The Value of the Computer in Learning Algebra Concepts

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The Background

The problems and difficulties which many secondary schoolchildren have with algebra (generalised arithmetic) are well known and have been the subject of much research investigation. Many of these relate to the conventions of the notation and the inability of children to interpret the meaning of the use of letters (Booth 1983a). Faced with a new and daunting cognitive situation, many fall back on their previous experience and make use of a one-to-one correspondence between the natural numbers and the letters of the alphabet (eg Wagner 1977), feeling a strong need for a numerical 'answer'. Booth (1983b) reported encouraging success using an imaginary 'Maths Machine' which the children had to 'program' to produce answers. The value of computer programming in understanding algebra has already been shown (see, for example, Tall 1983) and the natural extension of Booth's work was to provide the children with actual 'maths machines' to program.

The psychological framework of the research is based on constructivist Piagetian theory, with its idea of abstraction from experience, Ausubel's theory of meaningful learning, and the relational understanding of Skemp. All these theories emphasise the importance of the 'framework of knowledge' which the individual constructs in any cognitive area, and the need to build on the existing knowledge structures of the child by conceptual rather than rote means.

The experiment

a) equipment

To enable the children to construct a mental model for a variable in algebra, and the manner in which it is manipulated, a concrete model was provided consisting of a 'box' containing the current numerical value of the variable and an attached label with the variable name (figure 1). Although this model does not fulfil all the mathematical uses of the concept of variable (see e.g. Wagner 1981), it proved to be of great value to the children.

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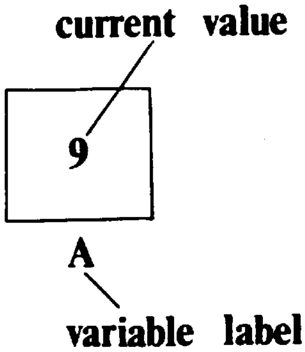

Figure 1

Two ‘Maths Machines’ were devised. The first was a cardboard model consisting of two large sheets of card (figure 2), one of which was blank (the ‘screen’) and the other with six rectangular boxes to store the variables. To carry out commands placed on the screen, the children performed individual casks such as carrying messages, looking after variable labels, inserting values on cards into the stores and performing the arithmetic calculations. (See Thomas 1985 for further details.)

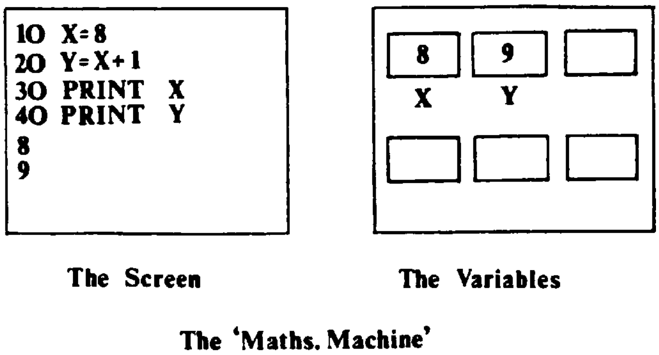

Figure 2

A second. problem-solving tool, designed specifically for the programme was a software ‘Maths Machine’ for use on the BBC computer. This program, which allows normal algebraic input (with implicit multiplication), was also based on the fundamental mental model of a variable discussed above. The screen consists of a series of ‘boxes’, initially empty. Some are labelled with variable names and contain the current value of the variable under consideration, others are for algebraic expressions which can be calculated and compared (figure 3).

The two ‘Maths Machines’ were designed to enable the children to develop their understanding of the general concepts of algebra, through structured exploration of practical examples. Each is a ‘generic organiser’ in the sense of Tall (1985). Through practical experience, specific examples are seen to be generic examples (representatives of a class of examples) from which the general concept may be abstracted. Each turned out to be extremely popular and successful.

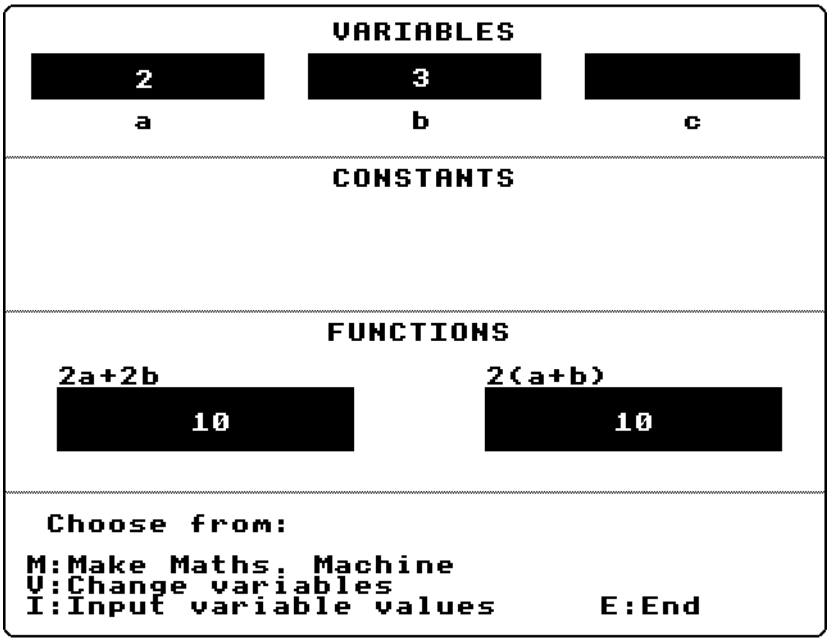

Figure 3

b) The Experimental Method

The subjects of the main experiment were a group of 42 mixed ability 12 year olds from the top year of a middle school, with no previous experience of algebra. They were divided into matched pairs using the results of an algebra pre-test based on the Concepts in Secondary Maths and Science (CSMS) algebra test. The teaching programme given to the experimental group consisted of about twelve hours of work replacing their normal mathematics periods. The module of work consisted of a variety of activities, using the equipment described above. The children were divided into groups of about three and were rotated each session between the computers available (three) and the 'Maths Machines'. The use of small groups was found to have beneficial effects. Peer group interaction, in helping and correcting each other, certainly seemed a valuable means of intelligent learning (Skemp 1985). The pupils started with an introduction to simple programming in BASIC and this was built into some investigations using short programs. An example of the sort of thing looked at would be a comparison of the outputs of these programs for three different values of each of *a* and *b*:

```
10 INPUT a              10 INPUT a
20 INPUT b              10 INPUT b
30 c=2*(a+b)            30 c=2*a+2*b
40 PRINT c              40 PRINT c
50 GOTO 10              50 GOTO 10
```

In this way concepts such as commutativity, the use of brackets and equivalence of expressions were all investigated unobtrusively and linked to practical experience through everyday problems.

The final part of the programme of activity involved the use of the software ‘Maths Machine’ to find the ‘solution’ to relatively difficult inequalities such as

For what value or values of x is $2x+1>5$?

This was achieved by inputting the formula $2x+1$ as a function and then choosing values of x to input. The ‘Machine’ displayed the value of the function for this value of x and so values giving a result greater than 5 could be recorded. It was not expected that many would obtain a result such as $x>2$ from their lists, although some did.

The set of five worksheets used in the programme will be made available at PME10.

A test based on the CSMS Algebra test, but different from the pre-test, was given as both post-test and delayed post-test ten weeks later.

The Results

The main question under test was whether or not the teaching programme had improved the children’s understanding of the use of letters in algebra, with particular reference to their use as generalised numbers and variables. The results (Table 1) showed that both the post-test and delayed post-test results of the experimental group were significantly better than those of the control group.

Test	Experimental Mean	Control Mean	Mean Diff.	S.D.	*N*	*t*	*df*	*p*
Post Test	32.55	19.98	12.57	10.61	21	5.30	20	<0.0005
Delayed Post-Test	34.70	25.73	8.47	11.81	20	3.13	19	<0.05

Table 1

The questions involved an understanding of all four of the levels of difficulty identified by Kuchemann (1981). The experimental group were significantly better than the control group on questions requiring an understanding of the use of letters as a specific unknown and as a generalised number or variable (Kuchemann’s levels 3 and 4). It was also encouraging to see that in some areas, where comparison was possible, the experimental group results were comparable with or better than those of children up to three years older on the published CSMS results. There were also many very encouraging examples of great individual improvements in understanding of the use of letters in algebra.

The children enthused over all the work, and were still talking about it a year later. The teacher who taught it was equally enthusiastic commenting that it “was a very worthwhile project which proved to be very pupil orientated. It is enjoyable, interesting and thought/discussion provoking between pupils and between pupils and the teacher."

It was concluded that the programme had been successful in its aim and that work of this sort using the computer and presented to secondary school children before they do any formal algebra could have wide ranging benefits.

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