

La conjecture d'André–Pink : Orbites de Hecke  
et sous-variétés faiblement spéciales

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**Errata**

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**2.1 Period matrices of abelian varieties, p. 32**

The definition of the period matrix of  $A$  is not the standard one and is not consistent with my formula for the action of  $\mathrm{Sp}_{2g}$ . I need to look up again the correct definition, but I think it should be: The **period matrix** gives the coordinates of  $\{e_1, \dots, e_g\}$  in terms of  $\{f_1, \dots, f_g\}$  (i.e. swapping  $e_j$  and  $f_j$  relative to what I wrote in the thesis).

**2.2 The Siegel modular varieties, pp. 33–34**

The algebraic geometric description of the Siegel modular variety is wrong: they do not satisfy part (a). (The cited source, [Mil08] Theorem IV.7.3, contains no proof and the statement of the theorem there is incorrect.)

Instead these paragraphs should say:

The Siegel modular variety (equipped with the bijection between its complex points and isomorphism classes of principally polarised complex abelian varieties) can also be characterised purely in terms of algebraic geometry by the following properties:

- (a) for every complex algebraic variety  $T$  and every principally polarised abelian scheme  $A \rightarrow T$  of relative dimension  $g$ , if  $\varphi$  denotes the map  $T \rightarrow \mathcal{A}_g$  which sends  $t \in T$  to the point of  $\mathcal{A}_g$  corresponding to the principally polarised abelian variety  $A_t$ , then  $\varphi$  is a morphism of algebraic varieties;
- (b) for every complex algebraic variety  $\mathcal{A}'_g$  equipped with a bijection with the set of isomorphism classes of principally polarised abelian varieties as above, if  $\mathcal{A}'_g$  satisfies (a), then the map  $\mathcal{A}_g \rightarrow \mathcal{A}'_g$  obtained by composing the associated bijections is a morphism of algebraic varieties.

These properties are equivalent to the definition of a coarse moduli space for principally polarised abelian varieties given in [Mum65], Definition 7.4. The variety  $\mathcal{A}_g$  is not a fine moduli space because there is no principally polarised abelian scheme over  $\mathcal{A}_g$  which has the correct fibre at each point of  $\mathcal{A}_g$ .

### 6.1.2 Proof of Proposition 6.2, p. 100

Gabriel Dill found a gap in the proof of Lemma 6.3. The height bound from Proposition 5.3 is not sufficient to be able to apply [PT13] Lemma 3.2 – a bound for the periods is needed as well. Gabriel also found how to fix this gap, by going back to Proposition 5.4 instead of Proposition 5.3.

Gabriel’s corrections can be found in the [arXiv](#) version of my paper *Families of abelian varieties with many isogenous fibres* ([arXiv:1209.3653](#)). Lemma 3.3 of this paper is the same as Lemma 6.3 in the thesis, and the additional work needed for the correction is in section 4.A of the paper.

Note that the version of this paper published in *Crelle* contains the same gap as the thesis.