# Professional development through lesson study: teaching the derivative using GeoGebra

Nellie C. Verhoef, Fer Coenders, Jules M. Pieters, Daan van Smaalen, and David O. Tall
Faculty of Behavior Sciences, University of Twente, Enschede, The Netherlands
University of Warwick, Coventry, United Kingdom
n.c.verhoef@utwente.nl

This study focuses on mathematics teachers' professional development through elements of Japanese lesson study. The teachers designed a research lesson with regard to the sense making of derivative using the integration of GeoGebra. In the second year of the four-year lesson study project, seven secondary school teachers – from different Dutch schools - worked cooperatively building on the first-year experiences with the introduction of the derivative. The teachers designed a lesson that focused on the encapsulation of the conceptual understanding of the derivative before solving problems with operational symbolism later in the course. They tried to make sense of the calculus using GeoGebra as a tool to realize surprise in which conceptual embodiment and operational symbolism blends together. The teachers integrated GeoGebra to consolidate the derivative using the visual idea of zooming in on the graph to see its local behaviour (where a differentiable function looks 'locally straight'). The teachers reported that they have learned to use visualizations and experienced the importance of student interaction. The teachers realized that this approach of the derivative –integrating GeoGebra – encouraged them to reflect on how the students made sense of learning activities in general.

**Keywords:** Lesson study; teachers' professional development; making sense of the calculus; the derivative; GeoGebra.

## 1. Introduction

In this study teachers' professional development was stimulated through lesson study and this was related to recent findings of Japanese lesson study research (Hart, Alston, & Murata, 2011). In the 1880s, Japanese lesson study began with 'open classes', that were held to encourage the introduction of new teaching methods and teaching curricula, producing the first interactive lesson study groups initiated by the government. The Japanese approach aims at making sense of mathematics in a way that not only improves students' understanding, but maintains longer-term success (Tall, 2008).

In the last ten years there has been a growing interest in Japanese lesson study experiences to realize positive changes in the classroom. Lesson study is consistent with the main effective and successful professional development characteristics, recently formulated by Desimone (2009; 2011): aimed at subject matter and student's learning strategies; active teacher participation and learning; and collaboration in communities. In lesson study teachers choose a topic and plan lessons collaboratively, subsequently observe these unfolding lessons in actual classrooms, and finally discuss their observations (Lewis, Perry, & Murata, 2006). Lesson study is chosen as a professional development strategy because research shows that within a context like lesson study, changes in knowledge and beliefs are to be expected (Lewis, 2009; Lewis, Perry, & Friedkin, 2009). There are many barriers to overcome in relation to the Japanese situation. In Europe as well as in the US, there is no widespread history of a changing process from individualized instruction to whole classroom instruction (Isoda & Tall, 2007). Moreover, the barrier in Europe is its focus on renewing instead of improving teaching and learning approaches. As a consequence, European teachers focus on preparing for the exams and often work in isolation in their own classrooms.

The aim of this study is to investigate the effects of lesson study on Dutch mathematics teachers' professional development with the focus on sense making in the calculus. The emphasis is on teaching the concept of the derivative because of its importance in science studies, and students' inclination of using symbolic operations without conceptual understanding. The use of GeoGebra builds both on the human sense of perception and action and links this to the 'making sense' of the symbolic operations (Tall, 2012). Research outcomes show that improvements of teaching and learning depend on structured conditions of required curriculum changes that build on the experiences over years (Bakkenes, Vermunt, & Wubbels, 2010; Buczynski & Hansen, 2010). Improvements in instructional practices to foster student learning take time (Borko, Jacobs, & Koellner, 2010; Vescio, Ross, & Adams, 2008; Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). This means that the impact of lesson study on teachers' professional development in this context of mathematics teaching builds gradually over the years.

In this study, the second year of a four-year project, four new teachers participated alongside the three first-year teachers. The first-year study revealed teachers' difficulties in implementing the approach because they were impeded by the Dutch culture of following the textbook closely, the strict school guidelines and finally the pressure for high exam results (Verhoef & Tall, 2011). This study intends to identify teachers' professional development in a lesson study team integrating a sensible approach to the calculus by using GeoGebra.

## 2. The context of mathematics teaching

### *2.1 A sensible approach to the calculus*

A sensible approach originates from educational psychology in which knowledge develops in communication with others (Bruner, 1966). In education, direct experiences are needed to learn (Bruner's enactive level). To make experience communicable (and thus available to other people for learning), a selection is needed for what is important and what to direct attention to (Bruner's iconic level). In our teaching we keep as close as possible to the concreteness of the enactive level, but make connections between ideas, discerning common and contrasting themes. When communication is about abstract attributes, symbols are needed (Bruner's symbolic level).

A sensible approach to mathematics takes account of the structures of mathematics. It also takes account of the increasing levels of sophistication as learning progresses from sense through *perception*, then through the relationships of *operation* and a developing sense of *reason*. Tall's framework combines both Bruner's enactive and iconic levels together into a long-term development of *conceptual embodiment* in mathematics. Enactive gestures stimulate students' perception through visualization of direct experiences. Iconic images stimulate students' perception through direct intention in order to communicate. Both enactive gestures and iconic images together realize conceptual embodiment. *Operational symbolism* in mathematics develops from embodied actions such as counting and measuring encapsulated as symbols in arithmetic used for calculation, whose generic properties are generalized in algebra. (Tall, 2013).

A sensible approach to mathematics provides teachers and students a cognitive basis for the development of the calculus which begins by making sense of the visual changing slope of a graph $y=f(x)$ and links this to the symbolic calculation of the practical slope $(f(x+h)-f(x))/h$ which can be visualized and imagined to stabilize on the theoretical slope $f'(x)$. This requires more sophisticated techniques to compute the theoretical slope of combinations of functions.

This provides the reason for introducing the limit concept. A sensible approach to the calculus is characterized by the development through *perception*, *operation* and *reason.*

### 2.2 *The derivative*

The general method of making sense of the idea of the changing slope of a curve is to realize that many graphs will change direction steadily, so that, on magnifying a graph a small portion will ‘look straight’. This means that its slope can be *seen* through perception and the changing slope can be found through the operation of looking along the curve to perceive its changing slope. Looking along a (locally straight) graph $y=f(x)$ one may imagine the changing slope $(f(x+h)-f(x))/h$ in order to draw a graph of the slope function and reason that as $h$ gets smaller this stabilizes to give a new graph representing the slope of the original. Let us denote the operation by $D$ and denote the slope function as $Df$ (where $D$ stands for ‘derivative’, namely the slope function *derived* from the original). In this case, by shifting our attention from the embodied world of the graph to the symbolic expression for the function, we can reason that for $f(x)=x^2$ the theoretical slope function looks is $Df(x)=2x$. This conception of the derived function originates fundamentally using dynamic human perception and action. More generally, for a locally straight function, the ‘practical slope function’ $(f(x+h)-f(x))/h$ for small numerical values of $h$ stabilizes to enable the learner to see the stabilized slope function $Df(x)$. It is important to be aware of the fundamental idea that the derivative function is the result of a *global* operation $D$ that operates on the original function $f$ to give the derivative function $Df(x)=f'(x)$. With this fundamental idea in mind, it is time to relate the dynamic visualization to the corresponding symbolic operation, linking human embodiment to mathematical symbolism to give a meaningful symbolic formula for the operation $D$ in a range of different cases (Tall, 2009, 2013).

### 2.3 *GeoGebra*

As our society becomes more dependent on technology and employment shifts from manual to intellectual activity, there is an increased need to raise the quality of mathematics teaching and learning around the world. At present widely diverging performances in different countries and a widespread desire to raise levels of performance in mathematics can be observed. Research outcomes report that despite the numerous benefits of using technology in mathematics education, the process of embedding technology in classrooms is slow and complex (Cuban, Kirkpatrick, & Peck, 2001; Voogt & Knezek, 2008). GeoGebra is open-source, available free of charge, software for mathematics teaching and learning that offers geometry, algebra and calculus features in a fully connected and easy-to-use software environment (Hohenwarter & Preiner, 2007). By providing different forms of dynamic and interactive figures (e.g. Figure 1), GeoGebra constructions can be integrated into mathematics classes (Little, 2008).

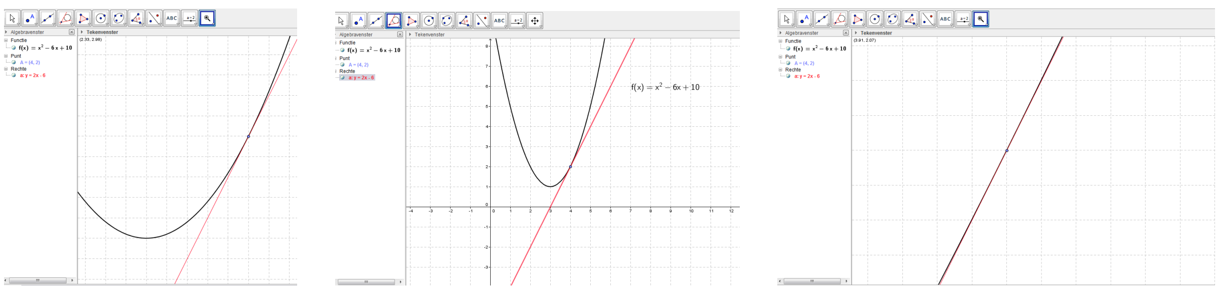


*Figure 1.* Zooming in using Geogebra to visualize local straightness

Recent studies accentuated students' use of GeoGebra and show positive student results in the self discovery of geometric theorems and the understanding of geometric transformations (Abumosa, 2008; Saha, Ayub, & Tarmizi, 2010). Integration of GeoGebra in teaching the derivative in a context of a sensible approach to mathematics seems to be supportive based on the realization of conceptual embodiment.

## 3. Theoretical framework

### *3.1 Lesson study*

There is general agreement in the educational research community about the importance of teachers' professional development to improve education. The Japanese form of *lesson study* offers a promising way in which the nature of teachers' and students' mathematical learning and thinking may be improved. In this approach a well-planned sequence of research lessons is designed that can be widely shared with other teachers. The designed teaching-learning scenario allows students to focus on specific mathematics learning and to make sense to the mathematics with predictable results for the full range of students in class. The lesson study process focuses on learning rather than teaching and offers excellent opportunities to evaluate the learning processes (Becker, Ghenciu, Horak, & Schroeder, 2008). Teachers collaboratively study their own practices through peer observation, evaluation and review. Lesson study so far has helped teachers to develop their subject matter and pedagogical content knowledge by having teachers review each other's practices, particular problems that arise in classrooms, and discuss how to improve teaching (Oshimaa, Horinoa, Oshimab, Yamamotoc, Inagakid, Takenakae, Yamaguchif, Murayamaa, & Nakayamaf, 2006). Collegiality and apprenticeship between experienced teachers are keys to the success of teacher development in the lesson study. The teachers carefully plan a research lesson and great thought is devoted to predicting how the students may react. When the research lesson is enacted in class, it is observed by other lesson study team members. The observers participate in the evaluation of the lesson where observations are shared, ways of refining and improving are discussed and the subsequent review of the lesson is planned. In most cases the research lesson is thereafter enacted in another classroom.

In a time of technological change and economic globalization, we need to use every resource in our power to improve the ways we organize our lives. Lesson study viewed within a framework of long-term growth of mathematical thinking is a powerful tool to improve how we learn to think mathematically (Isoda & Tall, 2007).

### *3.2 Assessing teachers' professional development*

In this study we describe teachers' learning in terms of Clarke and Hollingsworth's (2002) Interconnected Model of Professional Growth (IMPG). The model is represented in Figure 2 and briefly explained below.

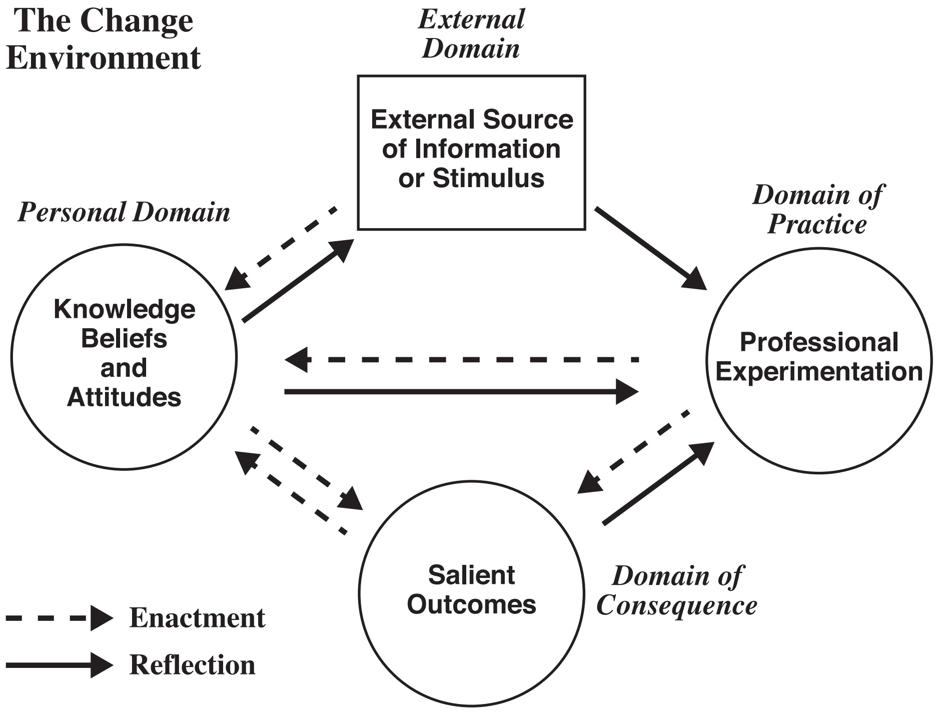


*Figure 2.* The Interconnected Model of Professional Growth (Clarke & Hollingsworth, 2002)

The IMPG suggests that teacher change occurs in recurring cycles through the mediating processes of 'reflection' and 'enaction' in four distinct domains. Three of these domains are situated in the teachers' daily world, the fourth (the External Domain) is outside this daily world. Teachers' knowledge, beliefs and attitude are situated in the Personal Domain. The External Domain is where a teacher meets new ideas. It consists of all kinds of sources for example research outcomes, scientific literature, or books. Discussions with colleagues or expert support are also powerful sources. The Domain of Practice involves all possible kinds of teacher classroom experimentations. The Domain of Consequence (salient outcomes) focuses on the consequences of student learning. This domain is colored by teacher's expectations beforehand. Clarke and Hollingsworth (2002) emphasized the effect of a change in one domain as a sequence of changes in the other domains. They identified temporal changes named 'change sequences'. When the change is more than momentary, this is seen as professional growth and the associated change sequence is termed a 'growth network'. We use IMPG to describe teachers' professional growth in terms of personal knowledge, beliefs and attitude, through external sources, classroom experimentations and salient outcomes. Using lesson study as a means of mathematics teachers' professional development in the context of teaching the derivative using GeoGebra, the research question in this study is: *How does lesson study effectively contribute to mathematics teachers' professional development?*

## 4. Method

### *4.1 Participants*

Seven secondary school teachers from different regional Dutch schools and five staff members of the University of Twente formed the lesson study team. Amongst the staff members were: two teacher trainers, one mathematician, one researcher (the first author) and a PhD-candidate. All teachers participated voluntarily. School management facilitated participation.

The research lesson was conducted in the teachers' secondary school where the students were approx 16 years of age. The male schoolteachers Alfred, Bobby and Carlo, participated for the first time in the school year 2009-2010. The teachers Dan, Freddy, Elena and Gwen joined them at the beginning of the school year 2010-2011. All teachers used digital boards in their classrooms. Table 1 lists teachers' relevant characteristics.

Table 1. *Description of participants*

| | Work experience in 2010 | Teaching in |
|---|---|---|
| Alfred | 17 years | lower level to upper level high school |
| Bobby | 14 years | mostly upper level high school |
| Carlo | one year | mostly upper level high school |
| Dan | 23 years | also mathematics teacher team leader |
| Elena | eight years | lower level to upper level high school |
| Freddy | two years | mostly upper level high school |
| Gwen | six years | also mathematics teacher team leader |

The five university staff members assisted in generating teaching ideas and also had their individual roles in the lesson study team. The researcher prepared the meetings at the university and distributed scientific literature. One of the teacher trainers chaired the meetings at the university. The researcher recorded the meetings at the university as well as the discussions at teachers' schools. The mathematician checked the mathematical correctness. The teacher trainers verified the practicalities in class. The PhD-candidate advised on the lesson study process in general.

### *4.2 Context of the study*

The three first-year lesson study team teachers Alfred, Bobby and Carlo studied Tall's (2008) theory in the first project year 2009-2010. Their experiences with the implementation of the introduction of local straightness in a lesson study context were disappointing. The teachers were not able to replace textbook approaches based on symbolic operations. They were hindered by the school based programming focusing on algorithms and automatisms. Alfred, Bobby and Carlo successively studied papers about procedural and conceptual knowledge in the year 2010-2011: Simpson and Zakaria (2006), Star (2005), Engelbrecht, Bergsten and Kågesten (2009). The four new teachers Dan, Elena, Freddy and Gwen studied papers on Tall's (2008; 2009) theory for the first time. The teachers were asked to present and to discuss the papers they had read in a lesson study team seminar at the university. The intention was to develop a collective jargon.

The lesson study team teachers formed three pairs, each consisting of a first-year participating teacher and a second-year participating teacher. This resulted in Alfred and Freddy, Bobby and Elena/Gwen, and Carlo and Dan working collaboratively. As a start each pair designed the research lesson. In a discussion the three pairs merged their ideas to one research lesson. The pairs implemented the research lesson successively: first Alfred (and Freddy), then Carlo (and Dan), and finally Bobby (Elena took the place of Gwen halfway the year). A plenary reflective meeting occurred before the implementation of the last pair's lesson.

The lesson study team teachers designed extra material to strengthen the textbook chapter about the derivative. The addition aimed to focus on students' conceptual understanding of the derivative before using differentiation rules in applications. The addition was used teaching the theory (including the differentiation rules) and before the applications.

### *4.3 Research instruments*

Below a description of the four instruments used, most were used several times over the year.

*Lesson preparation form.* The lesson preparation form consisted of teachers' intended interventions (using GeoGebra) and worksheets with student activities (assignments). The teachers' interventions focused on local straightness. The worksheets were used to uncover students' conceptual understanding of the derivative.

*Field notes of the student observations.* The observers in the classroom noted the students' communication of a nearby student group. These field notes, recorded during the lesson, characterized students' thinking processes.

*Reports of the discussions and the reflective meetings.* The reports of the discussions at teachers' schools and the reflective meetings at the university were used to uncover teachers' professional development.

*Exit-interview.* The university members of the lesson study team interviewed the teachers at the end of the lesson study. These exit-interviews were used to validate teachers' professional development in relation with their field notes of the student observations and the reports of the discussions and the reflective meetings.

Table 2 lists the data collection in time. The first column describes the involved teachers. The second column orders the used research instruments.

Table 2. *Time frame of data gathering*

| Teachers | Research instruments in order |
|---|---|
| Alfred+Freddy | lesson preparation form 1, fieldnotes 1, report of discussion 1 |
| Carlo+Dan | lesson preparation form 2, fieldnotes 2, report of discussion 2 |
| | report of reflective meeting at the university |
| Bobby+Elena/Gwen | lesson preparation form 3, fieldnotes 3, report of discussion 3 |
| All teachers | exit-interview at last |

*4.4 Procedure and data analysis*

The field notes of the student observations, and the reports of the discussions and the reflective meetings were summarized and categorized by the researcher in: (a) 'What was learned', and (b) 'From what sources was it learned' – lesson preparation, explanation, observation, discussion or reflection. For example, (a) Elena and Gwen learned to be alert on students' anxiety for mathematics, (b) seeing their own classroom experiments and hearing the student observations. This hearing originated from the discussion of the observers' field notes after their lessons. What was learned and from what sources was it was learned was checked for each member in the exit-interviews.

The lesson preparation forms were categorized by the researcher. Teachers' interventions (using GeoGebra) were categorized in stimulated communication by: (a) *enactive gestures* for example 'Telling a story', (b) *iconic images* (visualizations) for example 'Demonstrating Geogebra', or (c) *symbolism* for example by 'emphasizing differences with the textbook'. The student activities (assignments) were categorized in sense making to the calculus by: (a) *perception* for example 'Describe what you see', (b) *operation* for example 'Draw the graph of $f'(x)$ or link this to the calculation of the slope function, or (c) *reason* for example 'Explain what you did'.

## 5. Results

*5.1 Teachers' professional development from observations and reports*

Table 3 reports the teachers' professional development. The first column lists the teachers Alfred, Bobby, Carlo, Dan, Elena, Freddy and Gwen. The second column reports what the teachers learned based on their field notes of student observations and the reports of the discussions and the reflective meetings. The third column reports the sources the teachers learned from. Each element in the second column corresponds with the equivalent element in the third column.

The dotted lines divide the different pairs (Alfred-Freddy; Carlo-Dan; Bobby-Elena/Gwen). Before Bobby-Elena/Gwen started teaching, a reflective meeting at the university was held.

Table 3. *Teachers' professional development*

| | The teacher learned … | The teacher learned from… |
|---|---|---|
| Alfred | .. to design sense making activities using GeoGebra .. to test alternative teaching methods .. that his teaching role was too predominant .. his textbook approach is no longer a guide | .. observing the research lesson<br>.. experimenting in his own classroom<br>.. reflecting at the university<br>.. hearing Tall visiting the Netherlands |
| Freddy | .. how students learn by seeing the straight graph .. to make an abstract concept concrete .. to use sensible/visible and sensory/perceptible using GeoGebra, applets, pictures of the skyline, 'strings' along a graph | .. discussing after the observation<br>.. preparing the research lesson<br>.. reflecting at the university and<br>.. experimenting in his own classroom |
| Carlo | .. to introduce a sensible approach<br>.. to be aware of student learning<br>.. to be aware of his teaching method<br>.. to teach a sensible approach | .. re-reading Tall's articles<br>.. experimenting in his own classroom<br>.. observing the research lesson<br>.. reflecting at the university |

| | | |
|---|---|---|
| Dan | .. to be aware of students' misconceptions<br>.. to be aware of his teaching method<br>.. to stimulate students' interaction telling a story<br>.. to understand the approach of local straightness | .. reading Tall's articles<br>.. observing the research lesson<br>.. experimenting in his own classroom<br>.. reflecting at the university |
| Bobby | .. to use more visualization in his teaching method .. to teach rules of differentiation in a later phase .. to replace zoom in instead of zoom out .. to start the limit process with local straightness | .. re-reading Tall and his colleagues<br>.. reflecting at the university<br>.. observing the research lesson<br>.. discussing with his colleagues |
| Elena | .. to introduce differentiation rules intuitively | .. reading Tall's articles |
| Gwen | .. to be aware of the effects of appealing images .. to be alert on students' anxiety | .. talking with her colleague B<br>.. hearing her students and seeing her own classroom experiments |

The first-year disappointing experiences (Verhoef & Tall, 2011) had stimulated Alfred to use GeoGebra. Through the plenary meetings at the university he became more and more aware of the importance of student activities instead of his dominant teaching role. As an observer he discovered how students think. Freddy emphasized that he realized that the conceptual understanding of the derivative as a rate of change surpasses procedural operations. Through planning the lesson he became aware of strategies to teach a sensible approach of mathematics. After reading Tall's articles, Carlo changed to a sensible approach of the derivative. His observations made him aware of the difference between student learning and teaching. Dan reported that he had developed a feeling for students' misconceptions. He experienced successful student interaction telling a story of zooming in on the earth. Bobby learned to use more visualizations in his teaching method. He wanted to integrate geometry – seeing the tangent line and its slope - in teaching the derivative. He changed his teaching method by zooming in on a single point on the graph to gain a sense of local straightness through that point. Through the reflective meetings at the university he implemented local straightness using GeoGebra to introduce the concept of the limit definition. Bobby (who was procedurally oriented) alerted Elena (who was conceptually oriented) to use visualizations. Their discussions stimulated Elena to think about the introduction of differentiation rules. Elena and Gwen became aware of students' anxiety through the observations of their classroom experiences.

### *5.2 Teachers' professional development from the lesson preparation forms*

Table 4 shows the first lesson preparation that was implemented by Alfred and Freddy. The first column orders the assignments (student activities) on the worksheets and the additional teacher interventions (in italics), sometimes plus G(eoGebra). The second column shows short descriptions of these assignments (the student activities) on the worksheet and the additional teacher interventions (in italics). The last column holds the categorization of the assignments (student activities) (perception, operation, reason) and the categorization of the additional teachers' conceptual embodied interventions (enactive gestures, iconic images) (in italics). In each case, although one aspect is featured in the categorization, other aspects may also be involved.

Table 4. *The first lesson preparation form*

| | Description of the first lesson preparation | Categorization |
|---|---|---|
| 1a | Draw the graphs (Figure 3) | Operation |
| 1b | Explain what you did | Reason |
| 2a | Write down the number of tangent lines in one point | Operation |
| 2b | Explain your answer | Reason |
| *1* | *A story about zooming in from far away to the earth* | *Enactive gestures* |
| *2G* | *Zooming in P - tangent line through P (local straightness)* | *Iconic images* |
| *3G* | *Q nearby P on the graph - zooming out – PQ coincidences with the tangent line through P* | *Iconic images* |
| 3 | Describe what you mean by the derivative of a function | Perception |
| *4G* | *The derived function develops point wise (Figure 4)* | *Iconic images* |
| 4a | Do you expect a straight line? Why? | Perception/Reason |
| 4b | What happens if you translate the graph up(down)wards? | Reason |
| 5a | Your textbook uses a difference quotient. We zoom in and look. Describe the differences and the agreements between these approaches. | Reason |
| 5b | What approach do you prefer? Why? | Reason |
| 6a | Draw the graphs (Figure 3) | Operation |
| 6b | Explain what you did | Reason |
| 7a | Write down the number of tangent lines in one point | Operation |
| 7b | Explain your answer | Reason |
| 8 | Describe the graph (Figure 5) | Perception |

Note: G is the abbreviation of GeoGebra

Assignment 1a and 1b assessed students' conceptual knowledge of the derived function in contrast to the procedural textbook approach. The students had to draw the missing graphs (Figure 3). The students never answered such a question before, however they had been taught the needed knowledge earlier. The teachers characterized these questions as prior knowledge.

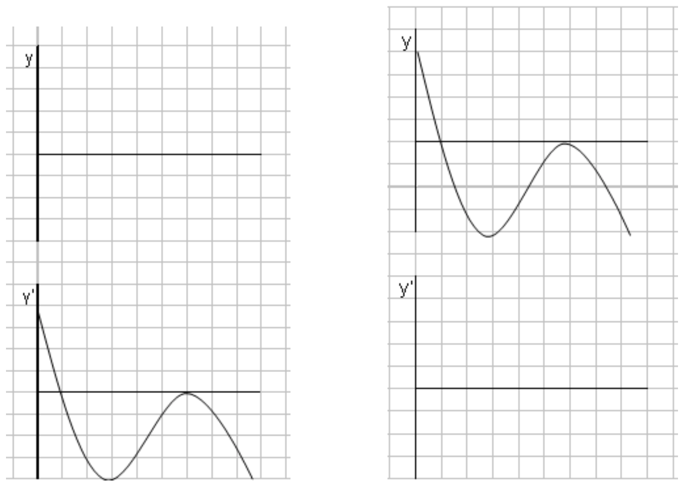


*Figure 3.* Students' presupposed prior knowledge (assignment 1)

Assignment 2a and 2b focused on the number of tangent lines in one point. This assignment was added as a consequence of the disappointing experiences of the first year. The teachers typified this knowledge as preliminary in general (earlier extensively taught), before thinking of the derivative at one point. The lesson continued with the teacher demonstrating local straightness using GeoGebra. The teacher introduced local straightness through a story about zooming in on the earth from a space shuttle far away. The teachers' demonstration ended in a visualization of the development of the derived function, Figure 4.

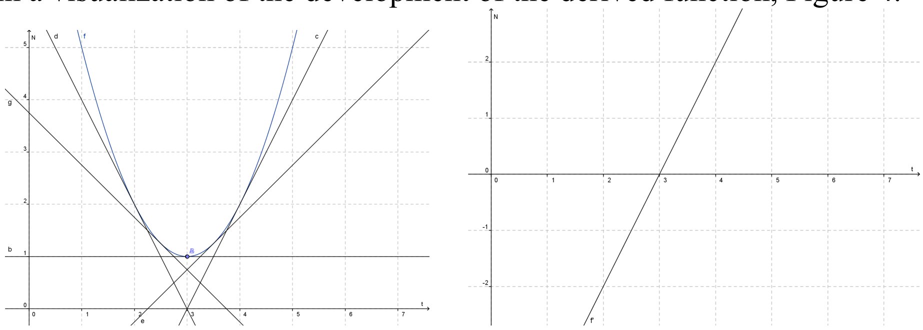

*Figure 4.* A visualization of the development of the derived function (assignment 2)

The teachers wanted to reveal students' conceptual understanding of the derivative. Therefore they repeated the first two assignments (assignments 6 and 7). They ended the lesson by asking the student pairs to describe the graph of an arbitrary function in their own words, Figure 5.

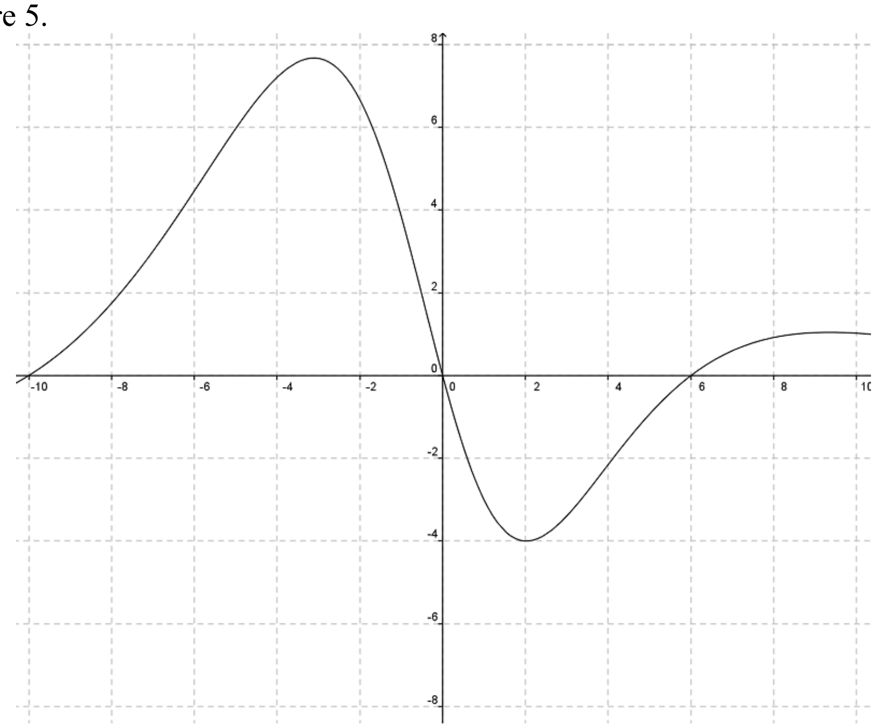


*Figure 5.* The graph of an arbitrary function (assignment 8)

The teachers realized the importance of assignment 8 which forced students' reasoning.

The lesson preparation from Carlo and Dan (lesson 2) hardly differed from that of Alfred and Freddy (lesson 1). Assignment 8 (perception) moved to the start of the lesson with the intention of encouraging students to reason about rates of change intuitively. Lesson 2 ended with assignment 7.

A plenary reflective meeting at the university preceded lesson 3, taught by Bobby-Elena/Gwen. The teachers realized that the reordering did not lead to the intended result. The students answered without thinking about it, they saw the assignment as a joke. As a consequence Bobby and Elena/Gwen reordered the last research lesson and added a new assignment ‘Summarize this lesson briefly’ (reason) after assignment 7, and ended with the original assignment 5. The teachers supposed that students’ reasoning would evolve from conceptual embodiment (learned in the research lesson) to operational symbolism (used in the textbook) by comparing a conceptual and a procedural approach. The teachers realized that this summary was a bad idea because the students did not have enough time. The teachers decided to plan the lesson strictly to create more time.

Table 5 relates each teacher professional development to their categorized lesson preparation. This was checked by the teachers in the exit-interview. The first column lists the teachers. The second column reports what the teachers learned. The third column shows what the teachers learned from their lesson preparations. The last column holds the categorization of the assignments (student activities) (perception, operation, reason) and the teachers' interventions (enactive gestures, iconic images, symbolism). The dotted lines divide the different pairs (Alfred-Freddy; Carlo-Dan; Bobby-Elena/Gwen). Before Bobby-Elena/Gwen started teaching, a reflective meeting at the university was held.

Table 5. *Teachers' professional development in relation with lesson preparation*

| | The teacher learned … | The teacher learned from the lesson preparation … | Categorization |
|---|---|---|---|
| Alfred | .. to focus on pure mathematics<br>.. to use more student interaction<br>.. to test alternative teaching method<br>.. to use the textbook no longer as a guide | .. using local straightness<br>.. asking descriptions:why?<br>.. demonstrating GeoGebra<br>.. emphasizing differences with the textbook | Enactive gestures<br>Reason<br>Iconic images<br>Symbolism |
| Freddy | .. to let students see graph becomes straight<br>.. to think in visualizations<br>.. to let see that an abstract concept can be made concrete | .. zooming in GeoGebra<br>.. demonstration GeoGebra<br>.. developing the derived function | Iconic images<br>Iconic images<br>Iconic images |
| Carlo | .. to become less algebraic (procedural) .. to foster intuition<br>.. to teach a sensible approach | .. asking stud preference<br>.. describing graph firstly<br>.. zooming in GeoGebra | Operation<br>Perception<br>Iconic images |
| Dan | .. to emphasize on zooming in<br>.. to feel for students' misconceptions<br>.. to be aware of a critical attitude to his teaching method | .. telling story zooming in<br>.. describing graph firstly<br>.. emphasizing differences with the textbook | Enactive gestures<br>Perception<br>Symbolism |
| Bobby | .. to visualize the teaching method .. to use more geometry, less algebra .. to differ zooming in and zooming out | .. demonstrating GeoGebra .. asking stud preference .. telling story zooming in | Iconic images<br>Operation<br>Perception |
| Elena/ Gwen | .. to be alert on students<br>.. to be aware of the use of GeoGebra .. to be aware of the role of differentiation rules | .. summarizing the lesson .. demonstrating GeoGebra .. emphasizing differences with the textbook | Reason<br>Iconic images<br>Symbolism |

The lesson study reflections stimulated Alfred to test alternative teaching methods using iconic images to stimulate communication. He became aware of the differences with his procedural textbook. The visualization using GeoGebra stimulated Freddy's creativity in his search for a sensible approach to mathematics. He had never used ICT before. Carlo fostered student intuition to make them sensible to mathematical concepts. He became aware of the limited procedures in his textbook. Dan introduced a story with the intention to stimulate student interaction with regard to zooming in on the earth. He tried to interact with his students. Bobby, who was procedurally oriented, became aware of the meaning of differentiation rules in combination with conceptual understanding. He tried to teach without his procedural textbook guidelines. Elena and Gwen focused on students' anxiety. They tried to increase students' interaction removing their anxiety by asking them to summarize the lesson briefly.

## 6. Conclusions and discussion

### *6.1 Teachers' professional development in the light of the IMPG model*

In the reported lesson study, the teachers prepared a research lesson using external sources. The focus of the lesson study was on mathematics teachers' professional development in the context of teaching the concept of the derivative. In the next section we discuss how the different IMPG domains contributed to the changes in the teachers' personal domain.

*External Domain*. The following extended sources influenced teachers' professional development. David Tall's visit to the Netherlands (June 2010) as well as his articles stimulated the teachers to reflect on their teaching methods. Their textbooks appear no longer to be a straight jacket. Through the plenary reflective meetings at the university the teachers intend to introduce a sensible approach using iconic images (visualizations) in their teaching methods. Through the discussions after the observations the teachers became aware of students' misconceptions. Preparing the research lesson they intend, in line with the Japanese lesson study results, to interact with the students trying to stimulate enactive gestures and operations with reason. The discussions at school and the plenary reflective meetings at the university highlight the effects of the use of Geogebra. Levine and Marcus (2010) argued the positive influence of collaboration in small teacher teams, and we see that the collaboration stimulates the teachers to teach with a focus on sense making of the calculus.

*Domain of Practice.* The teachers class implement alternative teaching methods. They stimulate sense making in the calculus introducing applets, pictures of the skyline and 'strings' along the graph using Geogebra. The teachers stimulate perception telling a story about zooming in on the earth and the demonstration of local straightness using Geogebra. The teachers stimulate reason asking students' descriptions, preferences and summaries. The teachers are aware of the fact that they need to make the derivative concrete as a rate of change: sensible/visible and sensory/perceptible, instead of a symbolic operation. They use of enactive gestures to visualize local straightness by zooming in. They introduce iconic imag*es* to stimulate direct intention to communicate.

*Domain of Consequence.* The students get a feeling of the rate of change by using a scissor along a graph. Enabling a scissor they are stimulated to a focus on perception and reason. Students' response on the question to describe an arbitrary graph stimulate them to interweave operation and reason. The students get more sense of the calculus using other variants of learning. For example: (a) the use of GeoGebra to show amazing local straightness continued by the development of the derived function, (b) applets to translate a function (and its derivative) upwards or downwards, (c) pictures of the skyline to show that a curve seems to be straight, or (d) the use of 'strings' along a graph to show rates of change – it all has to do with 'seeing'. The students' wondering is stimulated by these well-thought examples.

*Personal Domain*. The teachers realize that they established too little student interaction which leads to a lack of reason for students. The teachers become aware of the fact that the use of the last assignment – with the intention to apply knowledge – is typified as a start. The moved assignment changes from application to encouraging students to reason intuitively. Secondly, the teachers become aware of the difference between a story about zooming in and zooming out on the earth. Zooming out on the earth results in student misconceptions.

Zooming in on the earth stimulates students' intuition of local straightness. Thirdly, the teachers witness to the power of asking students to summarize the lesson. They aim that students are able to distinguish their preference to conceptual embodiment instead of operational symbolism. The teachers become aware of the difference between the textbook approach with an emphasis on operational symbolism and sense making of the calculus.

*6.2 Lesson study and professional development in the context of mathematics teaching*

Observing the research lesson stimulates teachers to reflect on their teaching methods which result in a focus on student learning. The teachers learn from their colleagues' classroom practices using enactive gestures like an account of zooming in on the earth. They observed how students discuss and hear their misconceptions. For example, Alfred remarked in the exit-interview: "the observation has touched me". Bobby indicated that the observations revitalized him as a mathematics teacher. The teachers experienced how to use iconic images, like zooming in on a graph with Geogebra. The lesson study motivates teachers to be an excellent mathematics teacher. The learning processes activated by lesson study clearly resulted in improved teachers' professional development. The teachers developed concrete ideas to implement enactive gestures and iconic images in the future. The teachers grow in developing a lesson study approach in this second year of the lesson study project (Verhoef, Coenders, van Smaalen, & Tall, 2013). This positive effect on professional development was demonstrated by the changes in the various domains of the IMPG related to personal growth: domain of practice, domain of consequences, and professional domain.

*6.3 A final remark*

Stigler and Hiebert's (1999) large-scale research to mathematics teaching approaches between US, Germany and Japan concluded that differences in teachers' competences were dwarfed by the differences in teaching methods seen across cultures. The researchers watched many examples of good teachers employing limited methods that, no matter how competently they are executed, could not lead to high levels of student achievement. For example, the use of ICT in US and only a blackboard in Japan. Their research outcomes showed how much teaching varied across cultures and how little it varied within cultures. Further research and an expansion of the descriptive knowledge based on lesson study are needed (Lewis, et al., 2006; Saito, 2012).

**References**


Abumosa, M. A. (2008). Using a dynamic software as a tool for developing geometrical thinking. *International Journal of Instructional Technology and Distance Learning,5*(12), 45-54.

Bakkenes, I., Vermunt, J. D., & Wubbels, T. (2010). Teacher learning in the context of education al innovation: Learning activities and learning outcomes of experienced teachers. *Learning and Instruction, 20,* 533-548.

Becker, J., Ghenciu, P., Horak, M., & Schroeder, H. (2008). A college lesson study in calculus, preliminary report. *International Journal of Mathematical Education in Science and Technology, 39*(4), 491-503.

Borko, H., Jacobs, J., & Koellner, K. (2010). Contemporary Approaches to Teacher Professional Development. In P. Peterson, Baker, E., & McGraw, B. (Ed.), *International Encyclopedia of Education* (Vol.7, pp. 548-556). Oxford: Elsevier.

Bruner, J. S. (1966). *Towards a Theory of Instruction*. New York: Norton.

Buczynski, S., & Hansen, C. B. (2010). Impact of professional development on teacher practice: Uncovering connections. *Teaching and Teacher Education, 26,* 599-607.

Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teacher and Teaching Education, 18,* 947-967.

Cuban, L., Kirkpatrick, H., & Peck, C. (2001). High access and low use of technologies in high school classrooms: Explaining the apparent paradox. *American Educational Research Journal, 38*(4), 813-834.

Desimone, L. (2009). Improving Impact Studies of Teachers' Professional Development: Toward Better conceptualizations and Measures. *Educational Researcher, 38*(3), 181-199.

Desimone, L. (2011). *A primer on effective professional development.* Phi Delta Kappan, *92*, 68-71.

Engelbrecht, J., Bergsten, C, & Kågesten, O. (2009). Undergraduate students' preference_for procedural to conceptual solutions to mathematical problems. *International Journal of Mathematics Education in Science and Technology, 40*(7), 927-940.

Hart, L.C., Alston, A., & Murata, A. (2011). *Lesson Study Research and Practice in Mathematics Education.* New York: Springer.

Hohenwarter, M., & Preiner, J. (2007). Dynamic mathematics with GeoGebra. *Journal of Online Mathematics and its Applications*. ID 1448, vol. 7, March 2007.

Isoda, M., & Tall, D. O. (2007). *Long-term development of Mathematical Thinking and Lesson Study*. Prepared as a chapter for a forthcoming book on Lesson Study.

Lewis, C. C. (2009). What is the nature of knowledge development in lesson study? *Educational Action Research, 17*(1), 95-110.

Lewis, C. C., Perry, R., & Friedkin, S. (2009). Lesson study as action research. In S. E. Noffke & B. Somekh (Eds.), *The Sage Handbook of Educational Action Research* (pp. 142-154). London: Sage Publications.

Lewis, C., Perry, R., & Murata, A. (2006). How should research contribute to instructional improvement? The case of lesson study. *Educational Researcher, 35*(3), 3-14.

Levine, T. H., & Marcus, A. S. (2010). How the structure and focus of teachers' collaborative activities facilitate and constrain teacher learning. *Teaching and Teacher Education, 26*, 389-398.

Little, Ch. (2008). Interactive Geometry in the classroom: old barriers and new opportunities. *Proceedings of the British Society for Research into Learning Mathematics*. University of Southampton, UK: BSRLM.

Oshimaa, J., Horinoa, R., Oshimab, R., Yamamotoc, T., Inagakid, S., Takenakae, M., Yamaguchif, E., Murayamaa, I., & Nakayamaf, H. (2006). Changing Teachers' Epistemological Perspectives: A case study of teacher–researcher collaborative lesson studies in Japan. *Teaching Education, 17*(1), 43–57.

Saha, R. A., Ayub, A. F. M., & Tarmizi, R. A. (2010). The effects of GeoGebra on mathematics achievement: enlightening coordinate geometry learning. *Procedia Social and Behavioral Sciences, 8*, 686-693.

Saito, E. (2012). Key issues of lesson study in Japan and the United States: A literature review. *Professional Development in Education*, 1-13.

Simpson, A., & Zakaria, N. (2004). Making the connection: procedural and conceptual students' use of linking words in solving problems. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Vol 4. (pp. 201–208).

Star, J. R. (2008). Re-conceptualizing procedural knowledge in mathematics. Retrieved June 2010 from http://isites.harvard.edu/fs/docs/icb.topic654912.files/Reconceptualize.pdf

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

Tall, D. O. (2008). The Transition to Formal Thinking in Mathematics. *Mathematics Education Research Journal, 20*(2), 5-24.
Tall, D.O. (2009). Dynamic mathematics and the blending of knowledge structures in the calculus. *ZDM – The International Journal on Mathematics Education*, *41*(4), 481–492.
Tall, D.O. (2012). Making Sense of Mathematical Reasoning and Proof. Plenary to be presented at *Mathematics & Mathematics Education: Searching for Common Ground: A Symposium in Honor of Ted Eisenberg.* Beer Sheva, Israel.
Tall, D. O. (2013). *How Humans Learn to Think Mathematically*. New York: Cambridge University Press.
Verhoef, N. C., & Tall, D. O. (2011). Teacher's professional development through lesson study: effects on mathematical knowledge for teaching. *Proceedings of the 35e Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 297-304). Ankara, Turkey.
Verhoef, N. C., Coenders, F. G. M., Smaalen, van D., & Tall, D. O. (2013). *International Journal of Science and Mathematics Education* (accepted for publication).
Vescio, V., Ross, D., & Adams, A. (2008). A review of research on the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education, 24,* 80–91.
Voogt, J. & Knezek, G. (Eds.) (2008). *International Handbook of Information Technology in Primary and Secondary Education. New York: Springer.*
Yoon, K. S., Duncan, T., Lee, S.W.Y., Scarloss, B., & Shapley, K. (2007). Reviewing the evidence on how teacher professional development affects student achievement. *Issues & Answers Report, REL 2007-No.033.* Washington, DC, U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southwest.