Why do two students, both of whom did well in one course, have vastly different experiences in the subsequent course? What is it about their cognitive structure, as evidenced by their problem-solving processes, that allows one to succeed while the other does not? Does the successful student merely have more available procedures, or is there a fundamental difference in his or her cognitive activities?

Nancy and Kathy both earned a “B” in college algebra, and both enrolled in pre-calculus the following semester. Nancy had little difficulty in pre-calculus, but Kathy had a great deal of difficulty. The course is a degree requirement, so she needed to pass it; she dropped once, re-enrolled and eventually passed, but with much hard work and a great deal of anguish. Why do “so many of the population fail to understand what a small minority regard as being almost trivially simple?” (Gray & Tall, 1994). Why do two students with apparently similar attainment go on to perform so differently?

To seek insight into these questions, we explored the cognitive structure demonstrated by the two students working problems involving graphs of linear functions from the first algebra course, and compared their problem solving approach to several of the problems.

We have previously studied the diffuse cognitive structure of a student who was less successful in a college algebra course (Crowley & Tall, 1999). This study compares and contrasts the work of a similar student—who struggled with very straightforward algebra concepts—with that of a student who attained the same grade and yet proved to be more successful in the succeeding course.

Barnard and Tall (1997) introduced the idea of “cognitive unit” as “a piece of cognitive structure that can be held in the focus of attention all at one time.” We see cognitive units as forming the nodes of a cognitive structure linked to other units using the web metaphor of Hiebert and Carpenter, incorporating the varifocal element of Skemp. If various elements are not connected sufficiently securely and fluently, the individual may not be able to consider the totality as a cognitive unit. Links that the individual is able to make are not made to a flexible conceptual entity, but to one procedure in a collection of them, the student’s “cognitive kit bag”.

Interviews revealed quite different cognitive structures. The successful student had a variety of approaches to problems, checking mechanisms, and an overall grasp of equations in one variable to build up links as if it were a cognitive unit. The student who struggled had a cognitive kit-bag of procedural techniques with no flexibility or checking mechanisms. She relied on her calculator to help her over difficulties with negative numbers and fractions. She had the same attainment but very different potential to cope with the ensuing course.