

DONALDSON'S QUESTION

Question 0.1. *Suppose J is an almost complex structure on a compact 4-manifold M . If J is tamed by a symplectic form ω , is there a symplectic form compatible with J ?*

To our knowledge, only a few cases are confirmed:

- (1) When M is $\mathbb{C}\mathbb{P}^2$ (Gromov [3], Taubes [5]);
- (2) When J is integrable (Li-Z. [4]).

There is an approach via the symplectic Calabi-Yau equation and the continuity method. The openness is proved in [1], and some closedness results are obtained in [7, 6]. In [2], we give a reformulation of Question 0.1 in terms of some pointwise estimates.

In the language of J -tamed (compatible) symplectic cones

$$\mathcal{K}_J^{t(c)} = \{[\omega] \in H^2(M; \mathbb{R}) \mid \omega \text{ tames (is compatible with) } J\},$$

Question 0.1 is simply stated as:

Question 0.2. *When M has dimension 4, if $\mathcal{K}_J^c = \emptyset$, is $\mathcal{K}_J^t = \emptyset$ as well?*

REFERENCES

- [1] S. K. Donaldson, *Two-forms on four-manifolds and elliptic equations*, Inspired by S. S. Chern, 153–172, Nankai Tracts Math., 11, World Sci. Publ., Hackensack, NJ, 2006.
- [2] T. Draghici, T.J. Li, W. Zhang, *Symplectic forms and cohomology decomposition of almost complex 4-manifolds*, arXiv:0812.3680, to appear in IMRN, doi:10.1093/imrn/rnp113
- [3] M. Gromov, *Pseudoholomorphic curves in symplectic manifolds*, Invent. Math. 82 (1985), no. 2, 307–347.
- [4] T.J. Li, W. Zhang, *Comparing tamed and compatible symplectic cones and cohomological properties of almost complex manifolds*, submitted, arXiv:0708.2520.
- [5] C. Taubes, *SW \Rightarrow Gr: from the Seiberg-Witten equations to pseudo-holomorphic curves*, J. Amer. Math. Soc. 9 (1996), no. 3, 845–918.
- [6] V. Tosatti, B. Weinkove, *The Calabi-Yau equation, symplectic forms and almost complex structures*, arXiv:0901.1501.
- [7] V. Tosatti, B. Weinkove, S.T. Yau, *Taming symplectic forms and the Calabi-Yau equation*, Proc. Lond. Math. Soc. (3) 97 (2008), no. 2, 401–424.