Exercise Sheet 3

0. (a) [H] Let X be a K3 surface, i.e., X is a compact complex surface with $K_X \cong \mathcal{O}_X$ and $H^1(X, \mathcal{O}_X) = 0$. Show that X is not the blow up of any other smooth surface.

   (b) Show that a smooth quartic hypersurface in $\mathbb{P}^3$ is a K3 surface.

1. Compute the Hodge numbers of the following varieties:

   (a) $C \subset \mathbb{P}^2$ a smooth curve of degree d.

   (b) $C \subset \mathbb{P}^3$ be a smooth complete intersection defined by two polynomials of degree $a, b \geq 1$.

   (c) $S \subset \mathbb{P}^3$ be a smooth hypersurface of degree d.

   (d) $S \subset \mathbb{P}^4$ be a smooth complete intersection defined by two polynomials of degree $a, b \geq 1$.

   (e) $Y \subset \mathbb{P}^4$ be a smooth hypersurface of degree 5.

   (f) $\mathbb{F}_a$, the Hirzebruch surface $\mathbb{P}(\mathcal{O}_{\mathbb{P}^1} \oplus \mathcal{O}_{\mathbb{P}^1}(a))$.

(*) You may use *Noether's formula*:

Theorem 0.1. *(Noether's formula) For a compact complex surface X,*

$$\chi(X, \mathcal{O}_X) = \frac{1}{12}\left(K_X^2 + e(X)\right)$$

where $e(X)$ denotes the topological Euler number of X.

References

[H] D. Huybrechts, *Complex geometry-an introduction*, Springer(2005).