MA135—VECTORS AND MATRICES
Example Sheet 2

All questions in Sections A and B must be handed in to your supervisor via the supervisor's pigeon loft by **3pm Monday, week 3.** Section C questions should be **attempted** by students who hope to get a 1st or 2:1 degree, but are not to be handed in.

Section A

A1. Let $z = \pi/6 + i\log 2$. Write e^{iz} in the form $a + bi$. (Careful! This is a trick question.)

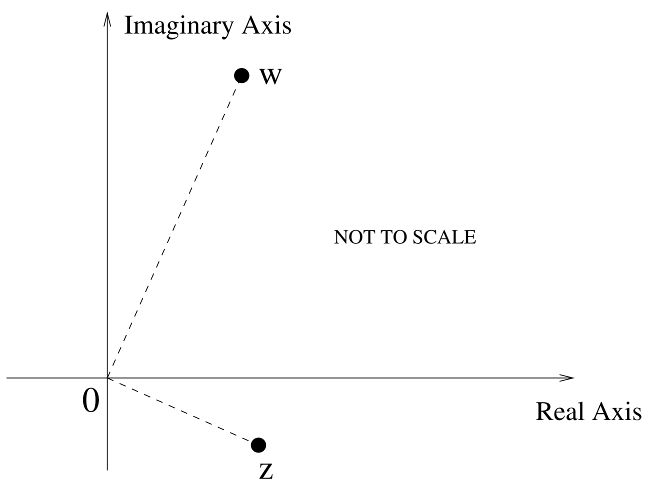
A2. Let $\alpha = \phi + i\theta$ where ϕ and θ are real.
(i) Simplify $|e^{\alpha}|$ and $|e^{i\alpha}|$.
(ii) Show that the conjugate of e^{α} is $e^{\overline{\alpha}}$.

A3. Find the cube roots of $2 + 2i$.

A4. Solve the equation $z^2 + 4iz + 1 = 0$.

A5. Find all roots of the polynomial $f(X) = X^4 + 2X^3 - X - 2$.
(**Hint:** You might start by looking for small integer roots).

A6. Consider the complex numbers z and w indicated on the complex plane as in the picture:



Which of the following is a good guess for the value of w/z: 2, -2, $2i$ $-2i$, $1/2$, $-1/2$, $i/2$, $-i/2$? Explain your answer in terms of the exponential form of complex numbers.

Section B

B1. Find all the 6-th roots of unity and plot them in the complex plane (only a rough sketch is necessary). What is their sum?

B2. Let

$$\gamma = 1 + i, \qquad \delta = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}\, i}{\sqrt{2}}.$$

(a) Represent γ, δ in (r, θ)-form.
(b) Use the (r, θ)-form to find δ^8, γ^3/δ, writing your answers in the form $a + bi$ (exact answers are required).

B3. Express $\sin^4 \theta$ in terms of multiple angles and hence evaluate

$$\int_0^{\pi/2} \sin^4 \theta \mathrm{d}\theta.$$

Section C

C1. Let α, β be non-zero complex numbers. Suppose that the points P, Q represent α and β on the complex plane. Show that OP is perpendicular to OQ if and only if α/β is imaginary.
(**Hint:** Use (r, θ)-form to prove this.)

C2. Prove that if α and β are complex numbers then

$$|\alpha + \beta|^2 + |\alpha - \beta|^2 = 2|\alpha|^2 + 2|\beta|^2.$$

C3. Recall the power-series expansion for e^x that you took at school:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots .$$

(i) Use this and the power series for e^{-x} to express the power-series

$$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

in closed form.
(ii) Express the power-series

$$1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \frac{x^9}{9!} + \cdots$$

in closed form. (Hint: Use the fact $1 + \zeta + \zeta^2 = 0$ for the cube-root of unity $\zeta = \exp(2\pi i/3)$.)
(iii) Do the same for

$$\frac{x}{1!} + \frac{x^4}{4!} + \frac{x^7}{7!} + \cdots$$