Changing Attitudes to Mathematics through Problem Solving

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University mathematics may be presented in a formal way that causes many students to cope by memorising what they perceive as a fixed body of knowledge rather than learning to think for themselves. This research studies the effects on students' attitudes of a course which encourages co-operative problem-solving coupled with reflection on the thinking activities involved. A pre- and post-test revealed that attitudes changed significantly during the course. Half the students stated beforehand that university mathematics did not make sense. A majority of these declared negative attitudes to mathematics as abstract facts and procedures to be memorised, reporting anxiety, fear of new problems and lack of confidence. After the course all measures investigated improved, confirming that appropriate problem-solving can alter students' perception of mathematics as an active thinking process.

The teaching of university mathematics may be so formal that it teaches "the product of mathematical thought rather than the process of mathematical thinking" (Skemp, 1971). Over the last decade, a ten week, 30 hour problem-solving course has been taught at Warwick University based on the framework of Mason, Burton & Stacey (1982) to encourage students to improve their mathematical thinking. In written responses, students had often commented on their difficulties with formal mathematics:

> Maths education at university level, as it stands, is based like many subjects on the system of lectures. The huge quantities of work covered by each course, in such a short space of time, make it extremely difficult to take it in and understand. The pressure of time seems to take away the essence of mathematics and does not create any true understanding of the subject. From personal experience I know that most courses do not have any lasting impression and are usually forgotten directly after the examination. This is surely not an ideal situation, where a maths student can learn and pass and do well, but not have an understanding of his or her subject. *Third Year Warwick Mathematics Student, 1991*

The aim of the problem-solving course is to provide students with an alternative view of mathematics as a living activity. Each week students solve problems together in a two hour session then reflect on their actions in a one hour seminar. They are never given solutions, but are encouraged to share ideas and reflect on their effectiveness. Skemp's theory (1979) of goals and anti-goals proves valuable in allowing them to focus on the cognitive source of difficulties rather than the emotional symptoms.

The Universiti Teknologi Malaysia has similar problems. There is a wider range of ability (from the 50th to 90th percentile, with the top 10% going abroad for their education). The students are dutiful and eager to please their teachers by working hard and learning the procedures to pass the examinations. As one student commented – "To be good in maths requires good memory and lots of practice".

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Based on a pilot study in the University of Warwick, it was hypothesised that the attitudes of UTM students could be fundamentally changed through a problem-solving course of the type outlined above. The first-named author translated the materials to Bahasa Malaysia (the language of instruction in the UTM) and taught a ten week, thirty hour course.

The 24 males and 20 females taking part in the research were a mixture of third, fourth and fifth year undergraduates aged 18 to 21 in SSI (Industrial Science, majoring in Mathematics) and SPK (Computer Education), covering the full honours degree range (see table 2 below). In the first two hour meeting each week, after an introduction by the instructor, the students spent the major part of the time working on the problems in their own in small groups of 3 or 4. After half an hour or so the instructor reviewed the situation, to see how well things are progressing, ensuring that everyone is solving the same problem and considering ideas generated by the students. She gave no clue nor made any attempt to lead the students towards a possible solution. They were encouraged to experience all aspects of mathematical thinking—formulating, modifying, refining, reviewing problems and their solutions, specialising to simple cases, generalising through systematic specialisation, seeking patterns, conjecturing, testing and justifying. During the one hour meeting, the students were encouraged to reflect on their mathematical experience and talk about their attempts to solve problems. The instructor commented on the effectiveness of the solutions, discussing where things may have gone wrong, where students may have failed to take advantage of certain things, and ended by summarising what the students had done.

The students' performance and attitudes were monitored by classroom observation, a questionnaire at the beginning and end of the course, and semi-structured interviews.

Classroom Observation

Initially, the students were very confused. They kept asking questions like "What shall I do now?" "Is this the right way of doing it?" when they became stuck after a frantic attack on the given problem. For a few weeks they showed enormous resistance. Little by little the resistance was worn away until, after four weeks, they were slowly able to make decisions and think for themselves. By this time they began to write a "rubric" commentary outlining their problem-solving activity.

Their knowledge of mathematics is sufficient to solve all the given problems. At first they were set simple problems which helped tremendously in giving them a sense of success to help build self-confidence, not only in those who were unwilling to tackle something new because they had failed in the past, but also those who were successful in regular mathematics courses. The instructor herself has not solved all the problems given in the course. On several occasions she solved a problem in front of the class, showing that even a mathematician does not produce a neat, straightforward textbook proof. This encouraged students to feel less reluctant to make conjectures which might prove to be wrong on the possible route to success. Their discussion became livelier as they moved into doing things that they could explain to their friends, rather than simply satisfying the course requirements or pleasing the instructor. Their problem-solving became " a more creative activity, which includes the formulation of a likely conjecture, a sequence of activities testing, modifying and refining," (Tall, 1991).

Questionnaire

An attitudinal questionnaire on mathematics and problem-solving, based on common responses in a Warwick pilot study, was given to the students in the first week of term and again following the course.

Section A: Attitudes to Mathematics

1. Mathematics is a collection of facts and procedures to be remembered.
2. Mathematics is about solving problems.
3. Mathematics is about inventing new ideas.
4. Mathematics at the University is very abstract.
5. I usually understand a new idea in mathematics quickly.
6. The mathematical topics we study at University make sense to me.
7. I have to work very hard to understand mathematics.
8. I learn my mathematics through memory.
9. I am able to relate mathematical ideas learned.
10. In a few sentences describe your feelings about mathematics.

Section B : Attitudes to Problem-Solving

1. I feel confident in my ability to solve mathematics problems.
2. Solving mathematics problem is a great pleasure for me.
3. I only solve mathematics problems to get through the course.
4. I feel anxious when I am asked to solve mathematics problems.
5. I often fear unexpected mathematics problems.
6. I feel the most important thing in mathematics is to get correct answers.
7. I am willing to try a different approach when my attempt fails.
8. I give up fairly easily when the problem is difficult.

Responses

Under each statement, students responded to a five-point scale: **Y, y, –, n, N** (i.e. definitely yes, yes, no opinion, no and definitely no). Their responses are given in Table 1. Rather than have separate columns for Y and y (which then have to be added to give the total "yes" response), the table has total "Yes" (Y+y) in **bold** print and a column for the subset "Y" who express the stronger definite opinion. The table reveals, for example, that 34 students (77%) regard mathematics as facts and procedures to be remembered, with a subset of 18 (41%) expressing this opinion strongly (a "definite yes").

Mathematics	**Yes**	Y	**No**	N	–
facts & procedures	**34**	18	**8**	2	**2**
solving problems	**27**	10	**16**	4	**1**
inventing new ideas	**21**	4	**21**	6	**2**
abstract	**25**	13	**17**	0	**2**
understand quickly	**9**	0	**30**	5	**5**
makes sense	**22**	4	**22**	5	**0**
work hard	**37**	15	**5**	1	**2**
learn by memory	**30**	1	**12**	2	**2**
able to relate ideas	**24**	8	**18**	2	**2**

Solving Problems	**Yes**	Y	**No**	N	–
confidence	**26**	7	**17**	2	**1**
pleasure	**43**	25	**1**	1	**0**
only to get through	**16**	4	**27**	8	**1**
anxious	**17**	1	**24**	4	**3**
fear unexpected	**30**	10	**12**	3	**2**
correct answers	**21**	4	**21**	3	**2**
try other approach	**42**	17	**0**	0	**2**
give up	**19**	3	**24**	9	**1**

Table 1 : Responses for 44 students on the Pre-test Questionnaire

The table in this form conceals valuable information about individuals. Responses to the statement that mathematics "makes sense" divides the students into equal groups, Group S consisting of the 22 students for whom it does and Group N, the 22 students for whom it does not. Table 2 shows the degree classification of the groups.

		Degree Class											
		Group N (22 students)						Group S (22 students)					
		I	II-1	II-2	III	P	F	I	II-1	II-2	III	P	F
Students	SPK year 5	2	5	2	1	0	0	0	3	3	0	0	0
	SPK year 4	1	5	3	0	0	0	2	6	4	1	0	0
	SSI year 3	0	3	0	0	0	0	1	2	0	0	0	0
	Total	**3**	**13**	**5**	**1**	**0**	**0**	**3**	**11**	**7**	**1**	**0**	**0**

Table 2 : Students for whom mathematics makes sense (Group S) and does not (Group N)

The two groups have almost identical distributions, so there is no correlation between examination success and whether the students consider mathematics makes sense. Table 4 shows the data for the two groups on the pre- and post-test for Part A. Items underlined in column 1 show a significant (<5%) or highly significant (<1%) change in the total **Yes** response (compared with **No** and **–**) using the χ^2 test with Yates' correction. Underlining in the **Yes** columns refers to significant changes for each group.

Mathematics		Group N (22 students)										Group S (22 students)									
		Yes		Y		**No**		N		–		**Yes**		Y		**No**		N		–	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
facts & procedures	↓	20	6	16	2	1	16	1	4	1	0	14	5	2	1	7	16	1	4	1	1
solving problems	↑	9	21	2	10	13	0	4	0	0	1	18	21	8	11	3	0	0	0	1	1
inventing new ideas	↑	6	18	1	8	14	2	4	0	2	2	15	19	3	7	7	3	2	0	0	0
abstract	↓	21	11	12	7	1	8	0	0	0	2	4	4	1	1	16	18	0	3	2	0
understand quickly	↑	4	8	0	0	13	12	2	1	5	2	5	12	0	3	17	9	3	1	0	1
makes sense	↑	0	14	0	2	22	7	5	0	0	1	22	21	4	3	0	0	0	0	0	1
work hard	↓	19	15	6	5	1	5	0	0	2	2	18	13	9	3	4	8	1	0	0	1
learn by memory	↓	17	6	1	0	3	15	1	3	2	1	13	5	0	0	9	16	1	4	0	1
able to relate ideas	↑	6	14	1	4	14	7	2	0	2	1	18	21	7	7	0	1	0	0	0	0

Table 3 : Responses to Part A of the Questionnaire

Figure 1 shows the percentage of total “Yes” responses (Y plus y) to the statement that “university mathematics makes sense”. Group S remains at 100% positive before and after, but 64% of Group N change from “No” to “Yes”.

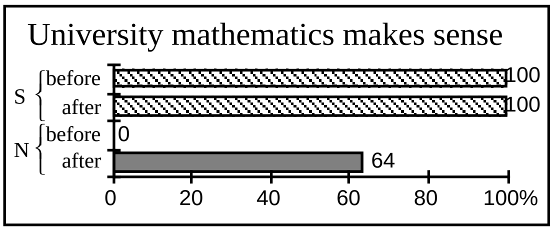

Figure 1 : University mathematics “makes sense”

Figure 2 shows the same bar-chart layout for the other statements, re-arranged to place related statements side by side.

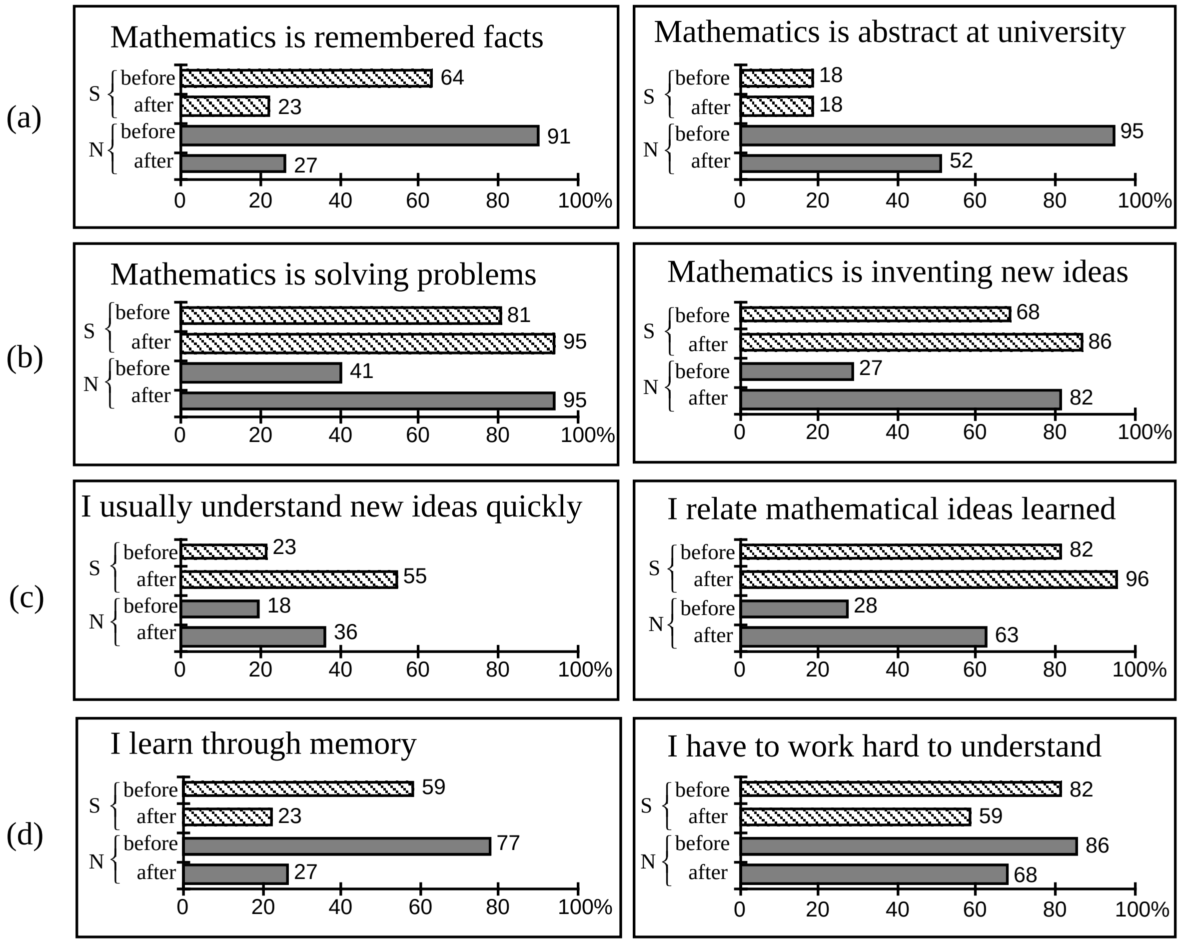

Figure 2 : Attitudes to Mathematics

Each graph tells a consistent story, supported by significance tests from Table 1.

(a) There is a significant decrease in the perception of mathematics as facts and procedures to be remembered (from 91% to 27% in Group N) and the perception that university mathematics is “abstract” remains low in Group S, whilst diminishing significantly in Group N.

(b) A significant overall increase in perception of mathematics as solving problems and inventing new ideas, changing mainly in group N.
(c) Significantly more students overall now claim that they understand ideas quickly and can relate mathematical ideas together (mainly group N).
(d) Significantly fewer students overall claim they have to memorise ideas and fewer claim to work hard to understand (not statistically significant).

Results of Section B : Attitudes to Solving Problems

The data from section B is given in table 4.

Solving Problems		Group N (22 students)										Group S (22 students)									
		Yes		Y		No		N		–		Yes		Y		No		N		–	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
confidence	↑	9	16	1	3	13	5	1	0	0	1	17	20	6	9	4	1	1	0	1	1
pleasure	↑	21	21	11	9	1	0	1	0	0	1	22	21	14	12	0	0	0	0	0	1
only to get through	↓	14	4	4	0	8	16	2	9	0	2	2	0	0	0	19	21	6	8	1	1
anxious	↓	12	6	1	0	8	15	1	1	2	1	5	0	0	0	16	21	3	8	1	1
fear unexpected	↓	17	6	7	2	3	14	1	2	2	2	13	4	3	1	9	17	2	7	0	1
correct answers	↓	13	2	3	0	8	19	2	5	1	1	8	3	1	1	13	17	1	6	1	2
try other approach	↑	21	21	8	9	0	0	0	0	1	1	21	22	9	11	0	0	0	0	1	0
give up	↓	13	3	3	0	8	19	3	8	1	0	6	2	0	0	16	18	6	12	0	2

Table 4 : Responses to section B of the Questionnaire

A subset of these results is displayed pictorially in Figure 3, re-arranged as before.

(e) Confidence increases overall from pre-test to post-test and the pleasure in solving problems starts very high and stays there.
(f) Anxiety when faced with problems is reduced to a low level in both groups (zero in Group S), whilst fear of the unexpected falls significantly.
(g) Willingness to try a new approach remains very high in both groups; the tendency to give up with a difficulty reduces significantly to a low level.
(h) Extrinsic pressure—solving problems only to get through the course—reduces significantly overall, and (possibly due to a willingness to try out ideas) concentration on getting correct answers reduces significantly.

Before the course, Group N display many negative qualities, 64% are motivated to do problems "only to get through the course" and, though all but one derives pleasure from solving problems, 55% feel anxious, 77% experience fear of the unexpected, 59% give up easily and only 44% feel confident. In contrast, Group S has a majority of positive responses everywhere except for a fear of the unexpected — they are confident, take pleasure in getting solutions, have low anxiety, are willing to try a new approach without giving up too easily, and see mathematics as more than getting right answers.

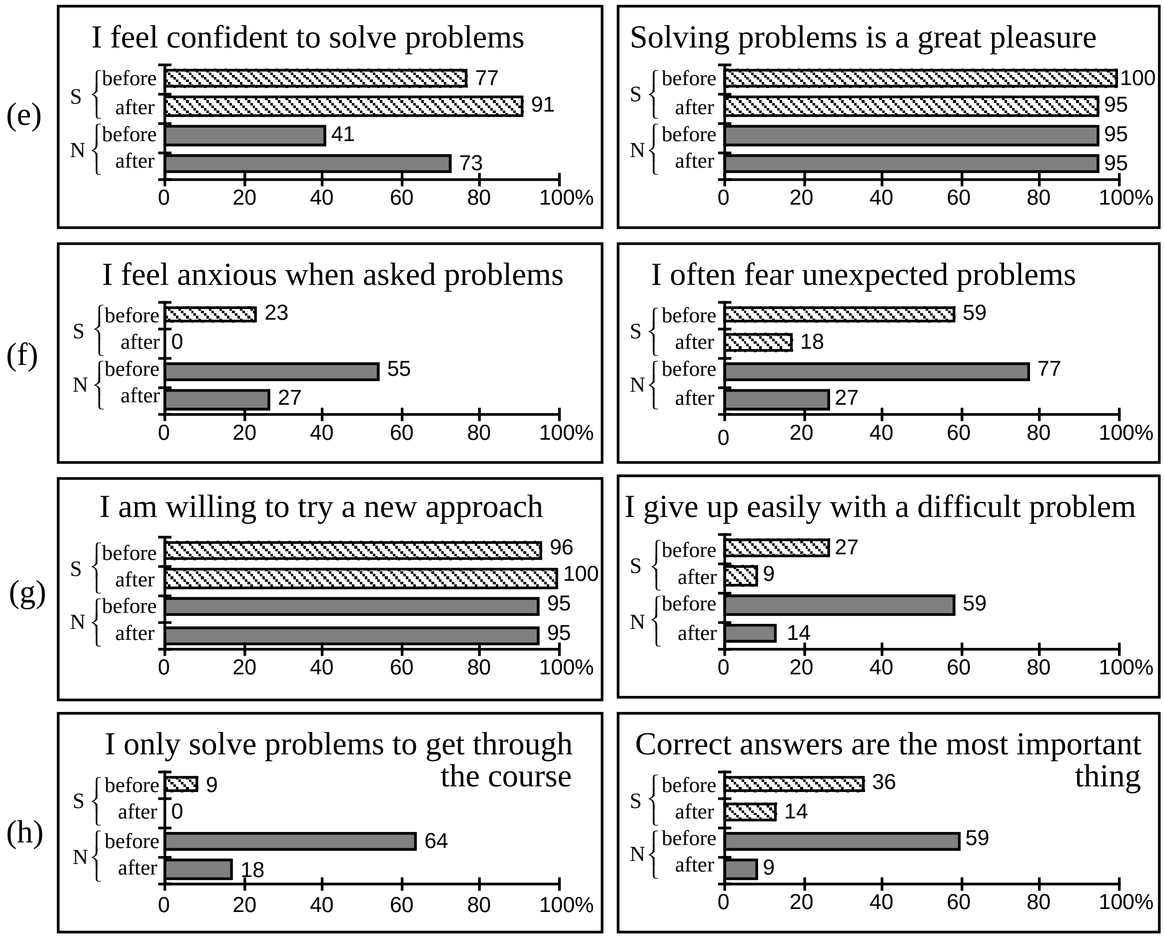

Figure 3 : Attitudes to Problem-solving

On all items in both section A and B there is a positive change, except for some so extreme that little change is possible. In many cases the marked distinctions between Group N and Group S is considerably lessened. In particular, increase in confidence (graphs (e)) is associated with viewing the task as a positive goal to be achieved, and decrease in anxiety and fear (graphs (f)) is associated with the diminution of the negative feeling of wanting to avoid failure (an anti-goal in Skemp's theory, 1979).

Student comments

In the questionnaire, the students were asked to write a few sentences describing their feelings about mathematics. In the pre-test, the feelings of both groups seemed very much influenced by three factors: the nature of mathematics, personal feelings (such as motivation, interest, pressure etc.) and teaching methods. In Group N, of 7 responses related to the nature of mathematics; 5 were negative saying it is 'too abstract', 'seems pointless', and 'theory more difficult than practice'. Thirteen responses relating to personal factors included 10 negative, such as 'lack of motivation', 'put off by amount that needs to be done' and 'puzzled by what is going on'. The positive feelings were mostly about it being 'enjoyable and challenging', 'great sense of satisfaction when able to understand new concepts and to solve problems' and 'effort put in is worthwhile'.

Five responses relating to teaching were all negative—such as ‘difficult to follow’ and ‘delivered in a dull atmosphere’. In Group S, there were 5 responses related to the nature of mathematics (3 positive), 14 responses about personal factors (12 positive) and 3 responses about teaching factors (all negative). Overall, in the pre-test, only 22% in Group N express positive feelings in comparison to 68% in Group S.

After the course the comments improve dramatically as is shown by Table 5. (The percentages do not add up to 100% due to responses including more than one factor.)

	Group N				**Group S**			
	Positive		**Negative**		**Positive**		**Negative**	
	Pre	**Post**	**Pre**	**Post**	**Pre**	**Post**	**Pre**	**Post**
nature of mathematics	9	23	23	5	14	32	9	5
personal	14	64	45	14	55	64	9	9
teaching	0	0	23	14	0	0	14	5

Table 5 Classification of written responses

Comments written after the course include:

> Maths has always given me a lot of problems because I don’t have the ability for memorisation. … Now that I know about mathematical thinking, my interest and desire to learn maths have increased. (male, 4SPK)

> This is the first time that I have actually used maths to think. Before I just learnt maths to pass the exam. (female, 4SPK)

> The way maths is taught here, it seems as though it is difficult and boring. … There is no opportunity to display one’s creativity. This makes it real dull and frustrating. (male, 5SPK)

> I am beginning to think instead of just doing the tutorial questions. … I think I am learning more because I understand what is going on. (female, 3SSI)

> The course should have been introduced earlier. … After following the course I am more confident to solve any maths problem that is given. (female, 5SPK)

These are consistent with the classroom observations and the changes intimated by the questionnaire, supporting the hypothesis that the course in problem-solving changes the attitudes of students from mathematics as a body of procedures to be learned to mathematics as a process of thinking.

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