"Mathematics 15-19 in a Changing Technological Age"

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1. Introduction

The arrival of the new technology brings us to the threshold of an exciting new phase of development in the history of mankind, which will affect every aspect of life, not least in mathematical education where it heralds the second major revolution in thirty years. However, there is a massive difference in scale between the introduction of "New Mathematics" in the 1960s and the current technological revolution. Whereas the former was largely a product of internal forces, caused by professional mathematicians dissatisfied with the mathematics curriculum, the latter is produced by technological forces beyond the control of the mathematical education establishment.

The sheer speed of the new technological revolution produces a problem of a new order of magnitude. At the last I.C.M.E. conference in Australia just four years ago, only a small minority of participants had even touched a computer. Yet technological change is so fast that what is unknown today will be commonplace tomorrow. Most of the papers (including my own) produced for the I.C.M.I. study on "The Influence of Computers" (Howson & Kahane 1986) were typed on electric typewriters. Two years later, in the developed countries, desk-top publishing using micro-computer and laser printer will soon render the electric typewriter virtually obsolete.

This bewildering pace proves exciting for those with the energy and finance to keep up with the latest developments, but it is threatening to many teachers bemused by the speed of change, and far beyond the current resources of those in third world countries. In practice, the spread of new technology to the wider public is far slower than the experts might hope. However, what is expensive to develop today is often cheap to mass produce tomorrow and it is prudent for us to be mindful of the far-reaching

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implications, not simply to see what is practical now, but also to develop a vision of the possibilities in the near future.

2. The Influence of Computers and Informatics on Mathematics and its Teaching

A number of international conferences have been held to try to divine the way ahead in the new technological age. The I.C.M.I. Conference at Strasbourg in 1985 devoted its attention to mathematics and teaching at university and senior high school level (Howson & Kahane 1986); the I.F.I.P. Conference in Sofia in 1987 considered the wider effects of informatics on the teaching of mathematics (Johnson & Lovis, 1987). In addition there have been national discussions on the the effects of computers on the 16-19 age range, in the United Kingdom, for example in Higgo (1985), Schwarzenberger & Johnson, (1985), and Everton (1987), and in the United States a task force of the Mathematical Science Education Board is developing a curriculum framework "appropriate to the needs of children and society in the year 2000" (Ralston 1987). What characterizes the international conferences is the exuberance and excitement of individuals putting forward new ways of approaching the curriculum. What must be born in mind, however, is the difference between the conception of an idea and its possible implementation in the real world curriculum.

> One other unusual factor makes curriculum development involving advanced technology more difficult than usual. It is the mismatch of time scales between technical change (one year) and curriculum change (ten years). The curriculum designer cannot assume a specific level of technological provision and sophistication in schools - it will vary widely both in time and from place to place. [Burkhardt 1986, page148]

Added to the technical considerations are the cultural and social factors related to real teachers operating in real classrooms. Winkelmann (1987) suggests that we should distinguish three different levels of computer use:

> Its use **in principle**: theoretical considerations of how I[nformation] T[echnology] might be used for teaching specific topics in a specific discipline; software developed for research purposes, but not available for the normal schoolteacher;
>
> Its use **in practice**: examples of actual use in the classroom, using normal equipment and generally available software, but maybe with the help of extra-ordinary motivated teachers, headmasters or parents;
>
> Its use **in reality**: observations on the percentage of the teachers of a certain discipline who actually use the new technologies in their teaching, on contents and themes which to a certain amount are really taught and/or changed by computers; considerations on necessary equipment, organisation, teacher training, curriculum development.

In the deliberations of our Action Group on mathematics in the 16-19 age group we should very much distinguish between these three levels: what we might like to see *in theory*, what evidence we have for what has proved possible *in practice*, and what we may expect *in reality*. For example, there are many examples of imaginative software that have been developed for use in the teaching of mathematics which are available in practice. However, a recent survey (Green & Jones 1986) in the United Kingdom reported that secondary schools had approximately one computer per 50 or so pupils; in reality 39% of the teachers used the computer very rarely in their mathematics lessons and 33% never used it at all.

3. The Challenge of the New Technology

Major problems that face us in the 15-19 age-group in mathematics education include:

> 1) Applications of mathematics are becoming more technologically diverse, leading to pressures to modify the mathematics curriculum.
>
> 2) A greater proportion of the population needs to be technologically aware because of the increasing use of technology and the decreasing birthrate in the developed countries.
>
> 3) Changes in technology are occurring so rapidly that individuals are required to be more flexible and capable of solving new problems as they arise.
>
> 4) The desire for new kinds of skill requires new methods of teaching and learning.
>
> 5) This in turn requires appropriate methods of assessment.

These needs for new skills and concepts may be counter-balanced by the the fact that:

> 6) Recent educational research is developing a growing understanding of students' conceptual difficulties and strategies for improving learning.
>
> 7) The computer changes our perception of the nature of mathematics, facilitating imaginative ways of approaching concepts that promise to make them more understandable to a new generation of students.

The breadth of technological change and the new need for flexibility suggests a move away from purely content oriented courses towards courses which balance content with experience in mathematical processes, including collecting, organising and interpreting data, mathematical investigations, modelling, problem-solving, and so on, with more accent on the active participation of the learner rather than the passive reception of information from the teacher. Moves have been made in this direction in the last decade to develop theory and practice but there is, as yet, little change in the reality of the classroom at the 15-19 level.

4.1 Curriculum Change

The pressures to modify the mathematics curriculum must be seen in their appropriate context. The mathematical education of students in the 15 to 19 age group varies widely in different countries. For instance, in Russia all 17 year olds study the calculus, in Hungary the figure is 50%, whilst in the U.K. only 10% (Howson 1987). Some countries offer only one mathematics course to all students, others offer different packages of subjects with mathematics appropriate to the choice: France has Baccalaureates in Natural Sciences, Mathematics/Physics, Engineering, Social Sciences and so on, each with its own appropriate mathematical content. In many countries the age-range 15-19 does not correspond to a natural administrative subdivision. In the United Kingdom, for example, the General Certificate of Secondary Education is studied in the years 14-16, after which compulsory schooling ends, with Advanced Level studies covering the age range 16-18.

Thus to attempt to speak of "mathematics 15-19" under a single heading is tempting fate. However, this is our brief at I.C.M.E. and it would be wise for us to focus on that period of education in school beyond the basic mathematics taken by the population, in the transition phase either to university life, or to more sophisticated applications of mathematics in society.

The original "New Mathematics" movement in Europe was inspired by the Royaumont Seminar of 1959, producing new curricula in many countries aimed:

> Firstly, to provide a better preparation for university study,
> secondly, to give all pupils an instrument for use in daily life.
> (Fehr 1961 p.105)

In the "New Mathematics", it was usually the professional mathematicians who led the way, with a structural approach to the subject. The curriculum was designed along mathematical lines, based on the major disciplines of mathematics, which in pure mathematics at this stage were taken to be algebra, geometry and the calculus, with leavenings of other topics including applications in mechanics, statistics and probability.

At the I.C.M.I. conference (Howson & Kahane 1986), algebra was seen to "remain of central importance in the secondary school curriculum" :

> The important thing, however, is not to have students achieve greater manipulative skill in algebra (e.g. in polynomial algebra), but rather to teach them to consider algebra as a natural tool for solving problems in many situations. (ibid page 13)

There was a note of regret at the loss of Euclidean geometry, though no consensus as to how this might best be remedied. It was considered that the loss of experience in understanding and constructing rigorous proof might have a detrimental effect on work in the new technology, as

> ... large numbers of people , particularly in scientific and technical professions, will need to handle statements of a logical or mathematical nature in a very precise fashion. (ibid page 21)

The major change in curriculum content suggested at the conference was to modify the ascendancy of the calculus by introducing computer science concepts of discrete mathematics:

> ... there is now a strong argument to provide a balance in the core curriculum between the traditional continuous mathematics topics and topics for discrete mathematics. (ibid page 15)

Other content changes suggested included those which become possible because of the power of the computer, for example, exploratory data analysis, in which the computer may be used to manipulate and display data in imaginative ways allowing the student to gain initial insight into statistical ideas.

There were diverging views expressed as to the value of programming, on the one hand, it was noted that the sheer technical considerations in implementing a program could sometimes overwhelm the mathematics, on the other hand, there is substantial research evidence that the cognitive act of engaging in programming could act as an aid to conceptualization. The nature of the language is an important factor. Programming can clearly help in practice (Dubinsky 1987, Tall & Thomas 1988), more empirical research is needed into its effect in reality.

4.2 Greater Technological Awareness

The need for more of the population to be technologically aware will mean that more students will need to study appropriate kinds of mathematics to a more advanced level. This will mean that courses will need to be suitably designed for a wider ability spectrum, from those with less academic ability to the most able. The degree to which this is necessary will depend on the current provisions. Some countries, who cater for mathematics to a minority in this age range may need to considerably widen their appeal.

The nature of the new mathematics required in the future is one which will require some thought. New technologies provide capabilities that relieve us from much mechanical drudgery and virtually all of the current algorithms taught in the 15-19 age range can be carried out automatically by computer. However:

> The availability of ready-made numerical and symbolic software will ... not result in a reduction in the required mathematical sophistication of the user. Quite the contrary ... much more demanding skills than purely algorithmic ones now have to be mastered. (Hodgson 1987, page 57)

On the other hand, methods of communicating with computers are being developed that allow the user to carry out sophisticated actions by intuitive decisions, pointing to a column of numbers that need adding, or moving information about onscreen, which means that, for the end-user, many complex activities will become simpler. The need to understand what is going on behind the computer facade is a suitable task for mathematics education to address. In theory this is a laudable aim: what will its effect be in practice and in reality?

4.3 Problem-solving

A cultural pressure is growing to respond to the speed of technological change by developing a resourcefulness in our children through a greater problem-solving approach in education. The underlying principle is that if we cannot predict what will be needed tomorrow, then the best response is to furnish the next generation with the mental tools to face new challenges. However, we should not underestimate the difficulties of implementing such an approach. Unlike the "new mathematics" of the 1960s, which was at least known to teachers with recent university training, the problem-solving approach is new to all but a minority.

In England the new General Certificate of Secondary Education for the most able 60% of the population at 16+ contains optional coursework which becomes compulsory in 1991. At present a minority of schools are taking the coursework option, but the need to cope with it will grow as the deadline approaches. The 16-19 curriculum, currently being developed by the School Mathematics Project, introduces a pragmatic balance of classroom teaching and individual student work using computer technology where appropriate.

In Holland a new 16-19 mathematics curriculum was developed in the years 1981-5 for students in the life and social sciences [Lange Jzn J. de, 1987]. This course advocates a *realistic* approach to mathematics by exploring real world problems intuitively before mathematizing them, in sharp contrast to the structuralist approach that dominated the "new mathematics".

There are suggestions that a greater element of problem-solving through *mathematical modelling* should be introduced into 16-19 mathematics:

> We surely want our students to be able to put their mathematical skill into practice, and it is only through active problem-solving that they will be able to do this - the prblems can be real or purely mathematical - what unites them is that they give students the chance to:
>
> Apply their mathematical skills
> Work in groups
> Show creativity
> imagination
> innovation
> critical judgement
> Motivate further mathematical study (Burghes 1987)

It will be interesting to see how the introduction of problem-solving, project work and mathematical investigations will be viewed in ten years time from a *realistic* point of view, just as the benefit of hindsight allows us to reassess the "New Mathematics". It is an exciting road to follow; the vital question is to what extent the increased ability to cope with processes will more than compensate for a possible decrease in knowledge of specifics. The correct balance between teaching mathematical content and encouraging the development of mathematical processes may not prove easy to find.

4.4 New Kinds of Teaching and Learning

It is clear that the changing demands will require different approaches to teaching and learning. Early hopes for computer environments were that children would learn "powerful ideas" if they were left to explore powerful software in their own way. The evidence suggests otherwise:

> Just as extremely open-ended learning environments rarely lead students to construct concepts that mathematicians and scientists needed centuries to devise, purely teacher-directed or computer-directed learning environments seldom convey those concepts with real understanding. (E.T.C. 1988, p. 8)

A balance seems to be needed, with the teacher offering suitable inspiration and guidance and the students having sufficient space to be able to develop their own conceptions.

The School Mathematics 16-19 curriculum is designed around the following pattern for each topic:

In this way it is hoped to be able to accommodate a broad range of abilities, providing extension materials suitable for the more able. This imaginative scheme works in practice, it will soon be tested in reality.

4.5 New Kinds of Assessment

It is clear that the emphasis on higher problem-solving goals will require radically different kinds of assessment from the traditional restricted-time written tasks. The assessment tests in the new Dutch curriculum were founded on five basic principles:

> - to improve learning
> - to demonstrate what students *know* rather than what they do not know
> - to *operationalize* the goals of the course (particulaly the higher order goals)
> - to accept competent judgement of scoring rather than objectivity
> - only to use tests that can readily be carried out in school practice.
>
> (paraphrased from de Lange Jzn 1987 pp.179-181)

The last principle meant that project-like tests were rejected as requiring too much investment in time. Assessment methods considered were traditional restricted-time written tasks, oral tasks, take-home tasks, two-stage tasks and essays. The innovative idea is the two-stage task: the first stage is a restricted-time written test of a more 'open' kind, which is marked and returned with comments for the student to take away and improve in their own time to obtain the second stage mark.

Reviewing the different methods it was concluded that:

> The best possibilities are offered by a combination of restricted-time written tasks, take-home tasks (or combined in one two-stage task) and oral tasks which clearly measure different aspects of learning.
>
> (de Lange Jzn 1987, page 265]

As an interesting side effect of the new course, it was noted that:

> It is an interesting outcome of the study that girls perform rather poorly on restricted-time written tasks when compared to boys. This difference disappears almost completely in the case of alternative tasks that were part of this study. [ibid.]

4.6 The Psychology of Mathematics Education

In the 1960s many mathematicians believed that they could clarify mathematical ideas for the wider population in compulsory education by introducing a structural approach to mathematics, already well-known at university level. It was hoped that by making the definitions clear and the deductions logical that a greater understanding would follow. Yet, despite the fact that many professional mathematicians had high hopes for the “New Mathematics”, the movement had only limited success, because a *logically* sound approach that satisfies the expert may prove to be *pedagogically* flawed for the learner.

At the International Conference of Mathematical Education in Karlruhe in 1976 a movement grew to form a working group to study the psychology of mathematics education and every year since that time regular conferences have been held under the auspices of the International Group for P.M.E. A recently formed subgroup at P.M.E. now addresses itself to the problems of "advanced mathematical thinking", which looks at mathematics from the 16+ age range through to university and research level.[1]

A major activity is the investigation of student's misconceptions and how these are often related quite naturally to their previous experience.

> Going to a higher level of knowledge usually consists in passing an "epistemological obstacle", that is an obstacle constituting the knowledge to be acquired. This is done by de-stabilizing an incomplete and insufficient knowledge. Such knowledge, up to now sufficient within a given range of problems, becomes insufficient. The pupil has to solve a problem where the new knowledge is a needed tool.
>
> (Cornu 1987, page 79)

It becomes apparent that student errors are not arbitrary:

> We know that the pupil's errors are in most cases the logical consequences of their knowledge structuring. Thus we can use them as symptoms, allowing us to find out, at least partially, the pupil's knowledge, and to diagnose the wrong (incomplete or false) knowledge. (ibid)

The knowledge structure is, in turn, partly the product of *our* structuring of the curriculum. By presenting students with simple ideas first and giving them plenty of experience in a limited context may unwittingly colour their conceptions in a way that causes cognitive obstacles later. Thus our desire to make things simple at the outset is causing some of the very problems we observe later. A possible way ahead is to provide more complex, but controlled, computer environments in which the student can gain a richer conception, more able to stand up to later developments. Hence the need for a balance between the guidance of the teacher and the exploration and problem-solving of the student.

In this way the new revolution in mathematics education may benefit from the cooperation of mathematicians and the new breed of mathematics educators with their interest in the cognitive side of learning mathematics.

[1]See the article by Gontran Ervynck in this collection of papers.

4.7 Changing Perceptions

Computer technology will change our views on what is difficult and what is easy:

> Things that would be over the heads of the kids and would be saved for a college course in many cases can be done, very simply sometimes, with a computer .- Teacher (E.T.C. 1988 page 10)

Teachers will therefore need to attack new curriculum materials with an open mind, for their intuitions of what students may achieve, based on a wealth of pre-computer experience, may not always prove to be correct. Research into student difficulties shows that the traditionally accepted pattern of difficulty in the concept of variable is changed when a computer is used and that complex ideas of the calculus, such as non-differentiability can be addressed far earlier in the curriculum than was previously considered possible (Tall & Thomas 1988). Thus research into cognitive difficulties carried out pre-computer will need to be replicated in a computer environment to remain of value.

5. Reflections

From several different directions the signs are developing in our human culture: towards a new marriage of human ingenuity and computing power, towards the development of problem-solving skills whilst still retaining the knowledge of specifics appropriate for the new age of technology. There is much to be done, in a positive spirit of enterprise to develop theory and practice and a sober tone of realism to judge what may be done in reality.

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