Gender and the versatile learning of trigonometry using computer software

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This empirical study tests the hypothesis that the versatile learning of trigonometry using interactive computer graphics would lead to a greater improvement in the performance of girls over boys. The experiment was carried out with 15 year old pupils in two schools with matched entry standards, each subdivided by ability into four corresponding mixed gender groups. In every case, experimental boys improved more than control boys and experimental girls improved more than control girls. However, whilst the control girls performance deteriorated compared with the control boys, the experimental girls performance improved in comparison with the experimental boys, eventually becoming superior in all but the least able group.

Difficulties in the learning of trigonometry

The initial stages of learning the ideas of trigonometry are fraught with difficulty, requiring the learner to relate pictures of triangles to numerical relationships, to cope with ratios such as sin*A*=*opposite/hypotenuse* and to manipulate the symbols involved in such relationships. Ratios prove to be extremely difficult for children to comprehend (Hart 1981), and modern texts have responded to the perceived difficulties by introducing the sine of an angle not as a ratio, but as the opposite side length in a right-angled triangle with unit hypotenuse which must be recognized with the triangle rotated into any position.

Further difficulties occur as the child must conceptualize what happens as the right-angled triangle changes size in two essentially different geometric and dynamic ways:

- as an acute angle in the triangle is increased and the hypotenuse remains fixed, so the opposite side increases and the adjacent side decreases,
- as the angles remain constant, the enlargement of the hypotenuse by a given factor changes the othe r two sides by the same factor.

The traditional approach uses pictures in two different ways, each of which has its drawbacks. Rough sketches of triangles may give the impression that the numerical procedures are the only way to get accurate results causing a possible schism between the use of pictures and numerical procedures. On the other hand, if the children draw an accurate picture, this focuses on the production of one static picture rather than the visualization of dynamically changing relationships.

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A computer approach can change all this by allowing the child to manipulate the picture and relate its dynamically changing state to the corresponding numerical concepts. It therefore has the potential of improving understanding. This ability to use the computer to carry out certain arduous constructions whilst the child can focus on specific relationships we call the *principle of selective construction*. We believe this to be one of the most powerful educational principles for the use of the new technology.

Gender differences in mathematical performance

Empirical evidence shows that although girls perform at least as well as boys in mathematics in the early years, as they get older a divergence in performance becomes apparent, particularly amongst the higher ability groups. For example, the percentage of boys and girls obtaining the highest grades (A,B,C) in the U.K. examinations taken at the age of sixteen (the General Certificate of Secondary Education) is as follows:

	% obtaining grade A		% obtaining grades A,B,C	
	1985	1987	1985	1987
Boys	9.4%	9.1%	41.2%	40.5%
Girls	4.8%	5.2%	33.7%	33.5%

(Source: D.E.S. 1989)

table 1

There is also evidence that girls of this age are less successful at visuo-spatial tasks than boys, particularly in the highest ability groups. We hypothesised that if software was designed to link together spatial concepts and numeric and symbolic data, then this may aid girls in perceiving linkages where the traditional UK curriculum leaves them with a perceived deficiency. In addition, observed differences in social behaviour in our earlier experiments (Tall & Thomas 1988), in which girls were seen to be more likely to cooperate, whilst boys would often compete, might enhance the girls' success rate at gaining conceptual insight.

Software designed to improve versatile learning of trigonometry

To improve versatility in relating numerical and visual cues a simple piece of software was designed linking numerical input for lengths and angles to a visual display of a right-angled triangle ABC . Any of the sides and angles can be specified and, once sufficient information is available, say two sides, or a side and an angle (in addition to the fixed right angle), the triangle will be drawn.

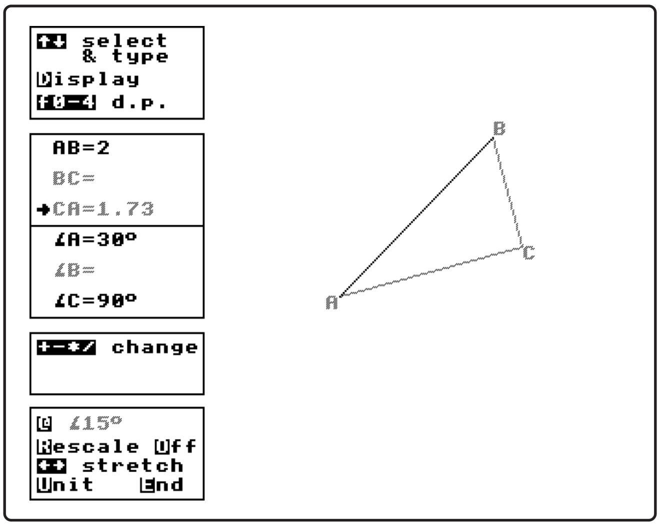


Figure 1: the screen display of the trigonometric software

The user may then choose to reveal any other sides or angles and manipulate the data in various ways, for instance to add, subtract, multiply or divide any length or angle by a given quantity. The triangle may be rotated and drawn either to a fixed scale or auto-scaled so that it remains the same size onscreen when all sides are adjusted by the same factor. By showing *examples* of trigonometric phenomena relating numeric and geometric representations, the software acts as a *generic organizer* in the sense of Tall 1986, allowing the teacher to demonstrate, and the pupils to explore, examples of the concepts being introduced. The teacher needs to act as an *organizing agent*, focussing the pupil on the important ideas, then encouraging pupil interaction with the software to enable them to make personal constructs of the interrelationship of the mathematical ideas. By using the principle of selective construction to focus on what are normally considered to be higher order relationships it was hypothesised that the students would gain more versatile understanding of trigonometric concepts.

Experimental design

The experiment was carried out in Kenilworth School which teaches the full range of ability and is organized as two parallel halls for pupils in the age range 12-16. In each hall the children aged 14-15 are streamed into four mixed gender classes according to ability. One school was selected as the control, with the classes taught by the normal class teachers, the other was given the experimental treatment by one of the authors (a former teacher at the school). The experimental teacher, travelling some distance to the school, had little time to do any more than arrive for his class, teach, then leave again, and any advantages, with the exception of the use of the computer, tended to accrue to the control groups.

A pre-test given to all but the least able confirmed that there was no significant difference between the groups being compared (see tables 5-8 below). All groups had 4 lessons of 1 hour 10 minutes and 4 lessons of 35 minutes in a two week period, followed by an immediate post-test and a second post-test eight weeks later.

After the first post-test the teachers of control groups 1 and 2 chose to discuss their student's mistakes and give them extra practice before the second post-test. This could be construed as giving these pupils an advantage over the experimental students but, as we will see, it did not improve their relative performance.

Aims in teaching trigonometry with the computer

The computer representation enabled the students to explore the relationship between numerical and geometric data in an interactive manner. For instance, at an early stage they used the computer to start with an angle of 10˚ and to tabulate the changing values of the opposite and adjacent sides as the angle is increased in steps of 10˚, giving them early insight into the complementary relationship between the increasing table of sines and the decreasing table of cosines. From the outset they were encouraged to make dynamic links between visual and numerical data which is less apparent in a traditional approach.

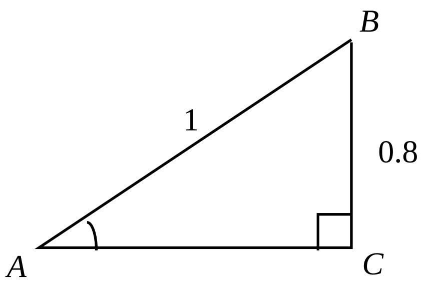
Over the two week period the experimental lessons developed in a versatile manner: using the computer to estimate numerical values before checking them with the computer and with a hand-calculator : estimating sides for given angles and angles for given sides, scaling triangles up or down in size, solving related problems. Three computers were available for group use, with care taken to give the girls their fair share. This was only a problem in experimental group two where the boys competed to try to monopolise the computer.

Students of all abilities found the experimental approach reasonably straightforward, with the less able encountering problems for instance in handling decimals less than one. Even the least able became adept at using the computer and, though they had some difficulty writing down their results, they had few difficulties with visualization. This has obvious implications for the rigidity of differentiated curriculum schemes, where it is decided that certain conceptual structures are too difficult to teach certain children.

Differences in responses of experimental and control pupils

The differences in conceptual development are shown most clearly on the delayed post-test which consisted of 5 problems to test more standard trigonometric techniques and 6 to test versatility in handling conceptual ideas.

In some routine questions, the control students were able to perform as well or better, but overall the experimental students had a significant advantage. (Figure 2, Table 2).



The sine of ∠A is
a) 90˚
b) 1.6
c) $\frac{1}{0.8}$
d) 1
e) 0.8
f) don't know

Figure 2

Responses to Figure 2	Performance of Groups (%)			
	1	2	3	4
Experimental	84	88	62	59
Control	72	50	14	-

Table 2

In less routine examples the more able experimental students regularly performed better than the corresponding controls, with the less able experimental students more than holding their own (figure 3, table 3):

Draw a sketch of a triangle *ABC*, where $\angle C = 90^\circ$, $AB=4$ and $\sin A = \cos A$. Use a ruler, and mark the values of angle *A* and angle *B*.

Note: this means the sine of angle *A* is the same as the cosine of angle *A*.

Figure 3

Responses to Figure 3	Performance of Groups (%)			
	1	2	3	4
Experimental	90	84	44	50
Control	55	44	45	-

Table 3

Table 4 shows the mean marks attained by each group on a school exam (given at the end of the previous year), followed by the pre-test and post-tests 1 & 2, with the latter divided into standard questions (S) and more versatile questions (V). The experimental students perform at least as well on standard questions and outscore the control students on versatile questions. Those cases where the mean of the experimental score is greater than that of the control score at a level $p<0.05$ are denoted by *sig*.

	Exam	Pre-test	Post-test 1			Post-test 2		
			S	V	Total	S	V	Total
E1	61	37	80	78	79	89	83	86
C1	70	39	80	47	64	86	50	68
			–	*sig.*	*sig.*	–	*sig.*	*sig.*
E2	47	18	45	56	51	57	66	62
C2	48	20	46	38	42	39	41	40
			–	*sig.*	*sig.*	*sig.*	*sig.*	*sig.*
E3	47	20	47	52	50	57	66	62
C3	49	21	31	28	30	24	23	23
			sig.	*sig.*	*sig.*	*sig.*	*sig.*	*sig.*
E4	39	*not given*	22	48	35	22	37	29
C4	39	*not given*	5	17	11	*not given*		
			sig.	*sig.*	*sig.*			

Table 4: significance (p<0.05) of results of post-tests by group and type of question

Statistical differences between girls and boys

The performances of the various sub-groups are given in tables 5-8. In every group (except the least able, where full data was not gathered) from pre-test to (delayed) post-test, experimental boys improved more than control boys and experimental girls improved more than control girls. In each case the girls started out with a lower (or) equal score to the boys on the pre-test, but by the delayed post-test, the control girls improved *less* than the control boys, the experimental girls improved *more* than the experimental boys. In group 2 the experimental boys do not perform as well as the control boys on the first post-test. This was the one group where the experimental teacher found difficulty with the boys who both tried to compete in the use of the computer and also pressed the teacher to give them procedural ways of finding solutions. Even here, where the control pupils had extra teaching prior to the second post-test, the eventual superiority of the experimental pupils was shown. In particular, even though the control teacher tried the much-approved method of getting the pupils to reflect on and discuss their errors, he failed to bring about an improvement, indeed, the control pupils showed a marked deterioration in solving versatile questions involving the need to translate a word problem into a diagram.

Group 1	Experimental		Control	
	Boys (N=8)	Girls (N=17)	Boys (N=13)	Girls (N=16)
Pre-test	44 %	33 %	44 %	33 %
Post-test 1	75 %	82 %	70 %	61 %
Post-test 2	72 %	92 %	70 %	62 %

Table 5

Group 2	Experimental		Control	
	Boys (N=12)	Girls (N=13)	Boys (N=15)	Girls (N=11)
Pre-test	18 %	18 %	22 %	17 %
Post-test 1	43 %	57 %	50 %	31 %
Post-test 2	59 %	65 %	49 %	28 %

Table 6

Group 3	Experimental		Control	
	Boys (N=8)	Girls (N=13)	Boys (N =11)	Girls (N=10)
Pre-test	22 %	20 %	24 %	18 %
Post-test 1	56 %	46 %	24 %	36 %
Post-test 2	38 %	41 %	24 %	23 %

Table 7

Group 4	Experimental		Control	
	Boys (N=6)	Girls (N=6)	Boys (N=5)	Girls (N=9)
Pre-test	-	-	-	-
Post-test 1	34 %	34 %	15 %	8 %
Post-test 2	33 %	25 %	-	-

Table 8

Table 9 summarises the significant differences (*sig*) between experimental and control students using a two-way analysis of variance ($p<0.05$) to compare raw score in the two post-tests (in all cases the experimental students score more except for group 2 boys in post-test 1).

	Post test 1		Post-test 2	
	Boys	Girls	Boys	Girls
Exp v. Control 1	n.s.	sig	n.s.	sig
Exp v. Control 2	n.s.	sig	n.s.	sig
Exp v. Control 3	sig	n.s.	n.s.	sig
Exp v. Control 4	sig	sig	-	-

Table 9

This shows the difference between control and experimental boys on the delayed post-test to be statistically non-significant in all three groups tested and the difference between girls to be statistically significant. Only in the immediate post-test of control group 3 is this pattern not repeated.

Only in group 4 do the experimental boys outperform the experimental girls on the delayed post-test

Conclusions

The experiment confirmed the hypothesis that the experimental treatment using the generic organizer on the computer helped the experimental students to improve their performance compared with the control students. It also showed greater improvement in the experimental girls than the boys (except in the least able group).

The reversal of the hypothesis with the least able may be part of an overall trend: that the computer helps students (of either gender) lacking versatility in linking numerical to visual skills. This is consistent with the generally observed tendency for boys' performances to have a greater standard deviation, with more boys performing well at the top end and more performing badly amongst the low achievers than the girls who cluster nearer the mean. It may be that those who gain an advantage from the computer are the less versatile girls (at or above the mean ability range) and boys at the bottom of the ability range.

Whilst there was some evidence in experimental group 2 that the boys attempted to assume a dominant role in the group, there is no evidence that this aggression translated into superior performances. The question as to whether the differences are social or genetic or due to other factors remain open. What is evident is a real improvement in the average and above average girls compared with the corresponding boys using the computer, which shows itself in an increased ability to think in a versatile way linking visual and numerical skills.

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