

# MAKING SENSE OF MATHEMATICS THROUGH PERCEPTION, OPERATION & REASON: THE CASE OF TRIGONOMETRIC FUNCTIONS

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This paper builds on the three-world framework presented in PME30 by Tall (2004) in which mathematical thinking in the individual evolves through embodiment, symbolism and formalism. It is here refined to consider how ideas become sophisticated through *perception*, *operation* and *reason*. It is based on foundational ideas developed by Bruner, van Hiele, Fischbein, Skemp, Collis and others relating to the full development of mathematical thinking from child to all forms of adult mathematics. In particular, it considers the mismatch between the longer-term ideas developed in succeeding levels of sophistication by graduate students destined to become teachers and the ways of thinking of the learners that they will teach in school. It is a significant evolution of ideas developed throughout the history of PME.

The empirical data focuses on three successive levels of learning in trigonometry, namely *triangle trigonometry* involving lengths that are magnitudes without sign, angles in right-angled triangles strictly between 0 and 90°, *circle trigonometry* involving angles of any size, sides that have signs and functions that vary dynamically, and *analytic trigonometry* that gives new insights unavailable to school children involving infinite power series and complex numbers.

It builds theoretically on the notion of *supportive* and *problematic met-befores* where some experiences of mathematics that the individual has encountered before either support or impede learning in new situations. This applies both to the learner and to the teacher. It leads to a completely new way of planning the education of teachers to enable them to teach in a way that can make sense to learners. We consider this to be an essential long-term evolution of ideas highly relevant to the teaching and learning of mathematics fully consistent with the long-term aims of PME. The submitted paper is a blend of highly relevant theory and specific empirical detail, considered 'excellent' by one reviewer and criticised on methodological grounds by others. You may your own mind up by reading the full version from

<http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot2012c-Chin-making-sense.pdf>

The ideas arise in the PhD of Kin Eng Chin (in preparation) and are included in *How Humans Learn to Think Mathematically* by David Tall, (forthcoming from CUP: NY).

## Reference

Tall, D. O. (2004). Thinking through three world of mathematics. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Bergen, Norway, 4, 281–288.

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