# Concept Maps & Schematic Diagrams as Devices for Documenting the Growth of Mathematical Knowledge

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*The major focus of this study is to trace the cognitive development of students throughout a mathematics course and to seek the qualitative differences between those of different levels of achievement. The aspect of the project described here concerns the use of* ***concept maps*** *constructed by the students at intervals during the course. From these maps,* ***schematic diagrams*** *were constructed which strip the concept maps of detail and show only how they are successively built by keeping some old elements, reorganising, and introducing new elements. The more successful student added new elements to old in a structure that gradually increased in complexity and richness. The less successful had little constructive growth, building new maps on each occasion.*

## Introduction

A *concept map* is a diagram representing the conceptual structure of a subject discipline as a graph in which nodes represent concepts and connections represent cognitive links between them. The use of concept maps in teaching and research has been widely used in science education (Novak, 1985, 1990; Moreira, 1979; Cliburn, 1990; Lambiotte and Dansereau, 1991; Wolfe & Lopez, 1993) and in mathematics education (Skemp, 1987; Laturno, 1993; Park & Travers, 1996; Lanier, 1997).

This study focuses on how concept maps develop over time. Students taking a sixteen-week algebra course using the function concept as an organising principle were asked to build concept maps of FUNCTION on three occasions at five-week intervals. In addition to qualitative analysis of the successive concept maps, we use a simple pictorial technique to document the changes.

Given a sequence of *concept maps*, a *schematic diagram* for the second and successive maps is an outline diagram for each distinguishing:

- items from the previous concept map remaining in the same position,
- items moved somewhere else, or recalled from an earlier map,
- new items.

Ausubel *et al* (1978) placed central emphasis on building meaningful new knowledge on relevant anchoring concepts familiar to the student. Using the schematic diagrams we investigate whether fundamental concepts persist in the development of successful students' concept maps and what happens to the less successful. This will be triangulated with other techniques of data collection and analysis. Given the extensive literature on the difference between those building a powerful conceptual structure and those remaining with inflexible procedures, we expect to find these differences reflected in the concept maps and schematic diagrams.

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## Concept maps and cognitive collages

The question as to whether a concept map actually *represents* the inner workings of the individual mind has long vexed the mathematics education community. Here we are not so much concerned with this issue as to how the individual *chooses* to represent his or her knowledge. It involves a wide range of technical, cognitive and aesthetic issues. Davis (1984, p. 54) used the term *cognitive collage* to describe the notion of an individual's conceptual framework in a given context. As one of us was for many years a professional artist and the other a practising musician, we warm to the rich inner meaning of the term "collage". For a child it may simply consist of a collection of pictures cut out of magazines and stuck on a piece of card, but for the artist it has a theme or inner sense that binds together distinct elements in a meaningful way. So it is with concept maps drawn by students. Some are seemingly arbitrary collections of items, others use all kinds of artistic and other devices to hold the ideas together. Figure 1, for instance, shows the first concept map of student MC drawn in the fourth week of term. The original is in colour, with colour coded inputs red and outputs representing links that are lost in a greyscale reproduction. By triangulating the development of these maps with students' written work and interview data we will explore how they provide a means of documenting cognitive growth over time.

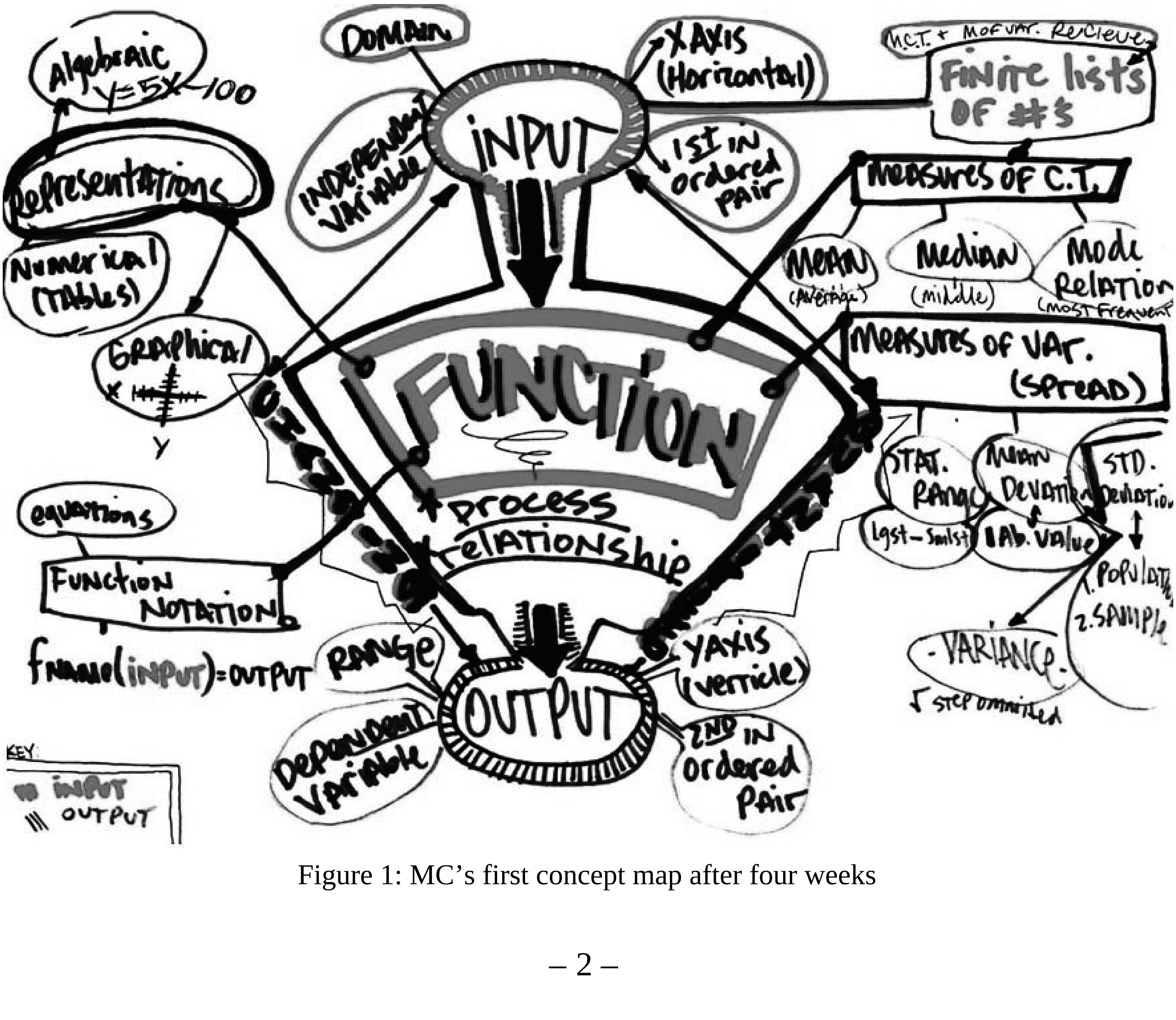


Figure 1: MC's first concept map after four weeks

## Methodology

The subjects of the full study were twenty-six students enrolled at a suburban community college in a developmental Intermediate Algebra course. The curriculum used a process-oriented functional approach based on linear, quadratic and exponential functions supported by graphing calculators. Data was collected throughout the semester on every student including concept maps requested in weeks 4, 9 and 15 of the sixteen-week semester. Students were advised to use "post-it" notes to allow them to move items around before fixing the map. The maps were collected a week later, reviewed with each individual student to gain further information on the intended meaning, and then retained by the teacher so that at each stage the student was invited to draw a concept map anew.

Results of pre-and post-test questionnaires—together with results of the open-response final exam and departmental multiple-choice final exam—were used to rank the students. Two subgroups were selected, the four "most successful" and four "least successful" in the rankings, for more detailed profiling using follow-up interviews.

The concept maps of the eight selected students were analysed to document the processes by which they construct, organize, and reconstruct their knowledge. Schematic diagrams were constructed for the sequence of concept maps produced by each of them. The full analysis of the concept maps and schematic diagrams (McGowen, 1998) was triangulated with other data (Bannister et al, 1996 p. 147). Here we focus on two students, MC (in the most successful group) and SK (in the least successful).

## Visual representations of students' cognitive collages

The second concept map of MC (figure 2) should be compared with the first (figure 1). Although the overall shapes change a little, the second is an expansion of the first. Some topics not studied in the interim (e.g. measures of central tendency and variability) remain unchanged, some are extended (representations, equations), and new items (finite differences) added.

The final concept map, created during week 15, was drawn on a very large piece of poster-board, too large to reproduce here. The topics included on the three maps followed the sequence of organization of the course and the connections shown are successively based on the main ideas of earlier maps. In his final interview, MC commented:

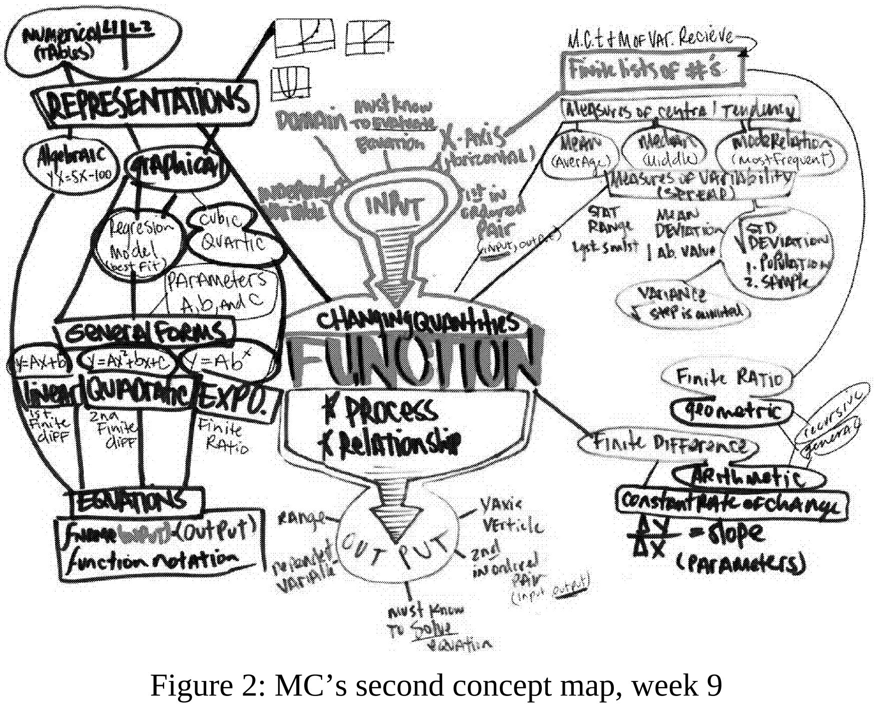


Figure 2: MC's second concept map, week 9

> While creating my [final] concept map on function, I was making strong connections in the area of representations. Specifically between algebraic models and the graphs they produce. I noticed how both can be used to determine the parameters, such as slope and the *y*-intercept. I also found a clear connection between the points on a graph and how they can be substituted into a general form to find a specific equation. Using the calculator to find an equation which best fits the graph is helpful in visualizing the connection between the two representations. I think it's interesting how we learned to find finite differences and finite ratios early on and then expanded on that knowledge to understand how to find appropriate algebraic models.

This final map is a rich collage, focusing on concepts and links between them, for instance, between graphic and algebraic representations, relating finding zeros in the first to factorising in the second.

The maps of SK provide a sharp contrast (figures 2, 3, 4). Week 4 includes definitions (in speech balloons). Week 9 is a bare skeleton with little in common with the earlier map. In week 15 the three basic function types (linear, quadratic, and exponential) become linked not to the central function concept but to *parameters.* The final map reveals procedural undertones by concentrating on routines (find slope, find constant common ratio, simplify, solve, evaluate, etc.).

Triangulating these concept maps with other data confirms that SK's knowledge is compartmentalized. She seems to have assembled some bits and pieces of knowledge appropriately, but others are missing, preventing her from building a cognitive collage with meaningful connections. When confronted with situations in which she is unclear what to do, she defaults to using remembered routines. She usually focuses initially on the numerical values stated in a problem. When confronted with a task for which she has no appropriate schema, she can only retrieve a previously learned routine. Her concept image of linear and exponential function on her week 15 concept map, for instance, is

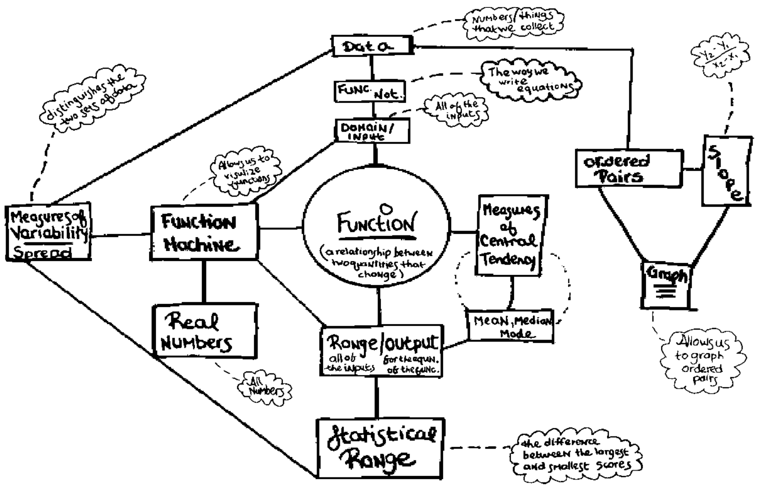


Figure 3: SK's concept map,week 4

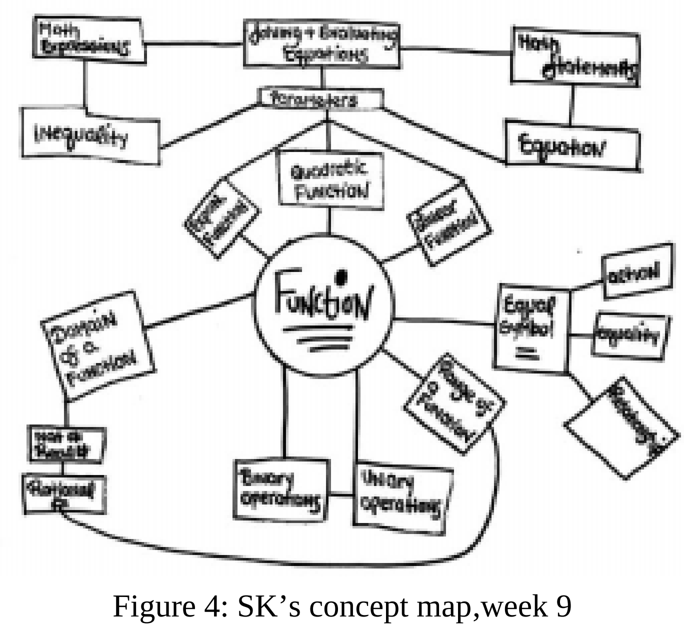


Figure 4: SK's concept map,week 9

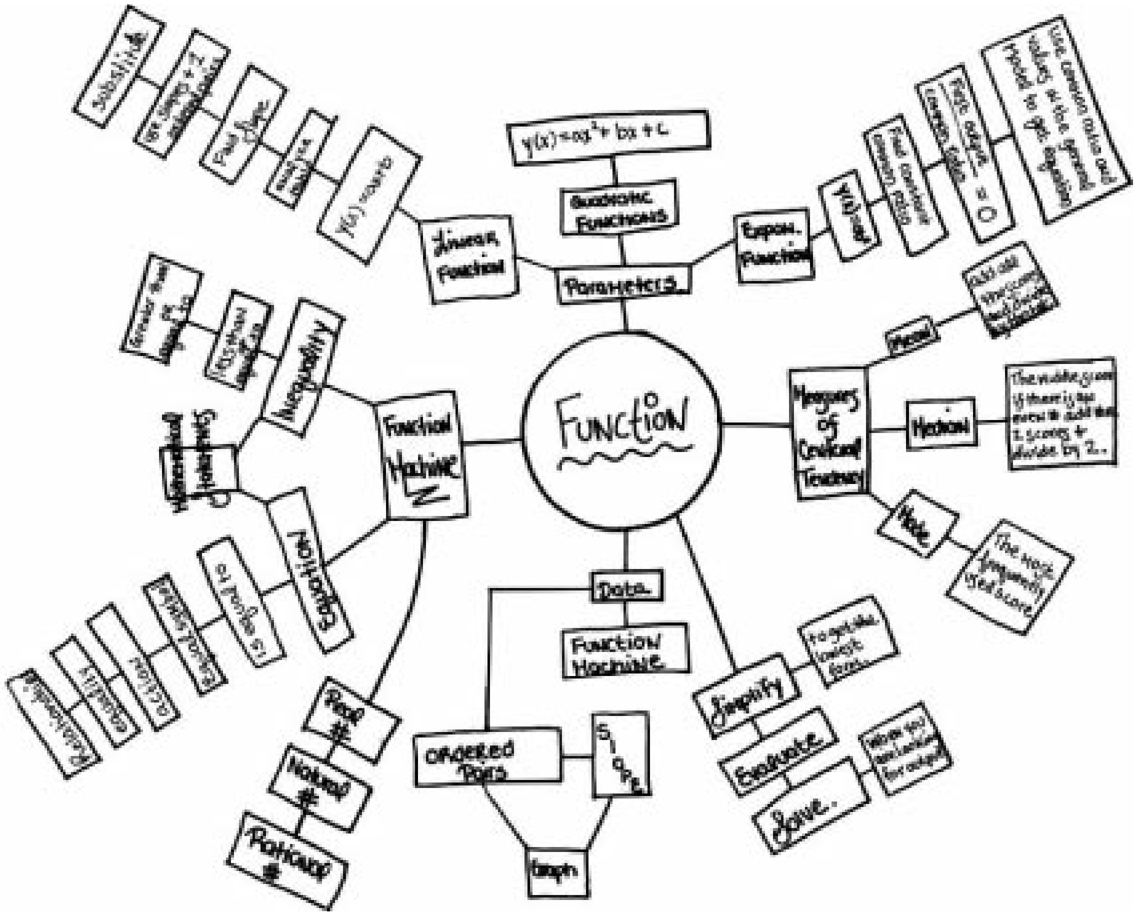


Figure 5: SK's concept map, week 15

limited to the computational procedures used to determine the parameters. Neither her classification schemes, nor her concept maps, reveal any interiority to these concepts or links to other concepts, to graphical representations or to alternative strategies for finding parameters. She demonstrated no ability to reverse a direct process in any context at any time in the semester. On at least two occasions, she retrieved and used two different approaches without realising her responses were inconsistent. She readily admits she is unable to distinguish between a linear, quadratic, and exponential function, even after sixteen weeks of investigation of these three function types. Confidence in the correctness of her answers decreased over the semester, and her attitude became increasingly negative.

The more successful student MC began with considerable lack of confidence in his algebraic skills. Despite this, he was able to select an appropriate alternative strategy when necessary, using the list, graphing, and table features of a graphing calculator. His ability to translate among representations is documented. His work suggests that he has formed mental connections linking the notions of zeros of the function, $x$-intercepts, general quadratic form and the specific algebraic model appropriate to the problem situation. He relates new knowledge to his previously acquired knowledge, building on the cognitive collage he has already constructed. The interview data indicates that he was able to deal with both direct and reverse processes, and recognizes them as two distinct

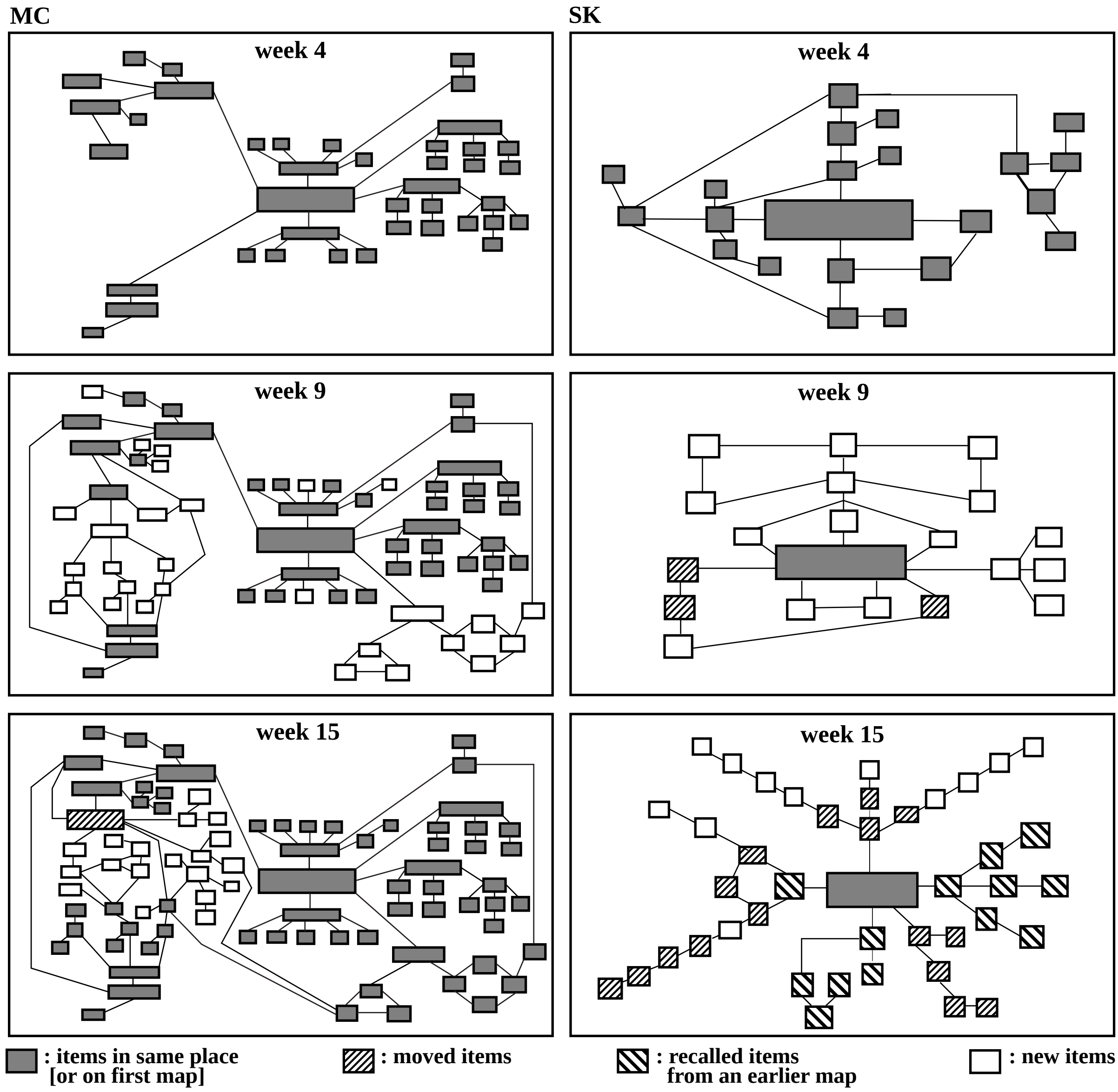


Figure 6: First concept map and successive schematic diagrams for MC and SK

but related processes. He was able to translate flexibly and consistently among various representational forms (tables, graphs, traditional symbolic forms, and functional forms). Confidence in the correctness of his answers increased over the course of the sixteen weeks. An examination of his work suggests that he initially focuses on the mathematical expression as an entity, then parses it as necessary, exhibiting the flexibility of process and concept necessary for more sophisticated study.

The use of schematic diagrams reveals these radically different developments (figure 6). It is immediately apparent that the basic structure of MC's first concept map is retained and extended in week 9 then further extended in week 15. However, the concept maps of SK seem to start almost anew each time, with few similarities and almost no basic structure that remains intact. Whilst MC builds from solid anchoring concepts, developing a strongly linked cognitive collage, SK builds on sand and each time the weak structure collapses only to be replaced by an equally transient structure.

## Comparison with other students

Analysis of the other selected students reveals striking similarities among the schematic diagrams for each group. *Each student in the more successful group produced a sequence of concept diagrams whose schematic diagrams retained the basic structure of the first within a growing cognitive collage.* Each set of schematic diagrams for the least successful also exhibited a common characteristic: *a new structure replaced the previous structure in each subsequent map, with few, if any elements of the previous map retained in the new structure*. *No basic structure was retained throughout.*

Triangulating this information with other data reveals that the basic classification schemes used by both groups of extremes confirm the concept map and schematic diagram analyses. The more successful have processes of constructing, organizing, and restructuring knowledge that facilitate the building of increasingly complex cognitive structures with rich interiority. Their basic anchoring structures, are retained and remain relatively stable, providing a foundation on which to construct cognitive collages whose concept maps are enhanced by imaginative use of layout, colours, and shape.

The concept maps and schematic diagrams of the least successful reveal the fragmentary and sparse nature of their conceptual structures. No category appears on all three maps of any individual student, nor even was there a single category common to all four of these students. As new knowledge was acquired, new cognitive structures and new categories were formed, with few, if any, previous elements retained. Even those that were retained were reclassified and used in new categories with a different classification scheme.

## Reflections

This study reports a wide divergence in the quality of thinking processes developed by remedial algebra students using graphing technology. High achievers can show a level of flexible thinking building rich cognitive collages on anchoring concepts that develop in sophistication and power. The lower achievers however reveal few stable concepts with cognitive collages that have few stable elements and leave the student with increasingly desperate efforts to use learned routines in inflexible and often inappropriate ways.

There remains the question of whether we are looking at these students through suitable lenses. Recent research offers new insights into the roles of perception and categorization (Rosch 1976), Lakoff (1987) which resonate with modern neuro-psychological theories of how the brain functions (e.g. Edelman, 1992) and how the evolution of the brain supports certain kinds of brain structure more than others (Dehaene, 1997). Within such wider frameworks we must ask “What if students like SK are organizing their knowledge according to a classification scheme which is not currently recognized or understood?” There exists the possibility that some students have different ways of knowing—ways of perceiving, categorizing, constructing, organizing, and restructuring knowledge—which those of us engaged in the teaching and learning of mathematics are unfamiliar with and have failed to consider. When one considers the significant improvement of the most successful students, the conundrum remains of how and why students like SK—who claim to want to connect new knowledge to old—appear unable to integrate new knowledge into existing structures.

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