USING JAPANESE LESSON STUDY IN TEACHING MATHEMATICS

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The teaching and testing of mathematics is now a global phenomenon. It cannot escape our attention that there are some countries that appear to be more successful than others. In particular, Japan and other Asian countries seem to score higher marks in international tests consistently. While it may be sensible to question what such tests actually measure, it is also important to see if we can learn anything from these experiences.

In Japan, particularly in primary schools, there is a long-established practice of lesson preparation that has been in existence for one hundred and thirty years. It is called Lesson Study.

Lesson Study involves teachers working together to design, test, and improve lesson sequences that are likely to improve learning and be useable by other teachers. The teacher who will teach the class, usually in collaboration with others, will take a mathematical topic, think through how to build the new ideas based on the learners' previous experiences, design a problem that can be used to illustrate the principles involved and reflect on the various ways that children might react to the new experience. This involves writing a plan that fits in with the overall development of the syllabus, specifying the main aims of the lessons and the detailed development of the lesson sequence, including a prediction of the different ideas that the learners may offer so that they can be taken into account during the lesson itself.

Part of the design is to structure the ideas that the students might suggest by organising the lessons in sequence so that an appropriate range of ideas have already been encountered in previous lessons. In this way the possible solutions are orchestrated so that the teacher can organise a coherent discussion based upon them. The lesson is taught by the teacher, with a group of supportive observers who discuss the lesson immediately afterwards. As the observers may have focused on different parts of the action, this has the advantage that the lesson is seen from several viewpoints by a team who have the ideas fresh in their minds. How the observations are carried out depends on the individuals concerned. It could be a small group, or, as in my experience, there may be many observers at the same time (Figure 1). In the lessons I attended, observers stayed at the back and sides of the class as the communal parts of the lesson proceeded, but when the children were working on the problem, they were able to walk around and observe how different children operated. The main purpose is to obtain a coherent overall view of the lesson to discuss possible improvements that can be carried out in a new classroom by the same or a different teacher.

Figure 1: Observing a Japanese Lesson as a chlld (front left) explains his ideas

In recent years it has been my privilege to work with a developmental project in the APEC communities where Lesson Study is being incorporated in very different societies. It is organised in Japan and Thailand, in cooperation with government-supported participants from Australia, Canada, Chile, China, Indonesia, Malaysia, Mexico, Philippines, Chinese Taipei, USA, and Viet Nam.

Seeing how very different societies adapt to Japanese methods of Lesson Study is a real eye-opener. Different cultures have very special ways of teaching, some by whole-class teaching, some by group work, some in which the class recites instructions given by the teacher. What is absolutely clear to me is that Lesson Study has genuine benefits that would be of value to us in the United Kingdom, as long as we think reflectively about what it is we are trying to do in teaching mathematics.

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At this point it will be helpful to think about a particular lesson taught in Japan to gain some specific insights into the techniques being used. The lesson I have chosen is in a primary class, but the principles involved have much wider implications for how the child learns in secondary school and later. In particular, I believe that it will help children learn to be flexible with number concepts rather than just rote-learning rules and procedures. For this reason, I have chosen a lesson designed to introduce long multiplication.

A lesson in a sequence of lessons

The 50-minute lesson concerned was taught in Sapporo City Maruyama Elementary School, to a class of 40 third grade students by Hideyuki Muramoto. It is the fourth in a sequence of 13 lessons.

The preceding lesson was based on the problem of calculating the number of black circles in an array that has three rows, each with the same number of elements:

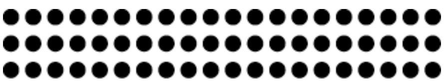

Figure 2: How many black circles?

To calculate the total number, the children first had to count the number of black circles in each row. Their previous experience involved learning all the one-digit products from 1×1 to 9×9. Their new problem was to use this previous knowledge to calculate the total number of discs. In the lesson, the children suggested various ways of subdividing the rows into smaller parts for which their previous knowledge could be used. Suggestions included $10+10$, $5+5+5+5$ and $9+2+9$. In each case, the total was calculated by multiplying each number by 3 and adding the results to get the answer 60.

The whole lesson of fifty minutes was devoted to this single example. In our Western classes the children would spend much more time doing a larger range of examples. The Japanese classroom focuses much more on the flexibility of ideas. The teacher uses the board at the front of the classroom to build a picture of the whole lesson, putting the problem up on the far left, sticking up students' solutions for all to see, and recording the lesson activity from left to right. The children write up the activities in their work books, sticking in the individual copy of the picture of the black disks and working on it, copying the teacher's writing on the board and adding their own observations.

Writing is an important part of Japanese education. While we English speakers have an alphabet of 26 letters but need to cope with all kinds of ways of spelling, the Japanese children have to learn to write thousands of different Kanji characters, so writing is a regular part of their learning. Nevertheless, given our own children's varied performances in reading and writing, regular writing in a mathematics class may not be out of place in our classrooms too.

The idea is not to just simply teach the children a single method of performing the calculation, but to encourage them to develop their own ways and then to compare them to see which methods are better than others. The class is not streamed and has a wide range of ability. In practice it would take a more able child to see $9+2+9$ as a solution and this method is harder to work out. However, the child who is struggling with arithmetic may see that, of the various possibilities, some may be too difficult and likely to lead to error, but others are appropriate for their purpose. Different children will get different experiences from the lesson, but for all of them, the main idea is to see that there are different ways of working out the problem and some are simpler than others. This focuses on the need to develop the most appropriate operations to carry out a calculation with a richly connected knowledge structure.

The Japanese Professor

To encourage the learner to think about how to choose good ways of working, the teachers use the Japanese word Ha-Ka-Se, which means Doctor or Professor. Each syllable also links to a separate meaning. The Japanese word ***ha***yai means fast, ***ka***ntan means both easy and understandable, and ***se***ikaku means accurate and logical. *Fast*, *easy* and *accurate* are useable words for primary school children to compare various processes. Using Ha-Ka-Se in the everyday classroom, especially in whole class work discussion, helps children become accustomed to comparing alternative procedures to seek those that are faster, easier and more accurate.

In the lesson just described, the children found the subdivision $10+10$ to be easier to handle than $9+2+9$, so, while the latter may be suggested by a more able child, the former is likely to be seen by most children as being faster, easier and less likely to lead to error.

The Lesson

We now move on to the next lesson, which was the object of the Lesson study itself. I was very privileged to be present and was subsequently involved in working with the team to edit a full lesson video into short episodes and place it on the web at http://apec.pbwiki.com/Multiplication+Algorithm+Grade+3+(Japan)

It is available for anyone to download and to study at leisure. Included on the web-page is a very full *Lesson Plan* that discusses the method and a *List of Episodes*, to enable quick access and repeated study. It will be helpful to download this lesson to see the various episodes discussed here. Indeed, the use of the lesson within the framework of this article could form a good basis for an extended in-service session.

The lesson began with a new problem. After the children have written the date and the title of the activity in their books, the teacher holds up the problem on a piece of paper placed on the board at the front, covered up with another piece of paper. Slowly he lowers the cover to reveal the problem step by step. The children are very excited. One boy declaims a dirge '*daa*, da, da, da, *daaaaaa*' as a sinister signal of expectancy. Others look in awe and begin making verbal suggestions: 'it's appearing', 'too many', 'too many', 'maybe 90'. Two rows of circles are now visible, and voices call 'it's only halfway, but I guess there are twenty circles', 'half of them is twenty', '40, 80'. Three rows are now visible and the guesses go on: '100, 120, …' The problem is finally revealed as being precisely three rows and a voice calls, 'you have just added two twenties'. Another reminds the teacher 'we need a smaller one for counting' and smaller copies are handed to each child in the class. As this happens, the counting begins and when the teacher speaks to the class as a whole, he clarifies the problem for the children to think about. There are three rows and each row has 23 discs, the problem is to calculate the total.

The class are now encouraged to work on the problem and we observers are able to walk around, and even take photos of the children's work (Figure 3.)

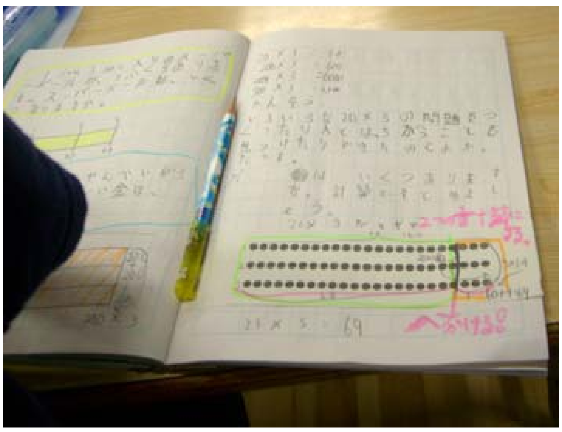

Figure 3: One child's work

Given the experience of the previous lesson, it is not surprising that some children see the breakdown as $20+3$ or $10+10+3$ and others are more adventurous with $9+5+9$. Interestingly, no child considered $5+5+5+5$ this time. The solutions seemed to polarize into the sensible simple subdivisions, such as $20+3$ or $10+10+3$, and more exotic examples such as $9+5+9$ with several other interesting possibilities.

As the children work, the teacher goes round the class talking to various individuals. However, this is not just to find out what is going on. He is quietly determining who has done what so that when he begins the discussion he can start with the simpler methods. After five minutes of group activity, the teacher checks that everyone has finished and then begins the process of collecting ideas from the class. As each idea is declared, several are placed on the board to build up possible solutions (Figure 4).

Figure 4: Initial solutions to counting the number of black circles

After solutions involving $20+3$ or $10+10+3$, solution $10+3+10$ is met with gasps from some of the children who seem to be amazed by its symmetry. This reminds us as adults that some things that we regard as simple are revelations that give enormous aesthetic pleasure to a child.

One child seems, in a sense, to be working at a higher conceptual level than the others. He sees the array broken down into three groups, first a nine, then the rest divides into two thirties, so that 69 is made up of 30, 30 and 9.

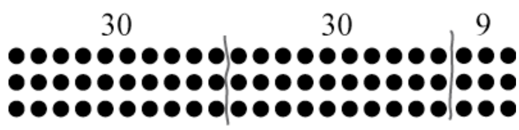

Figure 5: 69 is 30 + 30 + 9

His classmate suggests that this is not the way to do it because, to use his method, you need to know the answer (69) before you do the calculation. This particular case is interesting as the child clearly sees the array as 3 rows of 10, 3 rows of ten and 3 rows of 3 to get 30, 30, 9 so he can ‘see’ the whole picture. However, he has not articulated explicitly how he gets to this by building up from smaller calculations. The teacher takes a decision ‘on the hoof’ and speaks quietly to him rather than open up the discussion to the whole class, shifting to another case that will build the story.

The next child suggests that he does the calculation in a similar way to all the others, but thinks of 23 as two 10-yen coins and three 1-yen coins. The teacher asks him to go through his calculation and writes it on the board as other children enter the discussion about seeing 23 as two tens and three ones.

A child now stands and says he has noticed something. The class waits expectantly as he explains that everyone has made 60 and 9 and ends up with the answer 69. No one calculates, say, 39 and 30.

The discussion opens up as further solutions are presented, such as subdividing 23 into $11+12$, $9+9+5$ and $11+11+1$. Of these $9+9+5$ is particularly complicated as the child calculates 9×3 and 5×3 and has to add $27+27+15$. This is done by breaking down the 27s into $20+7$, adding the two 20s to get 40, the two 7s to get 14 and adding these to get 54 to which the 15 is added by a similar separation into tens and units. The calculation is written on the board by the teacher. It is *complicated* (Figure 6).

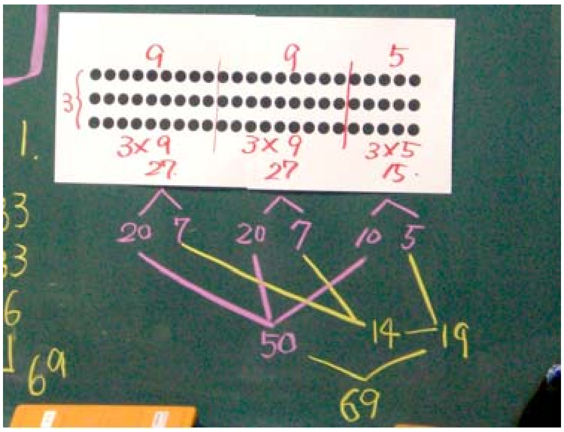

Figure 6: Calculating 9 plus 9 plus 5 is complicated

Although the term ‘Ha-ka-se’ is not mentioned explicitly, various children talk about what methods are easy and what are not.

The class is now ripe for the *coup de grâce*. Some children have already encountered column multiplication and come to the front to suggest that 23 times three can be calculated as three threes in the units place, then three twos in the tens place. The teacher carefully writes it up, following the children’s instructions, clarifying matters by asking more questions as he goes.

The column multiplication is discussed with the class and linked to the idea of two 10-yen coins and three 1-yen coins. Then the teacher takes the earliest picture breaking 23 into 20 and 3 and places it beside the column multiplication to display the link between the physical problem and the symbolic layout of the multiplication algorithm (Figure 7).

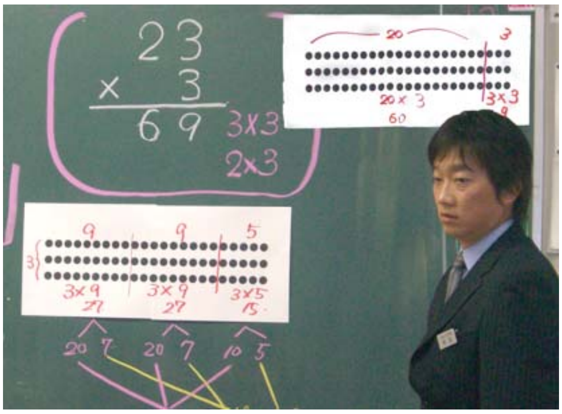

Figure 7: Linking the symbolic calculation with the visual picture

The teacher now begins to summarize by asking the class if they have seen any good ideas. The first comment is how good it is to think in terms of 10-yen coins. There is a discussion about the relative merits of the various ways of performing the calculation and the teacher makes some notes on the board suggested by the children, for the children to write in their books. He then summarises with the children first reciting the problem together and then using the layout on the board to recall the main ideas.

Reflecting on the lesson

Looking back over the lesson, we see that the idea of column multiplication is prepared by building on the pupils’ ideas of single-digit multiplication applied to practical problems that encourage several different possible solutions. Then they use a general principle to see which methods are more appropriate in terms of being quicker, easier and more likely to be accurate. From this discussion, multi-digit multiplication is seen to be easier if it is separated into tens and units, with each calculation pursued separately and then put together. It helps to imagine tens as 10-yen coins and units as 1-yen coins. The standard method of column multiplication is seen not only as one of a range of possible calculations, but also as the one that is simpler, easier and more likely to give a correct answer. This places the idea of column multiplication in a rich network of meaning instead of simply teaching it as a procedure to practise.

There are also deeper subtleties here. One relates to the distinction between thinking based on our human actions and perceptions, typically referred to as 'space and shape', and our technical ability to operate with symbols in arithmetic and algebra.

Many mathematics educators, and certainly government inspectors, focus on initial use of practical embodiments of mathematical ideas: Dienes' Blocks, Cuisenaire rods, and so on. However, for me, the notion of embodiment is more than that. It is about how we as humans perceive things, explore them, manipulate them and make sense of them, not only in the real world, but in our minds. The materials used in this lesson are physical pictures, and the embodied actions involve subdividing the array into smaller sections to be used to 'see' the problem broken down into products that they already know. Embodiment means more than the physical objects themselves, it refers to the human way we perceive things and act on them, and how we think about them in our mind.

I like to see embodiment and symbolism as *complementary* in school mathematics. Things 'make sense' in embodiment when we can 'see' the ideas and physically act upon them, in this case by drawing lines to separate the array into smaller arrays. On the other hand, symbolism gives us power of calculation and algebraic manipulation to specify quantities accurately and calculate a precise answer. Using embodiment of number properties gives meaning to them, which, in turn builds meaning into the symbolism itself.

The lesson given here is an admirable integration of embodiment and symbolism where the symbolism is not just taught as a specific algorithm, but as the 'best' way of solving the problem symbolically out of a range of related methods.

It also reveals a steady *compression* of knowledge, from lengthy counting procedures, to breaking a complicated problem into simple sub-problems that can be solved easily with current knowledge and developed into more powerful *and meaningful* methods of operation.

This is rather a subtle point. Many teachers in our western classrooms get their children to perform activities in a variety of ways to build connections. Constructivist techniques involve encouraging children to develop their own ways of working. However, here there is a genuine attempt to develop ways of working that are not just personal ideas, but focused on making the mathematics more meaningful, more efficient and paying explicit attention to the essential ideas. Long-term development of knowledge depends on compressing activities, say from counting to manipulation of numbers, from repeated addition to multiplication, from step-by-step operations to an overall grasp of principles so that the mathematics becomes not only more powerful, but essentially more *simple* to think about and to perform.

The lesson is given to a whole mixed-ability class, some of whom are highly fluent in arithmetic, others who struggle but are at least able to cope with some single-digit products. Some produce sophisticated, but rather complicated ways of solving the problem, which gives the same answer, but at a cost. The message, for all the children, is the same: that dealing separately with tens and units is a simpler and more meaningful way of proceeding.

It was a pleasure to be in this class where children were lively and active, yet showed respect to other children and listened to their explanations in an appreciative way. Each child is an individual but works constructively as part of the whole group. My perception is that the children give respect because they are shown respect. They are encouraged to form their own ideas but this is seen in a context where some ways of working are more powerful than others, so the responsibility for individual learning is framed by a desire for each child to seek the best methods available to them.

Can these ideas help us in the United Kingdom?

Being an Englishman writing an article for teachers in Scotland places me in an 'interesting' situation as the education system in Scotland is quite different from that in England and Wales. However, my experience in seeing Lesson Study carried out by individuals from a wide range of cultures around the Pacific gives a broader perspective. There are some countries where teaching is predominantly by rote learning, teaching specified rules to perform a particular operation. Given a culture based on respect for elders and the desire to please by passing tests, the desire to introduce Lesson Study is often tempered by the continuing need to teach to the test. The need to pass tests in the English National Curriculum has led to a continual increase in percentage passes over two decades and yet as I write this, *The Times* has an article claiming that the UK economy has lost a staggering £9bn because Britain is failing to produce enough good mathematicians. Big firms in Britain need to import mathematicians from overseas because countries like India and China are producing vast numbers of good mathematicians necessary in our new technological age.

At the other end of the scale is a report that only 2% of the five million adults who struggle with numeracy have taken up the offer to make up the deficit by studying under the British Government's Skills for Life scheme.

Every few years we have new initiatives to improve the teaching and learning of mathematics. But weaknesses still remain.

So what can Japanese Lesson Study teach us? For many proponents the major idea is to have young children learn how to learn by solving problems, not only as an addition to traditional learning of techniques, but also as an integral part of building up knowledge. As such, the techniques operate across the curriculum and Lesson Study is used in all subjects. If you look up Lesson Study on the internet, you will find sites that suggest how Lesson Study can be used as a general method of professional development.

However, it has a particular role to play in mathematics. In particular, it can shed a completely new light on how mathematics is learnt by children and how the learning experience can be organized to help children not only to carry out techniques but to make sense of the mathematics and focus on what is important to make mathematics both powerful and simple at the same time.

Mathematics builds up in a series of stages, each resting on the previous one, but often in new contexts where old methods are inadequate or even inappropriate, so that new techniques are needed. Lack of understanding at one stage can lead to successive stages getting more difficult to comprehend, leading to rote learning and even mathematics anxiety. Lesson Study involves the careful design of good lesson sequences that focus on helping children develop different methods of approach from which they can find ways of working that are fast, easy and accurate. The lesson spends a significant amount of time reflecting on what makes sense and focusing on finding better ways of working. For the teacher, it requires not only knowledge of mathematics, but also deep experience of how children think as they learn mathematics.

There are many general differences between the Japanese classroom and those in the United Kingdom. The classes in Japan are larger (40 in a typical primary class). However, given the same proportion of teachers, this gives more out-of-class time to prepare lessons. The techniques in the United Kingdom seek to support the individual child by having differentiated work in the same classroom, with extension materials for those making more progress and support materials for those who are finding difficulty. In Japan, approximately half the children attend Juku schools out of regular school hours, which can include intensive practice of arithmetic techniques.

A cursory comparison between Japan and Britain may focus on such differences and cause some to dismiss the Japanese experience as not appropriate for our culture. For me, what matters, is the deeper issue of the profound difficulties we all experience with the teaching and learning of mathematics which continue to plague us despite many government initiatives. These initiatives often attack the symptoms with tests to measure the changes in those symptoms rather than the underlying condition.

What Lesson Study offers, in its fullest incarnation, is a study of the deeper meanings that children give to mathematics and how we can work in the classroom to find better ways to make these meanings coherent and focused. While the lesson studied in detail here focuses on a primary class being introduced to long multiplication, it has a message that speaks to all levels of mathematics teaching at primary, secondary and tertiary levels. It teaches us how to look at mathematics anew, not just to teach techniques, nor even to set this within a broader context of conceptual understanding, but to make that wider conceptual understanding focus on how to help children to learn more powerful ways of thinking, linked coherently to their growing experience. This requires an understanding not only of the mathematics itself but how it can be organised to make sense to learners who are growing in their appreciation of its power. It requires a deep understanding of the long-term learning of mathematics and sensitive teachers who take their part in the growth of this learning. It offers insights to help us in our quest to teach mathematics meaningfully, to give pleasure to the learners not only through the attainment of higher marks on tests but also through the development of more powerful and insightful ways of thinking.

In Britain, it is 'cool' not to be able to understand mathematics. Adults have been so 'turned off' mathematics in school that they are happy to boast that they don't understand it and don't need it. What is necessary to change this situation is not just obligatory targets for the performance of techniques, but a genuine development of mathematical teaching that is aware of the nature of mathematics and has developed ways of teaching that encourage learners to make connected sense of the ideas.

In Singapore, for example, not only do they know that they produce better scores on tests than others, they work to develop the understanding of mathematics so that their success is not only maintained, but improved.

In most countries around the world there is a desire to improve mathematics teaching and the arguments are often political as much as educational. It has been my pleasure to work with many studies around the world and I see the desire for passing tests ingrained in so many cultures. In Britain we are encouraged to work within government guidelines that are controlled in a more directive way in England than in Scotland. A recent report of the Advisory Committee for Mathematics Education (ACME) revealed how teachers in England were responding to these pressures by focusing valuable resources into preparing for the tests. Yet this alone does not help teachers 'raise standards' as desired by the government. Nor does it help to produce adults who get pleasure from mathematics and use it in thoughtful and creative ways.

So what can we learn from the experience of Japanese Lesson Study? It has certain clear advantages, some of which are part of the Japanese culture. The children are enthusiastic, noisy and respectful in a heady mix of delight and obedience. The classes are larger than those in Britain with 40 children in each primary class. However, if teachers teach larger classes and have more time to prepare their lessons coherently, then this may offer different kinds of advantage. What matters in Japanese Lesson Study is the careful preparation of lessons so that the child can build mathematical ideas and develop a personal autonomy to make sense of the ideas in their own way. To make this happen requires an insight into how children learn, building on embodied activities being translated into increasingly sophisticated symbolic techniques.

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