prescribe, and it is also the position taken by modern information theory.

Contrary to Albert’s claim, the modern successor, and generalization, of PIR (in its legitimate uses) is not subjective Bayesianism. It is in fact the Principle of Maximal Information Entropy of modern information theory (see Shannon, 1948). There is nothing subjective at all about it. Subjective Bayesianism (which is rejected by many authors on the subject) depends on individual beliefs, not on objectively specified information. The Principle of Maximal Information Entropy is also widely used in the science of statistical mechanics (see Jaynes, 1968).

Concluding remarks

We found two of Albert’s criticisms of the Penrose theory to be worthy of detailed answers. First, we have refuted his charge that this theory is inapplicable to real-life situations; although it is an a priori theory, it does have both predictive power and prescriptive value. Second, we have argued that his wholesale rejection of the Principle of Insufficient Reason is unwarranted. Its use for certain finite spaces, as needed to justify the Penrose measure, is perfectly legitimate. All the rest of Albert’s critique consists of pedantic arguments about demarcation and the scientific status of non-empirical theories, which may be of interest to some philosophers but not to anyone seriously interested in studying and applying voting power.

2 The utility of the voting power approach

Dennis Leech

The voting power approach and power indices

From the foundation of the European Economic Community in 1958 to its first expansion in 1972, its six member countries used a system of qualified majority voting (QMV) for certain types of decisions. The voting weights
were as follows: France, West Germany and Italy had four votes each, Belgium and the Netherlands two votes, and Luxembourg one. From these figures one might assume that the smaller countries had a disproportionately large amount of influence in the Council of Ministers. For example, Luxembourg must have been overrepresented compared with West Germany because it had a quarter of its voting power with just over half of one per cent (0.57%) of its population. Alternatively, Luxembourg had one vote for its whole population of 310,000, whereas West Germany had only one vote for every 13,572,500 people. Of course, this disproportion was justified at the time on the grounds that Luxembourg was one of the six member states constituting the Community and the system of weighted voting in the Council of Ministers was chosen to ensure the representation of both states and populations.

In fact, however, Luxembourg had no voting power whatsoever over any decision taken by QMV. Since the threshold was set at 12 votes required to make a decision, it was mathematically impossible for Luxembourg’s one vote to be decisive. No matter how the other five voted, their combined total would never be equal to 11. Luxembourg therefore had zero voting power. This is a finding of much importance – not least to the citizens and government of Luxembourg, those of the other member countries, as well as political scientists studying the European Community. Moreover, it is not immediately obvious from a casual inspection of the weights. It can be understood only by using the voting power approach, which requires an analysis of the possible voting outcomes that can theoretically occur in terms of the capacity of each voter to decide the issue.

This example is well known in the voting power literature. The result was first pointed out by Brams and Affuso (1985) in a paper about power indices. However, it does not actually depend on the use of power indices, requiring no more technical equipment than simple arithmetic. It is sufficiently important and serious that a real institution could commit such an apparent error that I suggest this example alone demonstrates the utility of the voting power approach. That a sovereign member of the EEC was disenfranchised by the rules of qualified majority voting, despite all appearances to the contrary, should be widely known, if only out of historical interest. But the point is largely ignored in the literature on the Council of Ministers; a recent example is the important book by Hayes-Renshaw and Wallace (1997), which does not mention it despite containing a discussion of power indices.

It is easy to construct other examples of hypothetical voting systems that have similarly unfortunate properties by suitable choice of weights and decision rule. But there are also others in real institutions, not so well known
as the EEC example perhaps, that produce striking results. I will give two from the International Monetary Fund (IMF), another body where QMV is the constitutional basis of its decision-making (see Leech, 2002a). Member countries have weighted votes that are determined by their IMF quotas. The rules require that a country must cast all its votes in a bloc and therefore the system is one of weighted majority voting. The Executive Board has 24 members, some of whom are appointed by their governments (currently these are the USA, Japan, Germany, the UK, France, Saudi Arabia, Russia and China). The other directors are elected by groups of member countries arranged in so-called constituencies. Elected directors are able to cast a bloc of votes equal to the combined weights of the countries that elected them; in fact they are required to do so and cannot split them, and therefore the system formally resembles one of winner-take-all. In many constituencies the weights are such that one member country (whose representative is invariably elected the director) has more than half the weight and is therefore formally a dictator in voting terms while the other members in the constituency are powerless. This is the case in the constituencies represented by Canada, Brazil, India, Italy and Switzerland. The power analysis in these cases is obvious. However, in two constituencies, each with eight members, the structure of power is not immediately obvious and a voting power analysis reveals some surprising results (see Table 1).16

The Spanish-led constituency currently elects the Spanish representative to sit on the Executive Board as the representative of the eight countries in the group. A voting power analysis reveals that five members are completely powerless and that the three biggest members (Spain, Mexico and Venezuela) are all equally powerful. That is, it turns out that none of the five Central

<table>
<thead>
<tr>
<th>‘Spanish-led’ constituency</th>
<th>Votes</th>
<th>‘Nordic/Baltic’ constituency</th>
<th>Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costa Rica</td>
<td>1,891</td>
<td>Denmark</td>
<td>16,678</td>
</tr>
<tr>
<td>El Salvador</td>
<td>1,963</td>
<td>Estonia</td>
<td>902</td>
</tr>
<tr>
<td>Guatemala</td>
<td>2,352</td>
<td>Finland</td>
<td>12,888</td>
</tr>
<tr>
<td>Honduras</td>
<td>1,545</td>
<td>Iceland</td>
<td>1,426</td>
</tr>
<tr>
<td>Mexico</td>
<td>26,108</td>
<td>Latvia</td>
<td>1,518</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>1,550</td>
<td>Lithuania</td>
<td>1,692</td>
</tr>
<tr>
<td>Spain</td>
<td>30,739</td>
<td>Norway</td>
<td>16,967</td>
</tr>
<tr>
<td>Venezuela</td>
<td>26,841</td>
<td>Sweden</td>
<td>24,205</td>
</tr>
<tr>
<td>Total</td>
<td>92,989</td>
<td>Total</td>
<td>76,276</td>
</tr>
<tr>
<td>Threshold</td>
<td>46,495</td>
<td>Threshold</td>
<td>38,139</td>
</tr>
</tbody>
</table>

Table 1 Voting weights in two IMF constituencies
American members of the group could ever swing a vote to reach the threshold of 46,495 and that the ability of each of the biggest members to do so is equal. The other example is from the so-called Nordic/Baltic constituency, which operates somewhat differently from the others by choosing its executive director on a rota basis rather than by election. However, when it comes to a weighted vote being taken in the group, it turns out that one member, Estonia, has zero power: its 902 votes could never be decisive.17

These results are of considerable interest in pointing to deficiencies in the current voting arrangements in the IMF and they demonstrate again the utility of the voting power approach. Moreover, as with the example of the EEC with which I started, the results are in the nature of solid mathematical facts and do not rely on any behavioural assumptions whatsoever. In particular they do not rely on power indices or any ideas about probabilistic voting. Alternatively, if we insist on using the language of power indices, we can say that they are the same whatever power index we use: both the Shapley–Shubik and the normalized Banzhaf index would give Spain, Mexico and Venezuela one-third each and the other five countries zero. Estonia gets zero on either index (although the powers of the other seven countries will be sensitive to the choice between the Shapley–Shubik and the Banzhaf index).18

I have highlighted these examples to illustrate the voting power approach because they are simple, in that they give completely unambiguous results that are far from obvious when we look at the data. Whether we wish to call this approach political science or not, the results are empirical facts that are unassailable and cannot be ignored by political scientists. However, the voting power approach does not only deal in clear-cut results such as these; its more common use is in the analysis of weighted voting systems in which differences in the influence of different voters arise in complex ways from their different weights and the decision rule. The voting power approach aims to quantify this by means of power indices, defined in certain ways for stated purposes and on certain assumptions. These are the subject of Albert’s critique.

Albert’s misplaced critique: Confusing a progressive research agenda with intolerable ambiguity

Professor Albert directs his attack (Albert, 2003) on his own definition of the voting power approach. His proximate target is the paper by Leech and Machover (2003) – of which he was the discussant when it was presented at a conference on ‘European Governance’ in Saarbrücken in October 2002 – which uses power indices to study the effect of changing the QMV threshold
with equitable weights. That paper is an exercise in the use of a priori power indices as a tool for analysing the properties of a voting system – and therefore as an aid for designing institutions whose properties conform to intended democratic principles.

Albert takes the a priori power indices approach as if it were synonymous with the voting power approach in general. Since, as I have shown in the previous section, these are not the same, he is, to an extent, attacking a straw man. Moreover, his principal arguments are not new, having been made many times before, as much by advocates of power indices as by critics. In doing this he is following in a tradition, mostly associated with the names of Garrett and Tsebelis, of attacking the voting power approach by attacking a priori power indices (e.g. Garrett and Tsebelis, 2001).

The substance of Albert’s complaint is that voting power indices are calculated on the assumption of what he calls simple random voting: that voters vote randomly and independently of each other and are equally as likely to vote for as against any proposal. This assumption is, of course, patently false as a general description of any actual voting behaviour. If actual voter behaviour is conceived in probabilistic terms, with respect to randomly selected issues, then simple random voting is an inadequate model because it fails to take account of the likelihood that certain voters tend to vote the same way whereas others tend to oppose each other on average. But this distinction between actual behaviour and simple random voting cannot be the basis of a critique of power indices, let alone the voting power approach, since almost every writer on the subject has also made it. I quote below from the seminal works by Shapley and Shubik, Banzhaf and Coleman, to emphasize the point.

The values . . . do not take into account any of the sociological or political superstructure that almost invariably exists in a legislature or policy board. They were not intended to be a representation of present day ‘reality’. It would be foolish to expect to be able to catch all the subtle shades and nuances of custom and procedure that are to be found in most real decision-making bodies. Nevertheless, the power index computations may be useful in the setting up of norms or standards, the departure from which will serve as a measure of, for example, political solidarity, or regional or sociological factionalism, in an assembly. To do this we need an empirical power index, to compare with the theoretical. (Shapley and Shubik, [1954] 1988: 46)

It is important to recognise that this technique measures the voting power of the individual which is inherent in the rules governing the voting system and the distribution of population, and does not reflect the actual ability that any given individual voter has in a particular election to affect the outcome. The latter would depend to some extent on factors which are not inherent in the system, such as
the relative power of the political parties in different geographical areas, and conditions which may be peculiar to the voter himself (e.g. whether as a sign of protest he decides to vote for a minority party candidate who has no chance of winning). Thus, a critical distinction must be drawn between inequalities in voting power which are built into the system (e.g. the old county unit system in Georgia or the distribution of votes in the Electoral College) and those which result either from the free choice among citizens as to how they use their voting power (e.g. the political impotence of a Republican in a solidly Democratic state) or from factors outside of the legal rules governing the process (e.g. voter intimidation, weather, the televised prediction of election results, etc.). Concededly, these and other external factors may affect a citizen’s ability to affect the outcome of any particular election. The voting power measure here is that inherent in the system and necessarily represents an average of a voter’s effectiveness in a large number of equally likely voting situations. However, it is only with respect to those inequalities which result from the rules of a particular system of voting on which we may properly focus attention in determining the basic ‘fairness’ of the system itself. (Banzhaf, 1968: 308)

By the device of counting each partition of the collectivity once, and adding the number of partitions in obtaining measures of power, it is implicitly assumed that each member has equal probability of voting for or against a collective action . . . This is appropriate for the analysis of formal power as given by a constitution, that is, for an analysis of organizational rules. It does not, however, provide a basis for behavioural prediction of the collectivity’s action, when further information exists about the members. In particular, the two assumptions made in the analysis of formal power, equal probabilities of positive and negative votes by each member, and independence of votes among members, may be empirically investigated, and the collectivity’s action predicted by the use of such information. (Coleman, 1971: 297)

I have quoted at length from these seminal papers to show that their authors were fully aware of the distinction between a priori power indices and measures of empirical power and were at pains to discuss it. Albert claims that the voting power approach is completely ambiguous about whether it is political science or political philosophy and ‘such ambiguity is not to be tolerated’. But the issue here is not ambiguity but the need for more research. All the quotations above show that proponents of power indices conceive of two kinds of measures of power: a priori power indices; and empirically based power indices that reflect the behaviour or preferences of voters. These two types of indices are fundamentally different entities that serve different purposes. The former enable us to analyse the properties of voting systems in purely constitutional terms and therefore are in a sense a tool of applied political philosophy that can tell us about things such as equity. They have a real use in helping to solve normative problems such as how to design
weighted voting systems as used by intergovernmental bodies. On the other hand, the latter rely on observed behaviour and belong to positive political science. They have been the subject of much less research than the former.

We can illustrate the point by thinking about the EU Council of Ministers. An understanding of where power lies requires us to take account of many relevant factors: the political complexions of governments, the Paris–Berlin axis, the commonality among the Benelux countries, the Nordic or Mediterranean members, the small states versus the large states, new Europe versus old Europe, the Eurozone, etc. Such political science would benefit from being able to use empirical power indices that take account of such factors as a tool of analysis.

But from the point of view of the design of the formal voting system in a Union that is expanding, with the admission of new members being quite a normal process, it would clearly be inappropriate to base constitutional parameters such as voting weights on such considerations. That might lead to, for example, allocating France smaller voting weight because otherwise its tendency to vote with Germany would give it more power, and the UK larger weight because of its tendency to independence. That would appear arbitrary and would fail to provide a guide for what the votes of new entrants should be. Far better to allocate the voting weights on the basis of general philosophical principles that can be seen to apply equally to all countries and citizens, to new members as well as old ones. A priori power indices are useful in this.

Thus, the voting power approach gives rise to different power indices for different purposes and there is a substantial research agenda to develop and study them.

The assumption of simple random voting has a different role in the two types of indices, as the quotation from Coleman indicates. It is conventionally a part of the definition of an a priori power index – although, in fact, assumptions about probabilistic voting behaviour are not fundamental to them at all. On the other hand, an empirical power index must be based on observed behaviour or preferences, and the use of simple random voting is merely a convenient but inferior first approximation. Coleman was acutely aware of this point and his work (especially his 1971 paper but also his 1973 paper ‘Loss of Power’) contains several pointers to directions for needed future research into empirical power measurement.

It is not actually necessary to assume simple random voting in order to define a priori power indices. All that is required is a consideration of all possible outcomes that can theoretically occur, taking into account that each voter has the right to choose how to vote. Then, as Banzhaf (1965) explained:
The measure of a legislator’s power is simply the number of different situations in which he is able to determine the outcome. . . . The ratio of the power of legislator X to the power of legislator Y is the same as the ratio of the number of possible voting combinations of the entire legislature in which X can alter the outcome by changing his vote to the number of combinations in which Y can alter the outcome by changing his vote.

Thus, voting power can be defined in terms of the rights of individual voters: we count up each outcome because each voter has the basic right, as a member of the institution, to exercise choice. There is no need, therefore, to invoke the principle of insufficient reason to justify simple random voting. A priori voting power can be defined on a more fundamental level in terms of voter sovereignty.

Conclusions

I have argued, first, that the voting power approach is basically a way of thinking about voting systems in terms of outcomes. The voting power approach is capable of discovering important empirical facts about power, of which I have given three examples. Secondly, the neglect of the voting power approach by political scientists means that these facts are often missed. Thirdly, the voting power approach is not the same as the use of power indices.

Albert’s critique is mistaken in two ways. First, it is actually an attack on power indices, though he claims to be criticizing the voting power approach in general. Secondly, his insistence that there can be only one type of power index for all purposes leads him to claim that this causes ambiguity, which ‘is not to be tolerated’. But there can be different types of power indices that have different uses: a priori power indices are used to address normative questions connected with the design of voting systems (for example, Leech, 2002b; Leech and Machover, 2003); empirically defined indices are used to answer positive questions about actual voting power.

The voting power approach is an emerging field of research with its own inherent research agenda. Most serious research in it so far has focused on a priori power indices, but that does not mean that there is not a need for further research on developing empirically valid approaches to the measurement of voting power.
The voting power approach: A theory of measurement

Christian List

Introduction

Power indices have received increasing attention in political science, especially in the field of European Union politics. They are frequently used for investigating, first, the present distribution of voting power among EU member states in the Council of Ministers and the European Parliament, and, second, the effect of proposed institutional changes or EU enlargement on that distribution (e.g. Felsenthal and Machover, 1997; Nurmi, 1997, 2000; Nurmi and Meskanen, 1999; Dowding, 2000; Aleskerov et al., 2002). In a recent article, however, Max Albert (2003: 351) argues that the theory of power indices ‘should not . . . be considered as part of political science’, and further that, ‘[v]iewed as a scientific theory, it . . . can safely be ignored by political scientists’ (p. 351). His argument rests on a particular diagnosis of the theoretical status of power indices. The theory of power indices, Albert argues, is not a positive theory, i.e. not one that has falsifiable implications. Rather, he suggests, depending on the interpretation, the theory is either an empirically vacuous branch of probability theory or an unconvincing branch of political philosophy. In either case, the theory ‘has no factual content and can therefore not be used for purposes of prediction or explanation’ (p. 351).

I seek to re-examine the theoretical status of power indices and to explain why, in my view, it would be unwise for political scientists to ignore such indices. I agree with Albert on what the theory of power indices is not. It is not, by itself, a positive theory. But I disagree with him on what it is. I suggest that, in terms of its theoretical status, the theory of power indices is similar to the theory of inequality indices. The theory of inequality indices is not, by itself, a free-standing theory. Rather, it is a theory of measurement that supplements other social-scientific theories. An inequality index is a statistical measure for summarizing certain properties of a given income (or other) distribution across a population. Inequality indices can thus supplement any theory that refers to such distributions, whether that theory is positive or normative. Analogously, the theory of power indices is a theory of measurement that supplements other social-scientific theories. A power index is a statistical measure for summarizing certain properties of a given voting game, as defined below. Power indices can thus supplement any theory that refers to such voting games, particularly cooperative game theory and its applications to modelling political institutions.
Power indices as statistical measures on the set of voting games

The general definition clarifies that power indices are statistical measures on the set of voting games (e.g., Laruelle and Valenciano, 2001). A voting game is a pair $< N, v >$, where $N = \{1, 2, \ldots, n\}$ is a set of players and $v$ a function mapping each subset of $N$ (a coalition) to either 0 (non-winning) or 1 (winning), such that:

(i) $v(\emptyset) = 0$ (the empty coalition is non-winning) and $v(N) = 1$ (the coalition of all players is winning);\(^{21}\)

(ii) there exists at least one subset $S \subseteq N$ such that $v(S) = 1$ (there is at least one winning coalition);

(iii) for all subsets $S, T \subseteq N$, $S \subseteq T$ implies $v(S) \leq v(T)$ (a superset of a winning coalition is also winning);

(iv) for all subsets $S \subseteq N$, $v(S) + v(N \setminus S) \leq 1$ (for any partition of the set of players into two disjoint coalitions, at most one is winning).\(^{22}\)

Each $n$-player voting game represents a particular voting procedure in an $n$-member electorate. For example, simple majority voting or unanimity voting in a 100-member electorate each corresponds to a particular 100-player voting game. Let $V_n$ denote the set of all logically possible $n$-player voting games. Then $V_n$ can be interpreted as the set of all logically possible (binary) voting procedures in an $n$-member electorate.\(^{23}\)

Now a power index is a function $\Phi$ (with domain $V_n$ and co-domain $\mathbb{R}^n$) that maps each $n$-player voting game to a vector of real numbers, $< p_1, p_2, \ldots, p_n >$, called a power profile. For each $i$, $p_i$ is interpreted as the voting power of player $i$.

The Penrose–Banzhaf (PB) index and the Shapley–Shubik (SS) index, discussed by Albert, are instances of such functions:

- PB: $\Phi_{PB}(< N, v >) := < p_1, p_2, \ldots, p_n >$, where
  
  for each $i$, $p_i := \frac{1}{2^{n-1}} \sum_{S \subseteq N : i \in S} \left( v(S) - v(S \setminus \{i\}) \right)$;

- SS: $\Phi_{SS}(< N, v >) := < p_1, p_2, \ldots, p_n >$, where
  
  for each $i$, $p_i := \sum_{S \subseteq N : i \in S} \frac{(s-1)!(n-s)!}{n!} \left( v(S) - v(S \setminus \{i\}) \right)$.

(For each $S \subseteq N, s := |S|$.)

Each index can be interpreted in multiple ways. For the PB index, we say that player $i$ is pivotal for a particular coalition if $i$'s leaving that coalition
turns it from a winning to a non-winning one. The PB index for each \( i \) can then be interpreted as the proportion among all logically possible coalitions for which player \( i \) is pivotal. For the SS index, consider all \((n!)\) logically possible sequences in which the \( n \) players can join a coalition one-by-one. We say that player \( i \) is *pivotal* for a particular sequence if \( i \)'s joining the coalition of all players preceding \( i \) in the sequence turns that coalition from a non-winning to a winning one. The SS index for each \( i \) can then be interpreted as the proportion among all logically possible such sequences for which player \( i \) is pivotal. Other interpretations of the indices are possible, e.g. in terms of players’ probabilities of being pivotal. But, although such interpretations help our intuitive understanding of a given power index, they are not definitions of the index. A more precise way to characterize a particular index is to state a set of axioms – minimal conditions on summarizing voting power – such that the given index is the unique function \( \Phi \) satisfying these axioms (Laruelle and Valenciano, 2001).

A power index is thus a statistical measure for summarizing each logically possible voting game into a corresponding summary statistic, namely a power profile across players. As each possible voting procedure in the Council of Ministers or the European Parliament (including relevant weights) corresponds to a particular voting game, a power index can serve as a statistical measure for summarizing certain procedural features of such voting procedures *taken in isolation*.

**The analogy with inequality indices**

To illustrate the usefulness of such statistical measures, consider the example of an inequality index. An *inequality index* is a function that maps each logically possible income (or other) distribution across a population into a single quantity: the level of inequality. Prominent indices are the Gini and Atkinson indices, but others have been discussed (Sen, 1997). Just as a power index summarizes each voting game into a single summary statistic (the power profile), an inequality index summarizes each income (or other) distribution into a single summary statistic (the level of inequality). Power indices and inequality indices summarize different items, and thus the resulting summary statistics have different interpretations. But the *theoretical status* of both kinds of index is similar. They are both functions aggregating relatively complex items into less complex summary statistics, and they can thus supplement any theory requiring such statistics.24

In the case of inequality indices, the resulting summary statistics are known to be useful from normative and positive perspectives. Normatively, ranking alternative socioeconomic policies in an order of desirability may
involve assessing the level of inequality under each policy, which requires using an inequality index. Positively, the level of inequality, measured by the Gini index, has been shown to be a predictor of several phenomena. For example, inequality of land distribution correlates negatively with the stability of democracy (e.g. Russett, 1968), and income inequality correlates negatively with voting turnout (e.g. Goodin and Dryzek, 1980).

In the case of power indices, the generated summary statistics may be relevant for normatively evaluating alternative voting procedures (or voting weights) in a given context. Although the distribution of voting power is unlikely to be the only normatively relevant consideration here, it is plausibly one of several such considerations (others being the avoidance of stalemate or the consistency of voting outcomes). Most of the recent applications of power indices to EU politics fall into this normative category. Power indices are used for evaluating alternative institutional arrangements in the EU and the effects of potential changes, and sometimes for making recommendations on how to equalize voting power across member states or across EU citizens.

The potential of using power indices in positive research, by contrast, has been largely unexplored so far. So Albert’s complaint that power indices are disconnected from positive research is correct to the extent that we have not yet seen much evidence of their usefulness in positive research. But there is no reason why power indices cannot in principle be used in such research too. Like inequality, voting power might plausibly serve as a regressor in models of certain empirical phenomena. For instance, it is conceivable (though still an untested hypothesis) that voting power might affect decision outcomes: policies preferred by agents with greater voting power might prevail more often than ones preferred by agents with less voting power. Similarly, the distribution of voting power might conceivably affect the dynamic of decision processes and perhaps the nature of deliberation in a collectivity: if there are significant inequalities in voting power, certain agents might frequently be agenda-setters and others might be marginalized. There are clearly avenues for positive research here. The results, to be sure, are open.

The informational poverty of power indices

Albert might grant that power and inequality indices are similar in that they are both statistical measures for summarizing certain items. But he might argue that their difference lies in the fact that inequality indices are useful such measures whereas power indices are not. Following the claims in his article, he might argue that inequality indices are useful because they capture certain social-scientifically relevant properties of the items they summarize, whereas power indices are not useful because they capture only very abstract, and
social-scientifically detached properties of the items in their domain: ‘The
definition of voting power . . . is disconnected from any positive theory and,
therefore, useless for purposes of political science’ (Albert, 2003: 359–60).

In particular, Albert criticizes the ‘assumption of simple random voting’
underlying power indices. In terms of the informal interpretation of the PB
and SS indices offered above, Albert’s point is a critique of the method of
‘brute counting’ across all logically possible coalitions (in the PB case) or
across all logically possible sequences (in the SS case), without considering
any potentially relevant facts on how likely it is that each such coalition or
sequence will arise. For instance, if a player’s voting power stems solely from
his or her being pivotal for coalitions that are unlikely to arise (e.g. ones
between libertarian and Marxist players), then his or her alleged voting power
seems a vacuous quantity. In short, the PB and SS indices are informationally
poor. By focusing solely on the formal structure of the voting game and not
the players’ behaviour, they screen out potentially relevant information.

This point is forceful, but we should be clear about what follows from it.
First, the fact that standard power indices are sensitive exclusively to the
formal structure of a voting game may sometimes be a virtue rather than a
vice. For some normative purposes, certain behavioural facts about the players,
such as their preferences, might be deemed normatively irrelevant. Veil of
ignorance arguments are based on this view. Albert criticizes such arguments,
but I think that the best response here is to point out that there exist several
influential normative theories that make use of veil of ignorance arguments
– whether or not one endorses them (e.g. Rawls’s, Harsanyi’s and Buchanan’s
theories) – and such theories can thus employ power indices as methodo-
logical tools. On the other hand, whether or not the informational restrictions
of standard power indices impair their usefulness in positive research remains
to be seen.

Second, it is conceivable that, for at least some purposes (whether norma-
tive or positive), the informational restrictions do pose significant limitations.
We might be interested, for instance, not in the proportion of logically possible
coalitions or sequences for which a given player is pivotal, but rather in the
proportion of realistically feasible such coalitions or sequences. Once we recog-
nize this point, as Albert does, one response might be to pursue Albert’s route
and to abandon power indices for the purposes of political science. But there
exists a more constructive route: namely not to abandon but rather to extend
the theory of power indices. Nurmi (2000) explains how this can be done. If
we assume that not all logically possible coalitions, but only some specific ones,
are likely to arise, we can easily accommodate this behavioural assumption
in the construction of a power index. In the definition of the PB and SS indices,
we simply need to replace summation over all logically possible coalitions S


\[ \subseteq N(\text{such that } i \in S) \text{ with summation over all coalitions } S \subseteq C(\text{such that } i \in S), \]
where \( C \) is the set of those coalitions that are assumed to be feasible. As an illustration, Nurmi (2000: 368, Table 3) computes the modified SS index for the Council of Ministers under the assumption that only four particular coalitions between member states are feasible (e.g. Franco-German, Mediterranean, Benelux, Neutral-plus-Nordic). Nurmi concludes that ‘the criticism of the power index studies that is based on the equiprobability of coalitions assumption misses the point in so far as various kinds of player groupings can be modelled using the same apparatus’. Formally, all that such an extension requires is defining power indices on a domain that is richer than the one traditionally used. Such a richer domain might for instance be the Cartesian product of \([\text{the set of all logically possible } n\text{-player voting games}]\) and \([\text{the set of all logically possible sets } C, \text{ as just defined}]\).

Again, the analogy with inequality indices is instructive. Standard methods of inequality measurement are often criticized for their narrow focus on income. Just as power indices screen out certain information, so inequality indices, applied to just one attribute such as income, screen out potentially relevant information, for instance about each person’s capacity to convert income into welfare. Someone with a medical condition might require more income to attain a particular welfare level than someone without that condition, and therefore what superficially seems like an equal distribution (in terms of income) might actually be an unequal one (in terms of welfare) (for a famous discussion, see Sen, 1980). But it would be unwise, as a consequence, to abandon inequality indices for social-scientific purposes. Rather, a more promising route (and one pursued by many welfare economists) is to extend the theory of inequality indices and to construct indices that are sensitive to a richer information set. For example, multi-attribute inequality indices have been developed to meet this demand (e.g. Koshevoy and Mosler, 1997; Tsui, 1999).

So, power indices and inequality indices can each be defined on informationally poor domains as well as on informationally rich ones, depending only on the required social-scientific application and on the amount of information that is available.

**Conclusion**

I have invoked the analogy with inequality indices to illustrate why Albert’s conclusion – that political scientists can safely ignore power indices – does not follow from his diagnosis of the theoretical status of these indices. The premises concerning theoretical status that Albert uses to support his conclusion seem to be met equally by the theory of inequality indices, and
yet, I think, we would not conclude that social scientists can afford to ignore inequality indices. The fact that something is not a free-standing (positive or normative) theory, but ‘merely’ a statistical measure, does not undermine its usefulness (for positive or normative purposes, respectively). Something may be useful precisely because it is a statistical measure.

Just as inequality indices usefully supplement theories that refer to income (or other) distributions, so power indices can play a potentially useful role in theories that refer to voting games. There is no doubt that our methodological toolbox would be poorer without inequality indices. Power indices are a more recent addition to that toolbox and have had less time to prove their value. But throwing them out at this point seems premature.

Notes

1 The Penrose measure has been mistakenly attributed to Banzhaf (1965) and is often referred to as the ‘absolute Banzhaf index’. The most important derivative of the Penrose measure is the relative (or normalized) Banzhaf index. An important refinement is the pair of measures – of a voter’s power to prevent action and to initiate action, respectively – defined by Coleman (1971).

2 For a more detailed discussion, see Felsenthal and Machover (1998: 19ff, 105ff, 112); Felsenthal and Machover (2004).

3 For detailed explanation of the I-power/P-power distinction, see Felsenthal and Machover (1998: 35ff, 171ff). For a summary see, for example, Felsenthal and Machover (2001a) or Machover (2000).

4 He also mentions more pragmatic objections to the Penrose measure, made by other authors; but he brushes them aside as playing into the hands of proponents of the Penrose theory by taking the latter too seriously.

5 Non-empirical truths can lead to empirically testable predictions. Consider, as a very simple example, the theorem that $2 + 2 = 4$. It asserts an a priori truth about abstract ideal entities (numbers). But it leads to the prediction that, if there are two philosophers ensconced in an ivory tower, and two other philosophers join them, then – unless they bore one another to death or procreate – there will be four philosophers in the tower.

6 Albert prefers the term ‘positive’ to the more usual ‘empirical’: it sounds so much more . . . well, positive!

7 This is the power of the collectivity to act, defined by Coleman (1971).

8 Other theories may also lead to a similar prediction, but the Penrose theory is arguably the simplest to do so. In any case, the issue here is not how to choose between the Penrose theory and rivals but whether or not it leads to any testable predictions.

9 Thus Morgenstern ([1949] 1968: 303): ‘The initial problem of the theory of games was to give precision to the notion of “rational behavior”.’

10 See Felsenthal and Machover (1998: 45) for a ‘vote-buying game’ in which an outsider stands to gain or lose a unit of transferable utility, depending on whether a given bill is adopted or blocked by a decision-making body. The
outsider knows the decision rule but has no behavioural information about the voters. The vote of one of the members is offered for sale. How much would it be rational for the outsider to pay for it? The answer is provided by the Penrose power of the seller.

11 Equality of a priori voting power is thus analogous to equality of rights or equality before the law; it is a formal rather than an empirical equality, which does not produce equality of actual attainment, nor is it intended to do so.

12 This probability is closely and simply connected with the a priori probability of \( v \) being successful; that is, of the outcome going the way \( v \) votes. For details see Felsenthal and Machover (1998 or 2000).

13 For a detailed critical discussion, see Keynes (1921).

14 According to PIR, each point of an infinite probability space should be assigned probability 0. If the space is denumerably infinite, this would imply that the whole space must also have probability 0, which is absurd. In the case of a continuous random variable \( X \), PIR is usually interpreted as assigning to \( X \) a uniform distribution in some finite interval. But if we transform \( X \) to \( Y = f(X) \), where \( f \) is a one-to-one continuous function, then by the same token \( Y \) should presumably also have a uniform distribution. If \( f \) is not linear, this leads to contradiction.

15 See also the more recent Howson (2000: 81–6).

16 Four other constituencies have a member whose voting weight is dominant while being just short of an absolute majority. These are the constituencies of Belgium (which has over 41% of the weight), the Netherlands (49%), Australia (45%) and Argentina (49%). These countries are not strictly dictators and the voting power approach does not give exact results without the use of power indices.

17 The voting figures are taken from the IMF Annual Report (IMF, 2003). I have assumed decisions are taken by simple majority voting – that is, with reference to a threshold of 50% of the total weight plus one – in both these analyses. The results have been obtained by computer search to analyse all possible voting outcomes. The details are available from me on request, although, since the computer program used is accessible from my web page, interested readers can confirm these results for themselves.

18 I do not consider such matters as the choice of an a priori power index here because they have been extensively discussed elsewhere. See, for example, Felsenthal and Machover (1998).

19 It would be interesting to know how Professor Albert thinks this should be done.

20 I thank Simon Hix for his invitation to write this response, and Robert Goodin for his helpful suggestions.

21 The condition \( v(N) = 1 \) is not strictly necessary, because it is already implied by the conjunction of \( v(\emptyset) = 0 \) and (ii), (iii) and (iv) below.

22 Technically, a voting game is a *simple superadditive game*.

23 Under this interpretation, conditions (i), (ii), (iii) and (iv) are minimal consistency conditions on such voting procedures.

24 Indeed, power indices and inequality indices can even be usefully combined to obtain a summary measure of *inequality of voting power*. Using a power index we can assign to each voting game a corresponding power profile, and using an inequality index we can then assign to each such power profile a
corresponding summary statistic capturing the level of inequality of voting power under the given voting game.

Although some standard power indices can be interpreted in terms of random voting, note that this is an interpretation and not part of their definition. Other non-probabilistic interpretations can be given (such as the ones in the second section of my contribution above). Thus these power indices are not strictly speaking based on an assumption of random voting.

References


**About the authors**

**Dan S. Felsenthal** is Emeritus Professor in the Department of Political Science at the University of Haifa, 31905 Haifa, Israel; he is also co-director of the Voting Power and Procedures Project, at the Centre for the Philosophy of Natural and Social Sciences (CPNSS), London School of Economics and Political Science, UK.

Fax: +972 4 8257785
E-mail: dfelsenthal@poli.haifa.ac.il

**Dennis Leech** is a Reader in the Department of Economics at the University of Warwick, Coventry CV4 7AL, UK; he is also co-director of the Voting Power and Procedures Project, at the Centre for the Philosophy of Natural and Social Sciences (CPNSS), London School of Economics and Political Science, UK.

Fax: +44 247 652 3032
E-mail: d.leech@warwick.ac.uk

**Christian List** is a Lecturer in Political Science in the Department of Government at the London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK; he is also a postdoctoral fellow in the Research School of Social Sciences at the Australian National University, Canberra, ACT 0200, Australia.

E-mail: c.list@lse.ac.uk

**Moshé Machover** is Emeritus Professor in the Department of Philosophy at King’s College, London, Strand, London WC2R 2LS, UK; he is also co-director of the Voting Power and Procedures Project, at the Centre for the Philosophy of Natural and Social Sciences (CPNSS), London School of Economics and Political Science, UK.

Fax: +44 20 7848 2270
E-mail: moshe.machover@kcl.ac.uk